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## DUŠAN TŘÍSKA

## SOCIAL CHOICE <br> AND BEHAVIOR; the nature of their design and management

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## ABSTRACT

The book is yet another attempt to overcome gaps between various branches of societal research, namely those between economic theory and legal scholarship. Its methodological thrust has been compressed into three postulates, where:
Thesis $A$ advices analysts to conceptualize social phenomena as outcomes of a relationship between Designers and their Designees.

Thesis $B$ claims that Designers do not design behavior of Designees but only their tasks that, then, may or may not be fulfilled by the Designees' factual behavior.

Thesis $C$ seeks to convince the reader that a Designee - as an agent - involves a triad of adjacent agents, namely a Beneficiary, Defendant and Manager.

The three theses are presented, somewhat paradoxically, as fundamental contributions of the intellectually inferior legal scholarship to the scientifically superior economic thought.

As regards practical impacts, the book would like to inspire designers of realworld processes including their support by information technology.

## Key words

In addition to the terms in the book's title: a contract - its validity and effectiveness, collective choice, uncertainty, delict, transfer of wealth, regulation. JEL classification
A11, A12, A23
C70
D01
D81, D86
K00

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## PREFACE

## Interdisciplinary bridges

The title of this BOOK is as immodest as its genuine objective. Similarly outrageous is its analytical method that can be briefly characterized as yet another attempt to bridge the infamous gaps between various branches of societal research, namely those between

> economic theory ("ET") and legal scholarship ("LS")

Still more astonishing could then appear the fact that it will be LS that will be taken as a source of the fundamental enrichment to the analysis - in spite of the author's respect to the intellectual superiority of ET and his prevailing view that LS, unlike ET, essentially lacks a genuine scientific underpinning.

## Organization of the argument

This BOOK's methodological thrust has been compressed into no more than three Theses $A, B$ and $C$ presented to the reader on no more than one modest chapter of the following introductory PART I.

The book-size PART II of the BOOK may then be taken as "only" a COMMENTARY to the article-size PART I. It is divided into 11 Comments within which the Theses $A, B$ and $C$ are imposed upon social contexts as diverse as, e.g., shoe-making, compulsory vehicle insurance and a wealth re-allocation. Every context is selected with the aim to disclose phenomena that need not be visible elsewhere. In other words, the 11 Comments seek to show that essentially every problem is detectable with a different level of an "analytical ease and comfort" depending on where and when it is being analyzed. It will be one of our key points to stress that scholars disregard certain phenomena only because - within specific social contexts - the phenomena are essentially invisible, as if nonexistent.

## Theory vs. practice

Another daring ambition of the author rests in his attempt to address the gap between the unquestionably admirable super-towers of Academia and the real world of an ordinary analytical labor of applied branches of research, not
only LS but also BPM (business process management, operations research and work-flow analysis). It is fully respected that, unlike ET, the applied analyses do not enjoy the luxury of virtual laboratories and thought experiments where all kinds of abstractions are allowed every time the unpleasant complexity of the real world may emerge.

The author's experience here can be traced to his attempts to translate constructions of ET and LS into a language that could be processed by IT-practitioners, namely software programmers or code writers. At the same time, this kind of an intellectual adventure made him appreciate the unique capacity of BPM to deal with alpha-numeric-vector strings, i.e. the kind of variables by which real-world phenomena are almost exclusively described and before which a genuine science would rather close eyes.

## Text-books

The gravest complication with the author's attempt to introduce his Theses $A, B$ and $C$ to a broader academic community will certainly stem from his failure to supply convincingly relevant references to the existing literature on the subject. To illustrate, the author simply has not found an economist who would, in the sense of his Thesis $B$, consistently differentiate a factual action from what is "only" a task (obligation) to act. Similarly, to his knowledge there has been no serious attempt among legal scholars to establish a universally acceptable structure of a task's development, namely should it take the form of a finite set of discrete stages of a task (obligation).

As a rather hopeless attempt to moderate this extremely unpleasant lack of relevant bibliography, the author frequently confronts his analytical proposals with what he sees as text-book concepts. In most such cases he refers, albeit impliedly, to the didactical classics such as:

Varian, Hal R.: Intermediate Microeconomics: A Modern Approach, Eighth Edition, W. W. Norton \& Company (2009), ISBN-10: 0393934241, ISBN-13: 978-0393934243.

Varian, Hal R.: Microeconomic Analysis, Third Edition, W. W. Norton \& Company (1992), ISBN-10: 0393957357, ISBN-13: 9780393957358.
Gravelle, H.; Rees, R.: Microeconomics, $3^{\text {rd }}$ Edition, Prentice Hall (2004), ISBN-10: 0582404878, ISBN-13: 9780582404878.

As to the LS text-books, the following selection from his library will be by far more arbitrary:

Black, K., Skipper H. D.: Life Insurance, Prentice Hall (1994), ISBN-10: 0135329957, ISBN-13: 9780135329955.

Klayman, E. I.; Bagby, J. W.; Ellis, N. S.: Business Law: Concepts, analysis, perspectives, Richard D. Irwin (1993), ISBN-10: 0256148899, ISBN-13: 9780256148893

White, J. J.; Summers, R. S.: White and Summers' Uniform Commercial Code, Edition 6, (Hornbook Series) West Academic Publishing (2010), ISBN13: 9781628103748

Moreover, the author must shamefully admit that - given the elemental level of the LS topics - he has by and large used Wikipedia as a rather rich and relatively reliable supply of legal definitions and concepts.

## Instead of standard bibliography

As said, the author has decided to fully resign to the usual firework of up-to date papers, articles and journals as were those in his previous monographs:

## K některým možnostem optimalizace smlouvy a závazkových vztahů. (Towards Optimization of Contracts and Obligation Relationships), Oeconomica (2005), ISBN 80-245-0916-4. <br> Ekonomická analýza smluv, systémů a procesů. (Economic Analysis of Contracts, Systems and Processes), Oeconomica (2009), ISBN 9788024515755.

Yet, the author will acknowledge a number of names that have influenced his life-long scientific interests and preferences. The list would certainly include Claude-Frédéric Bastiat, James Buchanan, Quido Calabresi, Ronald Coase, Kurt Gödel and Douglas Hofstadter, Friedrich von Hayek, Oliver Hart, Leonid Hurwitz, Harry Markowitz, Eric Maskin, John von Neumann, Richard Posner, Thomas Shelling, Oliver Williamson, ... Their contributions to this BOOK however indirect they may appear - would include such topics as (more or less in the order of the above great names):

- invisibility of economic phenomena,
- "what should economists do?",
- damage and liability,
- contract vs. hierarchy,
- infinite recursion,
- counter-revolution of science,
- incompleteness,
- portfolio optimization,
- social goals,
- expected utility,
- "what do judges maximize?",
- intra- vs. inter-temporal choice,
- boundary of the firm.

Even non-economic readers may have heard about some of the topics due to the Nobel Prizes awarded to many of the above scholars. Hence, detailed expositions of their work can be easily found elsewhere, including the Nobel Foundation's web pages. However, to provide a serious interpretation of the scholars' impact upon this BOOK would most probably exhaust a separate book for every single topic and author.

## Further apologies

The BOOK - namely towards the later Comments - is rather technical in the sense that some of its sections use formalism to which students from outside economics may not be accustomed. Hence, the prospective readership should be relatively fluent in the language of microeconomics, on its intermediary level at least. As a result, the author's attempts may crash into the wall of the ever present vicious circle that the formalism without which the looked-for methodological bridge cannot be built is exactly the major cause of the gap that is to be bridged.

The concluding apology will admit that theBOOKwasoriginally conceptualized as the author's testament into which he would simply ingather production of his $40+$ years in science, economic policy and IT-business. However, as is often the case, the original plan got a little out of control and the resultant product has turned out to be a brand new monograph based, among others, on a broadly innovated terminology and notation. The technical character of the text thus makes it entirely unrealistic to expect that the author could have avoided serious defects and inconsequentialities. Admittedly, every time he may dare open his BOOK he will uncover something that he would express today "more efficiently" or should not have written at all.

Hence, what the author must rely on is a highly improbable indulgence and good will of the prospective readers - if ever there may emerge this kind of a social group.

Dušan TŘÍSKA, Prague, September 2017

## PART I.

## DESIGNERS: THEIR DESIGNEES AND NOMINEES

## 1. INTRODUCTION

### 1.1 The analytical benchmark

As said, the author will frequently confront his analytical proposals with what ET usually accepts as its didactic standards. To illustrate this referential wisdom, it may be of value to briefly sketch how social choice and behavior is conceptualized in the elemental economics of the Firm.

Basic or intermediary text-books of microeconomics establish the Firm by its Owner's choice represented as a solution to a maximization problem:

```
\(\boldsymbol{\operatorname { m a x }} \psi(K, L, Q)\)
s.t.: \(\max 1\)
\(Q \leq a_{f} f^{0}(K, L)\)
\(K, L \geq 0\)
```

where:
$\psi(K, L, Q) \quad$ is the Firm's profit $\psi=\left(p_{Q} \cdot Q-p_{K} \cdot K-p_{L} \cdot L\right)$ assumed to be the (direct) utility function of the Firm's Owner,
$K, L, Q \quad$ are endogenous (choice) variables - capital, labor and output, respectively, whose combination ( $K, L, Q$ ) will be called a situation of the Firm's Owner,
$\vec{p}=\left(p_{K}, p_{L} p_{Q}\right) \quad$ are prices of $K, L$ and $Q$,
$Q^{\max }=a_{f} \cdot f^{0}(K, L)$
is a production function that associates to a given combination of inputs $K$ and $L$ a maximally attainable magnitude $Q^{\text {max }}$ of the Firm's output $Q$.

Coefficient $a_{f}$ will be - for simplicity - the only indicator of the Firm's technological efficiency. Prices $p_{K}, p_{L} p_{Q}$ and efficiency $a_{f}$ will be taken as exogenous (environmental) variables by which the Firm's environment is constituted. We will also say that, a combination ( $p_{K}, p_{L} p_{Q} ; a_{f}$ ) constitutes a state of the Firm, whereas the endogenous (choice) variables ( $K, L, Q$ ) represent the above established situation.

The structure of max 1 can be also interpreted so that the societal driving forces of production are constituted by the Firm's Owner's:
sphere of interests or a set of feasible (variant, attainable, affordable) situations ( $K, L, Q$ ),
preferences or value judgments according to which the variants can be evaluated by the Firm's Owner.

In sum, if the Firm is defined by max 1 the only thing that its hypothetical Owner prefers is a profit and the only constraint to this objectives rests in the Firm's technological capacity. In other words, the Owner will select as his-her optimal situation the combination ( $K^{*}, L^{*}, Q^{*}$ ) in which he-she will maximize the Firm's profit.

By a so-called demand-supply function

$$
\left(K^{*}, L^{*}, Q^{*}\right)=\boldsymbol{d} \boldsymbol{s}\left(p_{K}, p_{L}, p_{Q^{\prime}}, a_{f}\right)
$$

is then represented how the optimum depends on the developments in the Firm's environment - how the Firm's optimal situation will be affected by its actual state.

### 1.2 Demand-supply vs. legal norm and business rule

### 1.2.1 IF-THEN representation

As the first step towards our methodological proposals, we will re-write purely formally - the above exercises in microeconomics as follows:

A production function will be put as a technological IF-THEN rule:

```
IF: (K,L),
THEN: Q Q imax can be produced "at most".
IT 1
```

Similarly, the societal driving forces of production will be represented by a behavioral IF-THEN rule:

| IF: | $\max 1$, |
| :--- | :--- |
| THEN: | $\left(K^{*}, L^{*}, Q^{*}\right)=\boldsymbol{d s}\left(p_{K^{\prime}} p_{L}, p_{Q^{\prime}} a_{f}\right)$. |
| IT $2 a$ |  |

Graphically, the two rules are represented by the input-output schemes in (a) and (b) of Fig. 1, respectively. The element denoted JOIN is added so far only with the aim to prepare the reader for the formalism by which production is represented in BPM. Later we will show that by logic gates such JOIN and SPLIT will be constituted particular "organizational patterns" of the Firm's production process.


### 1.2.2 Domain and range

Our preparatory work will continue so that we will rewrite the demand--supply function $\left(K^{*}, L^{*}, Q^{*}\right)=\boldsymbol{d s}\left(p_{K}, p_{L} p_{Q}, a_{f}\right)$ into a mapping from the domain $\left\langle p_{K}, p_{L}, p_{Q^{\prime}} a_{f}\right\rangle$ to the range of respective optima $\left\langle K^{*}, L^{*}, Q^{*}\right\rangle$ as follows:

$$
\text { [ds: } \left.\left\langle p_{K^{\prime}} p_{L^{\prime}} p_{Q^{\prime}} a_{f}\right\rangle \rightarrow\left\langle K^{*}, L^{*}, Q^{*}\right\rangle\right] \quad \text { ds } 1
$$

Its graphical representation is in Fig. 2:


Trivially, then, the picture can be read as:

| IF: | $\left(p_{K}, p_{L}, p_{Q}, a_{f}\right)$, | IT $\mathbf{2 b}$ |
| :--- | :--- | :--- |
| THEN: | $\left(K^{*}, L^{*}, Q^{*}\right)$. |  |

Formula $d s \mathbf{1}$ is firstly more correct - mathematically, so to speak - than the text-book demand/supply function $\left(K^{*}, L^{*}, Q^{*}\right)=\boldsymbol{d s}\left(p_{K^{\prime}} p_{L} p_{Q^{\prime}} a_{f}\right.$ ). However, at this stage of the argument our emphasis will be only that the notion of a domain demonstrates explicitly, that the mapping $d s \mathbf{1}$ is defined (exists) for only such combinations of prices and efficiency that fall into the set $\left\langle\vec{p}, a_{f}\right\rangle$.
The case when the present Firm "stays still" because its future behavior is not defined will be later confronted with other circumstances under which an agent "does not move" from his-her actual situation.

### 1.2.3 Conditions and tasks

In ordinary language the behavioral rule $\boldsymbol{I T} 2 \boldsymbol{b}$ can be read as follows:

```
IF: an agent "falls" into a particular state,
THEN: his-her behavior will be prescribed correspondingly.
```

This - so far only intuitive - terminology is another step towards our first methodological bridge between the canonized concept of a demand-supply and the following seemingly different concepts of LS and BPM where:

- the notion of a legal norm is in LS established in the general structure of a hypothesis and disposition (the third part being a sanction),
- the term business rule is applied in BPM in the basically identical sense.

The structure of a legal norm (business rule) can be read so that IF a given state of the world occurs (hypothesis), THEN somebody can be made obliged to do something (disposition). In plain terms, the IF-component can be seen as a condition (legal or business) and the THEN-component can be called an obligation, duty, plan, objective, ... or a task as we will refer to it in this BOOK.

Let us stress that for the moment all that we wanted to uncover was the unexpected formal affinity between the concepts of a demand-supply and legal norm (business rule).

### 1.3 Natural vs. artificial language

### 1.3.1 IT-parlance

The bridge-building ambitions necessarily bring forward terminological problems. With respect to the above discussion we should launch a systematic search for a term that would comprise phenomena as - seemingly - diverse a "demand-supply" and "legal norm".

Before we set for this linguistic mission, the author should admit that he belongs to scholars who regard a so-called natural language as essentially useless for the sake of a serious analysis, including that of social choice and behavior. Among many other deficiencies, it simply does not contain large enough supply of words so as to differentiate social contexts in the manner that would be both consistent and nice.

As demonstrated, already for notions as "ordinary" as "interests and preferences" we have rather used a highly artificial language of microeconomics. Apart from this, we will introduce terminology whose roots mostly lie in the author's - highly adventurous - attempts to translate legal documents into a structuralized language applicable by IT-technology.
Admittedly, then, the further presented IT-parlance will include terms and phrases that can be easily disgraced as rather dogmatic, inappropriate or simply ugly. The following two examples should illustrate the scope and scale of the problem.

### 1.3.2 Example 1: A person in different roles

IT-parlance should primarily take hold of the common wisdom of namely LS, that a given person often performs a number of different roles so as to behave as different agents. Hence, in our IT-parlance and agent will be identified:

- firstly by name so as to identify him-her as a particular person and
- only then by the person's corresponding role.

To illustrate the resultant childlike language of the BOOK, a woman named Mary will be differentiated as

Mary-the Producer, Mary-the Driver, Mary-the Policewoman etc.
Ugly and grammatically incorrect as this language may be, one of its objectives is to demonstrate that the "same" Mary will be allowed to inter-act with - as if - herself.

For dramatic effect, we may let Mary-the Policewoman stop and arrest Mary-the Driver for drunken driving. Less anecdotic will then be our much deeper analysis of a present Mary who will transfer some of her wealth to herself in the role of a future Mary.

### 1.3.3 Example 2: An order as a carrier of will

Another prominent representative of our IT-parlance is the term order by which we will understand what would be elsewhere

> a vote, claim, proposal, offer, acceptance, motion, objection, (third party) action, (plaintiff's) complaint

Hence the term order will hereinafter stand for any articulation (expression, display) of an agent's will. We will say that an agent communicates by way of submitting or even exchanging orders. Various prefixes will then be applied so as to differentiate, e.g.

SELL-order, BUY-order, BEN-order, DEF-order, MAN-order, ...
Prefixes SELL and BUY clearly refer to the communication on a stock-exchange and hence also demonstrate that some of the routines of this kind of a market--organizer have extensively inspired content and terminology of this BOOK.

Due to this prominent position of the term order, we will attempt to avoid usages such as "an order of performances", "a warning is in order", "law and order", "in order to" etc.

### 1.3.4 The evil of professional jargons

To conclude this linguistic section, we may add that it is a commonplace for LS and BPM - and not only for them - to attach a new term to a whatever phenomenon whenever it appears to be "somehow different". The opposite and highly recommendable approach can be well illustrated by the remarkable achievements of ET once it managed to accept the notion of a price as a common denominator for phenomena so seemingly diverse as, e.g., interest rate, wage, rent and premium.

However, one thing is the academic search for common denominators and entirely different thing is the self-interest of a professional community. We will certainly not be the first who will complain that also LS and BPM often use their jargon only to hide the genuine limits to their know-how.

### 1.4 Associated notes

Returning to the artificial language of standard micro-economics, the following two notes will concern (a) representation of the Firm and (b) one of the many essential questions to be raised.

### 1.4.1 Profit function

For the sake of further analysis, IT $2 \boldsymbol{b}$ will be sometime reformulated with the help of a formula

$$
\psi^{*}=\left(p_{Q} \cdot Q^{*}-\left(p_{K} \cdot K^{*}+p_{L} \cdot L^{*}\right)\right) \equiv \vartheta\left(p_{K^{\prime}} p_{L^{\prime}} p_{Q^{\prime}} a_{f}\right)
$$

where $\psi^{*}$ is an optimal profit obtained by substituting optimal values ( $K^{*}, L^{*}, Q^{*}$ ) into the formula for an ordinary (direct) profit.
As a result, ds $\mathbf{1}$ can be replaced with a so called profit function:

$$
\left[\vartheta:\left\langle p_{K}, p_{L}, p_{Q^{\prime}}, a_{f}\right\rangle \rightarrow\left\langle\psi^{*}\right\rangle\right] \quad \text { pf } 1
$$

or the rule:

$$
\begin{array}{lll}
\text { IF: } & \left(p_{K}, p_{L}, p_{Q^{\prime}}, a_{f}\right), & \text { IT } 2 c \\
\text { THEN: } & \text { the optimal profit will be } \psi^{*} . &
\end{array}
$$

In words, the two monetary representations show how the optimal profit $\psi^{*}$ of the Firm's Owner will respond to the changes in his-her environment.

### 1.4.2 Integrability problem

Returning to the rule IT $2 a$ it can be somewhat generalized so that

$$
\begin{array}{ll}
\text { IF: } & \text { an agent's interests and preferences are given, } \\
\text { THEN: } & \text { we may calculate out the respective behavior. }
\end{array}
$$

Then, it may be of value to remind the reader of the inverse text-book question:

> IF: $\quad$ an agent's behavior (action, performance, ...) is given, THEN: $\quad$ what are his-her interests and preferences?

The inverse relationship is mostly discussed as a so-called integrability problem or the classic question "What does the agent maximize?".

## 2. MAJOR THESES

The following Theses $A, B$ and $C$ could be as well characterized as Three Contributions of Legal Scholarship to Economic Theory.

### 2.1 Thesis $A$ : Designer vs. Designee

So far we uncovered the formal affinity between the concept of a demand-supply and the notion of a legal norm (business rule). Now we will claim that they all are products of a concrete, real-world agent - a so-called Designer.

### 2.1.1 Example

For the sake of illustration, let us consider two natural persons John and Mary whose roles will be - purely formally - characterized as

> John-the Designer and Mary-the Designee

Let the empirical meaning of the two roles be - highly intuitively - specified as follows:

John as a father of two children "feels obliged" to cover costs of their university studies and hence, "under this social pressure", considers variant strategies how to fulfill his task,
Mary is an Owner of a shoe-making Firm who is selected by John as his best (optimal) strategy how to procure the necessary finance.

In other words, John - as a Designer - believes that he is "entitled" to design conditions under which Mary will have to - in his favor - launch her profitmaking activity.

### 2.1.2 The thesis

We will argue that the relationship between a Designer and his-her Designee (Designees) constitutes the common denominator of the variety of social contexts dealt with in this BOOK. Thesis $A$ in fact primarily states that the pair of agents

Designer and Designee
is the core of any societal phenomenon.

Should we smooth down the relatively harsh statement, we could say that the BOOK will confine to phenomena that can be derived from the above relationship between Designers and their Designees.

In the methodological sense we could also understand Thesis $A$ as a proposal to approach any societal phenomenon so that its Designer and Designee will be uncovered. Exactly this we will be doing throughout the BOOK so as to substantiate the thesis in a variety of social contexts.

### 2.1.3 Operational unit

Invoking the above example and in the analogy with the already discussed demand-supply function $d s$ 1, we will further claim that Mary, as any other Designee indeed, will be always designed in a universal form of a behavioral IF-THEN rule, namely the mapping:

$$
\left.[m:\langle\vec{m}\rangle) \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]
$$

where:

- by the IF-component $\vec{m}=\left(m_{1}, m_{2}, \ldots\right)$ are designed conditions under which
- the THEN-component $\vec{T}_{m}$ can be prescribed as Mary's task.

For thus established IF-THEN rule we will coin the term operational unit or, for short, OP-UN. Doubtless, the terms "Designee" and "OP-UN" will be then taken as synonyms.

### 2.1.4 Designer's choice

As said, John-the Designer seeks to select his best strategy how to fulfill his task. In what follows the strategy will be denoted as $s t r^{*}$ where the asterisk is the text-book mark of an optimum. By definition, then, $s t r^{*}$ is a solution to the respective maximization problem:

```
\(\max U(s t r) \quad M A X_{J}\)
s.t.: \(\operatorname{str} \in\left[\operatorname{str}(0)=\left\{s t r_{1}, s t r_{2}, \ldots, s t r_{N}\right\}\right]\)
```

where:
$\operatorname{str}(0) \quad$ is John's sphere of interests constituted by $N$ variant strategies $\left\{s t r_{1}, s t r_{2}, \ldots, s t r_{N}\right\}$ how to finance his task,
$U(s t r) \quad$ is John's utility function that represents his preferences over the set of variants.

Hence the objective of this BOOK can be also characterized as a search for an operational form of $M A X_{J}$ and $s t r^{*}$ - under this or that social arrangement.

### 2.1.5 Associated notes

1) The above example about John and Mary-the Shoe-maker illustrates the case when John's optimal strategy str* consists in only one Designee - only one operational unit. Later, various patterns of multi-unit strategies will be corroborated in depth.
2) For the moment we leave aside not only multi-unit strategies but also the social setting under which a strategy str* is designed collectively by an $n$-member organization.
3) In theory, John may design anybody into the role of a Designee. Yet, any such design can be efficient if only John is "somehow" superior to the agent under his design. As said, John "must" be "somehow" entitled to "impose tasks" upon his Designees.
4) Both components (IF and THEN) of an operational unit may consist in whatever John-the Designer may believe to enforce. To illustrate, the conditions designed by an IF-component may involve - apart from prices and technological efficiency - also diagnoses of injuries, trends in a climate change, human rights abuses, infidelity of women, juvenile criminality, ... Apparently, all such events may be designed as (deliberate, negligent or unintentional) outcomes of particular behavior of particular agents.

### 2.2 Thesis B: Task vs. behavior

Thesis $B$ will now address expressly what Thesis $A$ contained only impliedly, namely that a Designer never designs factual behavior of a Designee but only his-her task. LS will then immediately remind us that a task may enter different stages of its development and that its content in every stage is likely to be of a complex structure.

### 2.2.1 Task vs. operational unit

We shall begin with one more confusion that may result from a reckless usage of the natural language. Our warning will stress that the very notion of a task can have a "non-empty" empirical meaning only in association with conditions under which the task can be prescribed.

Put still more accurately, what can make sense is only the above established operational unit as a whole. However, with the aim not to lose contact with LS-parlance, also we will often speak about "a task's stage and content" as if it represented the operational unit as a whole.

Given this license of speech, we will only remind the reader - from time to time - that a task is no more than a THEN-component of an OP-UN and as such, likewise the IF-component, has - in itself - no reasonable interpretation.

### 2.2.2 Development

Our Thesis $B$ primarily states that a task's development (the development of the respective operational unit) consists in
a finite number of discrete and mutually exclusive stages
In this BOOK we will essentially confine to only three such stages denoted $M(1)$, $M(2), M(3)$ in the following table for Mary-the Designee.

| stage | characteristic |
| :---: | :---: |
| M(1) designed | $M(1)$ is simply an abbreviated notation $\left.M(1) \equiv[m:\langle\vec{m}\rangle) \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]$ |
| M(2) <br> prescribed | This stage is an outcome of a transition $M(1) \Rightarrow M(2)$. Depending on the particular state $\vec{m}=\left(m_{1}, m_{2}, \ldots\right)$, by $M(2)$ will be determined who will be to do what, where and when. <br> Put differently, $M(2)$ is a specific instance (realization) of the set of potential outcomes designed by $M(1)$. Formally, $M(2)$ is obtained by substituting values of $\vec{m}$ into the mapping $\left.[m:\langle\vec{m}\rangle) \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]$. <br> Whereas $M(1)$ has been read as an "IF-THEN" statement, the state $M(2)$ is a statement "BECAUSE-THEN". |
| M(3) completed | The stage is an outcome of a transition $M(2) \Rightarrow M(3)$. Once in the stage $M(3)$ Mary loses any possibility to affect whether and in what particular form the prescription $M(2)$ will be fulfilled. Two mutually exclusive instances of $M(3)$ will be: <br> $M(+3)$-fulfilled with the meaning that $M(3)$ is fully consistent with M(2), or that the task is duly completed, when, by contrast, $M(3)$ is not consistent with $M(2)$. |

In sum: $M(1)$ represents Mary in her designed stage, whereas $M(2)$ and $M(3)$ represent the same Mary but in "more mature" stages of her development.

### 2.2.3 Content

This BOOK is to a large extent also about the unmanageable complexity of the real-world. In the particular case of the mapping [m: $\langle\vec{m}\rangle) \rightarrow\left\langle\vec{T}_{m}\right\rangle$ ] it may be enough to consider the following example of the stage $M(2)$-prescribed:

BECAUSE: it rained in London for three days in a row and Mr. Strong (Mary's employee) is ill,
THEN: another employee of Mary, namely Miss Weak will have to take a car " $X$ " and deliver 15 pairs of shoes to the hands of Rock-the Receptionist of the British Parliament, on June $15^{\text {th }} 2035$, at 04.30 p.m.

In the BECAUSE-component of the prescription, "London" and "Mr. Strong" represent the particular form in which Mary's state has fallen into the domain $\langle\vec{m}\rangle$. The THEN-component of the example is to illustrate that Mary's task $\vec{T}_{m}$ has been designed as $\vec{T}_{m}=\left(K_{m}, L_{m}, \vec{Q}_{m}\right)$ where:

1) by $\left\langle L_{m}\right\rangle$ are designed potential Executors, e.g. Mary's employees,
2) the Executor may use only vehicles designed by $\left\langle K_{m}\right\rangle$,
3) the structure of the deliver is designed by $\vec{Q}_{m}=\left(Q_{a / m,} Q_{b / m \nu} Q_{c / m} Q_{d / m}\right)$, where: $Q_{a / m}$ is a specific kind of the delivery, e.g. shoes, $Q_{b / m}$ is a magnitude of the delivery, $Q_{c / m}$ is a place of the delivery, $Q_{d / m}$ is a time of the delivery.

### 2.2.4 Express vs. implied formulas

The above example clearly shows that operational units are in reality mostly designed by variables whose specific nature is often characterized as an alpha--numeric vector string.

The analytical unfriendliness of this kind of variables is more than obvious. Among others, they may allow for only primitive representations of the mapping $\left.M(1) \equiv[m:\langle\vec{m}\rangle) \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]$, e.g. that in the following table with the cumbersome enumeration of all the pairs of states and tasks that John decided to design as feasible.

| Observed state | Prescribed task |
| :---: | :---: |
| $\vec{m}^{1}$ | $\vec{T}_{\mathrm{m}}^{1}$ |
| $\vec{m}^{2}$ | $\vec{T}_{\mathrm{m}}^{2}$ |
| $\ldots$ | $\ldots$ |
| $\vec{m}^{\text {i }}$ | $\vec{T}_{\mathrm{m}}^{\mathrm{i}}$ |
| $\ldots$ | .. |
| Example: <br> - rain in London for three days at leas <br> - Mr. Strong is not available | Example: <br> - Miss Weak <br> - car "X" <br> - 15 pairs of shoes <br> - Rock-the Receptionist <br> - June $15^{\text {th }} 2035,04.30$ p.m |
| $\ldots$ | $\ldots$ |

Whenever John may attempt to replace this primitive enumeration with a more sophisticated representation, he will - most probably - pay dearly for this comfort by a loss of content. A simple example will be a linear function, through which John will determine that, e.g., the longer will last the rain the further will be postponed the time of delivery. The obvious benefit would be that the
function provides - impliedly - an infinite number of potential prescriptions. The loss rests in that John had to sacrifice, e.g., his ambition to regulate the relationship between a particular Executor and place of delivery.

The highest price is then paid by Designers who simplify their outcome into a constant mapping by which the same task is prescribed in any feasible state of Mary's environment.

### 2.2.5 Task vs. condition

Our first warning stressed that, contrary to what the natural language may suggest, a task is no more than a THEN-component of an OP-UN and as such has in fact no reasonable interpretation. Another note of this kind will be that, in ordinary speech, we may read the OP-UN so that: Should the task $\vec{T}_{m}$ be prescribed, the state of Mary's environment "must" be feasible. Similarly, one can say that
the state $\vec{m}$ "must" be from $\langle\vec{m}\rangle$
By quotation marks in "must" we stress that the word must represents here a condition rather than a task. In particular, highly confusing may then become arrangements under which the condition requires that somebody, e.g., Richard must fulfill his task. Here, we will have to carefully differentiate between cases when:

- Richard must fulfill his task because he has been so designed (by John or some other Designer),
- Richard "must" fulfill his task because Mary herself has been so designed (by John).

Needless to stress that legal documents in particular can be perfectly unclear about which of the two fundamentally different musts has been meant.

### 2.3 Thesis C: Designee vs. Nominee

Strongly inspired by elemental LS, again, our last Thesis $C$ will claim - somewhat mysteriously - that a Designee - as an agent - involves three agents, namely a Beneficiary, Defendant and Manager.

### 2.3.1 SP-representation of an operational unit

Put differently, Thesis $C$ states that John's design of Mary involves nominations of the above triad of Nominees.

We may use the mapping $M(1) \equiv\left[m:\langle\vec{m}\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]$ again and express Thesis $C$ so that Mary's environment $\langle\vec{m}\rangle$ will consist in three separate sets of conditions $\left\langle\vec{m}_{\text {ben }}\right\rangle,\left\langle\vec{m}_{\text {def }}\right\rangle$ and $\left\langle\vec{m}_{\text {man }}\right\rangle$ by which specific requirements will be imposed upon the three Nominees.

Hence, Mary's design will be put as

$$
M(1) \equiv\left[m:\left\langle\left\langle\vec{m}_{\mathrm{ben}}\right\rangle,\left\langle\vec{m}_{\mathrm{def}}\right\rangle,\left\langle\vec{m}_{\mathrm{man}}\right\rangle\right\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]
$$

and read so that Mary can be prescribed her task $\vec{T}_{m}$ if and only if the behavior of a Beneficiary, Defendant and Manager is consistent with what is required by $\left\langle\vec{m}_{\text {ben }}\right\rangle,\left\langle\vec{m}_{\text {def }}\right\rangle$ and $\left\langle\vec{m}_{\text {man }}\right\rangle$, respectively.
For reasons to be explained later the formula [m: $\left\langle\left\langle\vec{m}_{\mathrm{ben}}\right\rangle,\left\langle\vec{m}_{\mathrm{def}}\right\rangle,\left\langle\vec{m}_{\text {man }}\right\rangle\right\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle$ ] will be referred to as an SP-representation of the general mapping $M(1) \equiv[m:\langle\vec{m}\rangle$ $\left.\rightarrow\left\langle\vec{T}_{m}\right\rangle\right]$.

### 2.3.2 A Beneficiary

It is a common wisdom in LS that every task is, by definition, always designed in favor of a particular agent - a Beneficiary. For concreteness, the Beneficiary "assigned" by John to Mary will be personified by a young man named Benjamin. Let us stress that Benjamin-the Beneficiary is, by definition, the only agent entitled to demand that Mary's task be prescribed and hence also fulfilled.

Benjamin's demand will be called

## a BEN-order

and hence his role will then be characterized as his choice whether and in what form he will submit a BEN-order. As said, the BEN-order must satisfy conditions designed by the sub-domain $\left\langle\vec{m}_{\text {ben }}\right\rangle$. For the sake of our analysis, the conditions will be classified into:

- a trivial condition that Benjamin will ever decide to submit his BEN-order that he will not decide NOT to submit the order,
- the the order's correctness and justifiability.

At this stage of our argument, the notions of an order's correctness vs. justifiability will have to be accepted only intuitively. Their analysis will be postponed for later as it will still require some more preparatory work, and in fact we can do without it at the moment.

For the moment we shall rather stress that - in this particular exposition Benjamin is not the same person as John and that this fact is to demonstrate that John designs $M(1)$ in favor of a third agent, i.e. a person other than John or Mary. Put in LS-parlance, Benjamin is a third party Beneficiary.

In contrast, John may design $M(1)$ in his own favor - so that he himself will become the Beneficiary. Moreover, even Mary may be the Beneficiary who will, then, herself determine whether and in what form her own task should be prescribed.
In conclusion we will use the opportunity and claim that - given our bridgebuilding ambitions - we will take the roles of a Beneficiary and a so-called Plaintiff as of the same kind, in principle.

### 2.3.3 A Defendant

By a sub-domain $\left\langle\vec{m}_{\text {def }}\right\rangle$ are designed conditions under which an elderly gentleman, named Dave for illustration, will be entitled to submit

## a DEF-order

with the aim to affect the outcome of Benjamin's BEN-order. Hence, Dave-the Defendant is, by definition, the only agent entitled to modify or even nullify what would be prescribed otherwise - should Dave decide NOT to submit his DEF-order.

The conditions $\left\langle\vec{m}_{\text {def }}\right\rangle$ will be, again, classified into:

- a trivial condition that Dave will ever decide to submit the DEF-order and
- the order's correctness and justifiability.

In LS this kind of a Defendant's "defense" is associated to topics such as voidable act, defense, estoppel, statute of limitation, waiver etc.

However, the gravest confusion may arise from that the aim of a DEF-order is not to raise "objections" against BEN-order's incorrectness and-or unjustifiability. As established here, DEF-order can have a sensible empirical meaning if only a BEN-order is ever submitted and as such is "perfectly" correct and justified.

### 2.3.4 A Manager

What remains is the obvious question who and how will validate the two counter-orders, who will determine that the BEN- and DEF-orders satisfy the above established conditions. For that matter, Thesis C states that, John-the Designer "must" - within $M(1)$ - also nominate the respective "referee", judge, intermediary, fact-finder, assessor, ....

In this BOOK the role will be called a Manager and mostly personified by a young lady named Manuela. In short, Manuela-the Manager will be, by nomination, entitled to issue the ultimate verdict about whether and in what particular form Mary's task will be prescribed.

In our IT-parlance the verdict will be referred to as a

## MAN-order

and we will differentiate whether it is in the affirmative or negative.
By a negative MAN-order Manuela determines that Mary is to "remain where she is" - or, by far more accurately, to remain in her stage $M(1)$. As explained the MAN-order will be in the negative namely when Manuela determines that the respective BEN-order does not satisfy conditions $\left\langle\vec{m}_{\mathrm{ben}}\right\rangle$ or has been "successfully" nullified by the DEF-order.

In the full analogy with other Nominees:

1) By $\left\langle\vec{m}_{\text {def }}\right\rangle$ are designed conditions under which Manuela-the Manager may produce her verdict. The conditions will be again classified into a trivial condition and conditions of correctness and justifiability.
2) Into the role of a Manager may be nominated whoever - not only Manuela as an impartial third agent, but also - Benjamin, Dave, John, Mary or all of them as a four-member organization.

To conclude we should - as in many other cases of our IT-parlance - admit that it may appear inadequate to coin the frequently used term Manager for this particular role of a Nominee. The rationale for this term will be seen later, once our concepts of an ex ante and ad hoc regulation will be introduced.

## 3. APPENDIX TO PART I

Before we proceed to PART II the following chapter will seek to foreshadow the nature of what we regard as a "mere" COMMENTARY to our major Theses $A$, $B$ and $C$.

### 3.1 A Designer: the choice vs. behavior/action

To begin with, two points should be stressed. Firstly, a slash "/" in "behavior/ /action" will indicate that the two terms will be taken as synonyms. Secondly, a "behavior/action" of a Designer will be established on different grounds than that of a Designee.

### 3.1.1 Designer's "psychological portrait"

Invoking Thesis $A$ and the formula $M A X_{J}$ we will now in Fig. 3 assume, for concreteness and simplicity, that John-the Designer selects his optimal strategy from only three variants

$$
s t r_{1}, s t r_{2} \text { and } s t r_{3}
$$

and that each of them represents a profit-making Firm.
In words, John believes that fulfillment of his task $J(1)$ can be supported by money and that the money may be procured through his "investment" into one of the businesses in Fig. 3.


Put analytically, $M A X_{J}$ now obtains a somewhat concretized form:

```
max}\mp@subsup{U}{}{j}(str
s.t.: str }\in[\operatorname{str}(0)={M(1),M(1)2,V(1)}
    MAXX 
```

To illustrate the "rationale" underlying John's choice let us assume for concreteness that his optimal strategy will be

$$
s t r^{*}=M(1)_{2}
$$

and speculate that he may have been motivated not only by profit, but also:

- his inter-personal preference, e.g. the fact that he likes Mary better than Victor,
- his beliefs, e.g. that a competition in the grocery industry will be less severe than that in the shoe-making business.

This and other kinds of Designers' "psychological portraits" will be discussed with ever increasing emphases throughout the following PART II.

### 3.1.2 Axioms of rationality

It is a commonplace in ET to specify circumstances under which an agent can be seen as rational. In our IT-parlance, John-the Designer will be taken as a rational decision-maker if able:

1) to specify his social position as $J(1)$, i. e. to identify conditions under which his particular task may be prescribed,
2) to establish a set of strategies $\left\{s t r_{1}, \operatorname{str}_{2}, \ldots, s t r_{N}\right\}$ that he considers as feasible with respect to the support of his task,
3) to impose a preferential ordering upon the set of feasible strategies, i.e. establish their ranking " $\gtrsim$ " where the relationship $s t r_{r} \gtrsim s t r_{s}$ states that John regards the $r$-th strategy $s t r_{r}$ as no worse than the $s$-th strategy $s t r_{s}$,
4) to select from the set $\left\{s t r_{1}, \operatorname{str}_{2}, \ldots, \operatorname{str}_{N}\right\}$ : one and only one $s t r_{a c} \quad$ that he regards as his existing, actual strategy, one and only one str* that he regards as his best - optimal strategy.
Toward the end of PART II we will enrich the above list by a Fifth Axiom of Rationality - for the sake of the analysis of a so-called uncertainty choice.

### 3.1.3 Production cycle; re-design of a strategy

The distinction between $s t r_{a c}$ and $s t r^{*}$ suggests by itself that - under specific circumstances - John may decide to replace or re-design his actual strategy for the sake of a coming cycle of the production process.


In Fig. 4 we consider John-the Designer who:

- expects to be - at times $t^{1}, \ldots, t^{4}$ - repeatedly prescribed his task, e.g., monthly payments of alimony designed as $J(1)^{1}, \ldots, J(1)^{4}$ by the respective court of justice,
- subsequently selects his $\boldsymbol{i}$-th $\boldsymbol{i}=1,2, \ldots, 4$, optimal strategy $s t r^{* / i}$ how to fulfill every $\boldsymbol{i}$-th task.

For concretes and simplicity, we will assume that every time John selects his strategy from the above set $\left\{M(1)_{1}, M(1)_{2}, V(1)\right\}$. In sum, John repeatedly resolves the same $M A X_{J / a}$ so as to support the same, repeatedly prescribed monetary tasks.

To illustrate, let John's choices have in Fig. 4 the following meaning:
CYCLE 1 At $t^{1}$ John decides to invest into Victor-the Grocer $V(1)$. Unfortunately this investment leads to John's failure $J(-3)^{1}$ due to the Victor's failure $V(2) \Rightarrow V(-3)$.

CYCLE 2 At $t^{2}$ - as a re-action to the collapse in CYCLE 1 - John gets under pressure "to act". He terminates the cooperation with Victor and replaces it with Mary's grocery $M(1)$. However, also this "move" is not "successful" as the state of Mary's environment, accidently, does not allow for any grocery-type operations. The outcome $M(1)_{1} \Rightarrow M(1)_{1}$ leads to $J(-3)^{2}$.
CYCLE 3 At $t^{3}$ - as a re-action to the collapse in CYCLE 2 - John transforms Mary's firm into shoe-making $M(1)_{2}$. The resultant $M(2)_{2} \Rightarrow M(+3)_{2}$ leads, at last, to $J(+3)^{3}$.
CYCLE 4 At $t^{4}$, understandably, John decides to keep to the successful cooperation with Mary-the Shoe-maker.

### 3.1.4 Designer's behavior/action

The arrangement in Fig. 4 may be summarized so that at the beginning of every coming cycle John may choose whether or not he will re-place (redesign) his

$$
\text { actual strategy } s t r_{\mathrm{ac}}
$$

with some str*. According to this "binary" (yes-or no) choice:
John's action or "non-empty" behavior will be established as the case $s t r^{*} \neq s t r_{\mathrm{ac}}$
John's non-action or "empty" behavior will represent the opposite outcome $s t r^{*}=s t r_{\mathrm{ac}}$ representing John's choice "not to move", i.e. not to change the strategy that he applied in the directly preceding cycle.

### 3.1.5 A Designee: the behavior/action

As explained, of the two agents, a Designer and Designee, only the former one is a decision-maker. Contrariwise, by definition, whatever a Designee does is a "mere" fulfillment (or breach) of the respective task. Invoking Fig. 4, this specific nature of a Designee's behavior/action will be established as follows:
as an action or "non-empty" behavior will be taken the transitions:
$V(2) \Rightarrow V(-3)$ in CYCLE 1 and $M(2)_{2} \Rightarrow M(+3)_{2}$ in CYCLE 3,
as a non-action or "empty" behavior will be taken the case $M(1)_{1} \Rightarrow M(1)_{1}$ in CYCLE 2 when Mary's task has not been prescribed or, equivalently, when - for whatever reason - Mary stays in the stage $M(1)$-designed.
Summarizing, then:

1) Designee's behavior/action has been established regardless of whether the task is finally fulfilled or breached. The stage $M(-3)$-breached will be later interpreted a so-called delict or wrongful behavior/action.
2) We have generalized the reason for an "empty" behavior so that the respective verdict of Manuela-the Manager is in the negative.
3) We will also come across peculiar cases where:

- a Designee, as already noted, "stays still" because the respective mapping does not provide a unique prescription of his-her task,
- Manuela's verdict has the meaning of a so-called forbearance to act and hence the Designee is as if prohibited to "move" from his-her actual situation.

To conclude this section on a Designee we will also add that in our, admittedly dogmatic, IT-parlance the notion of a behavior/action will always involve some kind of a delivery - be it a delivery of "ordinary" goods and services, or an "extra-ordinary" delivery of such entities as a criminal verdict, freedom of choice, peace and quiet, ...

### 3.2 Infinite recursion of designs and nominations

The following examples can do no more than illustrate the nature of the mysterious topic of an infinite recursion.

### 3.2.1 Example 1: Manager of a Manager

Invoking the SP-representation $\left[m:\left\langle\left\langle\vec{m}_{\text {ben }}\right\rangle,\left\langle\vec{m}_{\text {def }}\right\rangle,\left\langle\vec{m}_{\text {man }}\right\rangle\right\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]$ of the general mapping $M(1) \equiv\left[m:\langle\vec{m}\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]$ we may recall that the

$$
\text { managerial sub-domain }\left\langle\vec{m}_{\text {man }}\right\rangle
$$

represents conditions under which the conditions $\left\langle\vec{m}_{\text {def }}\right\rangle,\left\langle\vec{m}_{\text {man }}\right\rangle$ will be validated. Put differently:

- what $\left\langle\vec{m}_{\text {def }}\right\rangle$ and $\left\langle\vec{m}_{\text {man }}\right\rangle$ primarily design is correctness and justifiability of a BEN- and DEF-order, respectively,
- analogously, then, by $\left\langle\vec{m}_{\text {man }}\right\rangle$ is designed correctness and justifiability of a MAN-order or the Manager's verdict.

Assuming that it is Manuela who validates the conditions imposed upon Benjamin and Dave, an obvious question arises who is to validate conditions $\left\langle\vec{m}_{\text {man }}\right\rangle$ imposed upon Manuela herself. In other words, we should ask "who is to manage the Manager?" - if we may paraphrase the classic question: "Who will guard the guardians?"

In this BOOK the classic problem of a "management of management" will be by and large based on the assumption that Manuela is not only nominated but also

$$
\text { designed as (OP-UN) })_{\operatorname{man}}
$$

by her own and fully independent Designer, e.g. Zeta-the Designer. Then, (OP-UN) $)_{\operatorname{man}}$ will be as any other operational unit and, among others, include conditions under which Manuela can be made to fulfill her managerial task. In
other words, if designed as (OP-UN) man Manuela can be prescribed to deliver her MAN-order with the verdict over the transition $M(1) \Rightarrow M(2)$.

In sum, the unregulated freedom of choice awarded to Manuela by John's nomination can be fundamentally regulated by Zeta in the form of the design (OP-UN) man .

It goes without saying that, by definition, also (OP-UN) $)_{\text {man }}$ involves nominations of the respective Nominees - the Beneficiary, Defendant and Manager.

Assuming that Barbra is Manuela's Beneficiary, we will attempt to illustrate the two levels of Nominees as follows:

- Benjamin is deciding whether he will demand that Mary's task be prescribed,
- Barbra, mutatis mutandis, is deciding whether she will demand Manuela's managerial task be prescribed.


### 3.2.2 Example 2: Designee turned Designer

Keeping to the example in Fig. 3 and the associated $M A X_{J / a}$ let the optimal strategy be $s t r^{*}=M(1)_{2}$ again. However, let it be designed in an extensively incomplete form

$$
M(1)_{2} \equiv[m:\langle\vec{m}\rangle \rightarrow\langle\infty, \infty, \vec{Q}\rangle]
$$

where $(K=\infty)$ and $(L=\infty)$ have the meaning "whatever" and "whoever", respectively.

The incompleteness can be read so that the only thing that matters to John-the Designer is that some grocery products are delivered in a prescribed magnitude and to a prescribed place. In other words, John, deliberately or negligently, "leaves it open" by what particular "technology" the task will be fulfilled.

Put in the LS-parlance, somebody will have to fill in the gap. We shall claim that it must be Mary herself, as - on a given level of the analysis - there is no-one else.

In our IT-parlance Mary will fill the gap by designing a lower-level strategy in the sense of Fig. 5 where we summarize the inter-level metamorphosis of Mary-the Designee into a Designer as follows:
on LEVEL ( $i$ ) John designs str* $=M(1)_{2}$ so as to support fulfillment of his task $J(1)$ designed on LEVEL $(i+1)$,
on LEVEL (i-1) Mary, in the full analogy with John, designs her own strategy $\operatorname{str}^{*}{ }^{*} \mathrm{M}=X(1)$ in order to support fulfillment of her task $M(1)_{2}$ designed on LEVEL ( $i$ ).


Fig. 5
Summarizing, then, John is solving his $\boldsymbol{M A} \boldsymbol{X}_{J}$ on LEVEL $(i)$, whereas on LEVEL ( $i-1$ ) Mary is selecting her optimal strategy by solving

$$
\boldsymbol{\operatorname { m a x }} U^{\mathrm{M}}(s t r)
$$

s.t.: $\operatorname{str}_{\mathrm{M}} \in\left[\operatorname{str}^{\mathrm{M}}(0)=\left\{\operatorname{str}_{1 / \mathrm{M}}, \operatorname{str}_{2 / \mathrm{M}}, \ldots, \operatorname{str}_{\mathrm{N} / \mathrm{M}}\right\}\right]$

$$
M A X_{M}
$$

### 3.2.3 Associated notes

1) It is by nature that no design can ever be complete. Hence:

- any Designee must fill in some gaps in his-her design;
- no Designee can avoid his-her "conversion" into a lower-level Designer;
- every Designer is a Designee designed by a higher-level Designer;
- a Designer designs tasks of others so as to establish strategies how to fulfill his-her own task.

2) To illustrate the general heterogeneity of variant strategies, Mary - herself designed as the above incomplete Grocer - may select her lower-level strategy from variants as diverse as, e.g.:

- a purchase of the grocery products from a nearby grocery shop or
- an armed robbery of the same or some other shop.

3) If Mr. Strong - an employee of Mary - is spotted inside a nearby grocery shop, a standard text-book integrability problem should be posed "who does maximize what". Put differently, we should separately ask what are the genuine interests and preferences of Mr. Strong, Mary and John.
4) For real-world Designers the phenomenon of an infinite recursions should be a never ending nightmare. One of the author's most daring statements will then be that the poor performance of current IT-systems can go to a large part to their Designers' negligence of the problem concerned above.

### 3.3 Two kinds of maximization " $I$ "

### 3.3.1 Calculation formula

Let us recall that with the aim to deliver her verdict about Mary's task Manuela--the Manager ascertains values for all exogenous variables $\vec{m}=\left(m_{1}, m_{2}, \ldots\right)$, substitutes them into the mapping $M(1) \equiv\left[m:\langle\vec{m}\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]$ and calculates out the resultant verdict.

To illustrate, let the collected data be $\vec{m}^{+}$and Manuela obtains the resultant task $\vec{T}_{m}^{+}$so that she finds out the blue shaded row in the table by which the design of Mary has been expressed.

| state | task |
| :---: | :---: |
| $\vec{m}^{1}$ | $\vec{T}_{\mathrm{m}}^{1}$ |
| $\vec{m}^{2}$ | $\vec{T}_{\mathrm{m}}^{2}$ |
| $\ldots$ | $\ldots$ |
| $\vec{m}^{+}$ | $\vec{T}_{\mathrm{m}}^{+}$ |
| $\ldots$ | $\ldots$ |

As said, John may attempt to represent the primitive enumeration of the table's rows by some more sophisticated method. In this BOOK we will extensively discuss cases when the Designer decides to express the design in the form of a procedure hereinafter called a "lower-case" maximization.

### 3.3.2 "Lower-case" maximization

In Fig. 6 we focus on LEVEL ( $i$ ) from Fig. 5 and - on this particular level differentiate:
"upper-case" maximization $M A X_{J}$ representing, by definition, John's interests and preferences,

[^0]"lower-case" maximization $\max _{M}$ representing "only" one specific form in which a solution to $M A X_{J}$ can be expressed.

Of the two maximizations $M A X_{J}$ and $\max J_{M}$ only the former $M A X_{J}$ represents genuine decision making. Contrariwise, $\max J_{M}$ is nothing more than a formula into which the respective Manuela-the Manager will substitute data so as to calculate the resultant verdict over Mary's task.


If we ask John about Mary's future behavior, he will probably provide us with the particular formula for $\max J_{M}$ and advice us to simulate her future behavior by substituting into it data representing variant developments of Mary's environment. In sum, if $\max J_{M}$ is known, we can derive from it what Mary may be prescribed to do.

As already noted, entirely different is the question "what made" John design Mary by this or that particular $\max J_{M}$, or what are John's particular interests and preferences, what is the particular form of $M A X_{J}$. In other words the inverse procedure is to "integrate" problem $M A X_{J}$ from the problem max $J_{M}$.

### 3.3.3 Profit maximization

Specifically, max $J_{M}$ can have the meaning of the above discussed max 1. In this particular case John's interests and preferences are such that his best strategy will be to make Mary maximize (for him) profit.

The integrability problem may then lead to $M A X_{J}$ that itself can be of the form $\max 1$. Both the Designer and Designee can then be - confusingly enough seen as profit maximizers, even though the arrangement concerned provides no information about Mary's "authentic" interests and preferences. All we can learn about her from $\max 1$ is that she has been made to work for John.

Needless to stress that the confusion can become quite serious once the roles of a Designer and Designed are performed by the same person as - we suspect assume most text-books.

### 3.4 A multi-unit strategy and a collective Designer

For simplicity we have so far confined to strategies that had only one-unit and were designed by an individual Designer. In what follows we will briefly skech how the two limitations can be relaxed.

### 3.4.1 The Firm as a set of operational units

For the sake of this section, let John design his overall strategy as a threeunit two-phase profit-making production str $^{*}=(K(1), L(1), Q(1))$ shown and interpreted in Fig. 7. For the sake of further analysis, the focus is on Mary who is to manufacture shoes from leather and labor procured "for her use" by Richard and George, respectively.


The following few notes may illuminate the multi-unit multi-phase concept:

- A set of OP-UNs of the above properties will be further classified as a system/ /process where the slash " $/$ " will indicate that the two terms are taken as synonyms (like in the case of a behavior/action).
- BPM would call the graphical representation in Fig. 7 a work-flow chart where the logic gate JOIN determines the processual nature of production. In LS-parlance the designed sequence/inter-dependence of operations will be referred to as an order of performances.
- The notion of a stage can be trivially extended upon a system/process. Yet, it may prove essentially hopeless to provide a simple and nice terminology for a mix of mutually different stages - e.g. their combination (K(+3), $L(2), Q(1))$.
- In reality hundreds of OP-UNs are likely to be interconnected with dozens of logic gates of various kinds. The logic gates though will not be taken as separate elements of the system/process - only as symbols of specific properties of the IF-components of the operational units behind the gate. Hence every system/process consists in operational units and only them.
- Hopefully a reader will not become confused by the purely formal affinity between Fig. 7 and the representation of a production function depicted in (a) of Fig. 1.


### 3.4.2 A collective choice; election vs. negotiation

The second obvious way how to enrich our Thesis $A$ will rest in accepting that a real-world Designer is in fact

## an $n$-member organization

whose members become co-Designers of the organization's collective strategy.
As any other collective choice, co-designing can be seen as a kind of an election whose participants seek to affect the resultant outcome by their votes or, in our IT-parlance, their

## DESIGN-orders

Put differently, a member of an organization - by his-her DESIGN-order demands that the resultant collective choice will involve at least some of his-her interests and preferences.

In this BOOK we shall deal with the concept of a collective Designer rather extensively. However, we will in fact confine to only one kind of a collective choice, namely that over a contract formation. In LS-parlance, we will mostly confine to the negotiation between contracting parties, i.e. agents seeking to design themselves into the roles of contract parties.

Hence, put in our IT-parlance, we will mostly analyze an election on the floor of a 2-member collective Designer. For concreteness, the 2 co-Designers will be personified by Richard and Mary and, consequently, the 2-member collective agent will be referred to as

## RM-the Designer

and the election will be characterized as a (two-party) negotiation.

### 3.4.3 Summary

The two above established - highly technical - criteria how to classify systems/ /processes are illustrated by six patterns (a) - (f) in Fig. 8. The reason for this particular selection is that in PART II the patterns (a) - (f) will be - one by one - applied within different social contexts so as to uncover peculiarities of different arrangements of social choices and behavior.


Fig. 8

### 3.5 System/process and its borders

### 3.5.1 System/process defined

Before we confront the diverse and mostly contradictory concepts of a border we should clarify what exactly is to be bordered.

In the first place, as already said, a system/process is a set of (one or more) operational units and only them. Reversely, a set of OP-UNs will represent a system/process if and only if:

- the OP-UNs are designed by the same Designer and, moreover,
- the OP-UNs - taken together - represent the Designer's particular strategy associated to the Designer's well defined particular task.

Under additional circumstances, e.g. those shown in Fig. 7, a system/process may be referred to as a collection of departments of the Firm. Other systems/ /processes will be said to represent, e.g., contract parties, family members etc.

### 3.5.2 Internal vs. external events

A closer comparison of the following Fig. 9 with Fig. 7 will show that a man named Richard is in both pictures designed as an element of the IF-component of Mary-the Manufacturer. In this sense Richard belongs to the respective system/process - is its part, i.e. the particular element of the interior of the system/process.


The fact that Richard's position is unquestionably inside the system/process under study, may be impugned by the following terminology:
in Fig. 7 Richard's behavior/action is often seen as an internal event in the sense that Richard has "the same status" as the other two members of the system/process - George and Mary,
in Fig. 9 Richard's behavior/action is an external event in the sense that he has the same status as e.g. prices $\vec{p}$ and efficiency $a_{f}$, that - let us recall - most text-books would take as exogenous/environmental.

### 3.5.3 START and END of a system/process

In BPM, borders of a system/process are often represented by so-called STARTand END-points of the respective work-flow chart. For example:

- Richard in Fig. 7 is a START-point due to the mere fact that his IF-component involves no internal event. In other words, Richard's task does not depend on any other member of the system/process under study,
- Mary in both Fig. 7 and Fig. 9 constitutes an END-point of the system/ /process as her development does not appear in any IF-component of any member of the system/process under study.


### 3.5.4 Multi-unit START and END

Fig. 10 goes a little further and depicts a system/process whose blue-shaded border consists in four START-points, whereas the pink border is constituted by three END-points. At least two complications or even confusions may arise here:

- in between the two border areas are four OP-UNs that can be seen as another kind of an "interior" of a system/process,
- if $t^{\mathrm{E}}<t^{\mathrm{S}}$, as in Fig. 10, the END of the system/process appears to precede its START.


Fig. 10

### 3.5.5 START and END of a production cycle

Fig. 11 is of the same kind as Fig. 4. Only every production cycle now consists in the three-unit two-phase strategy of the kind shown in Fig. 7.
The two different colors used for cycles are to suggest that the blue strategy applied in CYCLE 1 can be re-designed into the pink one for the sake of CYCLE 2. In this sense the time

$$
t^{1 /(2 b)}=t^{2 / 1}
$$

represents the moment at which one system/process ends and the other begins.


### 3.6 Default rules as gap fillers

### 3.6.1 Incompleteness of IE-representation

Unfortunately, it is mostly beyond the analytical capacity of real-world Designers to design an operational unit in its SP-representation established within our Thesis C. Instead, they will rather apply the IE-representation, where "IE" represents the fact that the IF-component of an operational unit is often divided into internal and external events.

The terminology is illustrated in Fig. 7, where the general formula $Q(1) \equiv\left[q:\langle\vec{q}\rangle \rightarrow\left\langle\vec{T}_{q}\right\rangle\right]$ for Mary-the Manufacturer has been concretized as

$$
Q(1) \equiv\left[q:\left\langle K(+3), L(+3) ; \vec{p}, a_{\mathrm{f}}\right\rangle \rightarrow\left\langle\vec{T}_{q}\right\rangle\right]
$$

and generalized into

$$
Q(1) \equiv\left[q:\left\langle\left\langle\vec{q}^{\text {int }}\right\rangle,\left\langle\vec{q}^{\text {ext }}\right\rangle\right\rangle \rightarrow\left\langle\vec{T}_{q}\right\rangle\right]
$$

where:
$\left\langle\vec{q}^{\text {int }}\right\rangle \quad$ are the internal events/conditions $K(+3), L(+3)$,
$\left\langle\vec{q}^{\text {ext }}\right\rangle$ are the external events/conditions $\vec{p}, a_{\mathrm{f}}$.
The incompleteness of this representation rests primarily in that it is "completely" silent about the triad of Nominees - a Beneficiary, Defendant and Manager.

### 3.6.2 Managerial gap-filling

By definition, all gaps "must" be somehow filled, e.g. "by action", i.e. so that somebody will simply take over the role of a Nominee and the others will accept it. For example, the lack of a "properly nominated" Beneficiary is in reality often remedied by a "self-understood" default rule that a task is "mostly" designed in favor of its Designer.

The following examples will illustrate two more cases of an incomplete nomination, this time of a Manager. We shall consider BEN-orders that are incomplete and conflicting, respectively. The common denominator of the two cases will be that the respective Manuela-the Manager may not be certain about what exactly is demanded.

Hence she will have to declare the BEN-order to be incorrect, unless she is equipped by an appropriate default rule designed within the managerial sub--domain $\left\langle\vec{m}_{\text {man }}\right\rangle$.

### 3.6.3 Incomplete orders; default rule I

Let $M(1)$ be designed so that:

- as the feasible places of delivery can be prescribed: Designer's home, British Parliament and Prague Castle,
- the respective condition, under which the place will be prescribed, is that this or that address is determined in the respective BEN-order.
Manuela's uncertainty may then arise, if the Beneficiary (incorrectly) demands some unfeasible "fourth" address or is silent about it.

One way how to prevent the order's incorrectness rests in designing $\left\langle\vec{m}_{\text {man }}\right\rangle$ so that Manuela will take Prague Castle as the place of delivery
unless the Beneficiary (correctly) states otherwise
In other words, should the BEN-order be incorrect due to the above incompleteness, Manuela will fill in the gap by "Prague Castle".

### 3.6.4 Conflicting orders; default rule II

Let the Beneficiary

- firstly submit a (BEN-order) ${ }^{1}$ demanding the delivery to the British Parliament and
- then submits a conflicting (BEN-order) ${ }^{2}$ with a demand for Prague Castle.

The solution is again in that $\left\langle\vec{m}_{\text {man }}\right\rangle$ will include a rule for Manuela how to resolve the ambiguity. Examples can be characterized as:
LIFO (last in first out) or - as lawyers would say - lex posterior derogat (legi) priori,
FIFO (first in first out) or the rule touch-move mostly applied by chess-players.

### 3.7 Miscellaneous notes

### 3.7.1 Regulation ex ante and ex post

In what follows the particular form of a design will be interpreted as John's choice how to regulate - ex ante - Designee's behavior/action. By "ex ante" we express the obvious fact that the regulation is determined "long before" the respective Beneficiary will demand the task to be prescribed.

However, the same Designer may understand that in real-life processes there must be always some limits to the rigidity of the ex ante regulation, namely with respect to the uncertainty of the world's development. Hence, for a real-life Designer there is no other way than empower the above established Manuelathe Manager to ad hoc amend or even completely revert the outcome of the ex ante regulation.

### 3.7.2 Properties of SP-representation

The SP-representation $\left[m:\left\langle\left\langle\vec{m}_{\text {ben }}\right\rangle,\left\langle\vec{m}_{\text {def }}\right\rangle,\left\langle\vec{m}_{\text {man }}\right\rangle\right\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]$ of the general mapping $\mathrm{M}(1)=\left[m:\langle\vec{m}\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]$ will be pushed ahead in this BOOK as the most advanced, sophisticated or even complete way how to formalize social behavior/action. Two additional notes may illustrate this qualification:

1) Other representations of operational units will be incomplete mostly with respect to our Thesis C. As stressed, real-world Designers often do not respect Nominees' relevance - if they ever recognize their existence.
2) The prefix "SP" in an SP-representation is to pin point one further methodological, interdisciplinary bridge towards LS, namely to its classification of Substantive and Procedural branches of law.

### 3.7.3 Demand

In economic text-books a demand function describes how optimal amounts of inputs change with the changing environment of the agent concerned. The reader may have noticed that the notion of a demand will be assigned a rather different meaning in this BOOK. With the aim to bridge the gap between ET and LS we have opted for a terminology according to which:

- a Beneficiary demands (proposes, requires, claims, ...) that a particular task be prescribed and hence also fulfilled,
- a Defendant demands (proposes, requires, claims, ...) that what the Beneficiary demands be altered or even nullified.

Analogously, a member of an organization will be said to demand - by his-her vote - that the resultant collective choice involves also some of his-her interests and preferences. Specifically then, a contracting party will be said to demand that a contract be formed in a particular specification.

### 3.7.4 Pseudo-beneficiary

It is noteworthy that apart from the five major categories of agents (a Designer, Designee and the triad of Nominees) at least two other kinds of agents have appeared in our analysis so far. Recall, then, that Richard-the Interventionist in Fig. 9 has been designed within the IF-component of Mary-the Manufacturer the respective OP-UN, whereas Mr. Strong-the Executor was designed as an element of her THEN-component.

This group of such other agents will be now expanded by an External Recipient of a delivery.

To begin with we will recall the example where Miss Weak was to deliver shoes to the hands of Rock-the Receptionist of the British Parliament. It may be of value to emphasize that "Rock-the Receptionist" is only an element of the THEN-component of the respective OP-UN. The gentleman named Rock is here nothing more or less than a particular value of the variable

$$
Q_{\mathrm{c} / \mathrm{m}}: \text { a place of delivery }
$$

and "Rock's hands" is only the formula in which is expressed the "address" onto which the shoes must "land" - if so prescribed.

In IT-parlance a person in this kind of a role will be called

## an external recipient of a delivery

As such, Rock will be strictly separated from the Beneficiary in whose favor the respective task has been designed. The major difference between the two roles is in that, by definition, Rock-the recipient has no control whatsoever over the respective task. Still more importantly, the recipient need not ever know that some delivery may be "on the way". Practical legal problems then arise when this uninformed recipient refuses to receive the delivery - for whatever reason.

In this context LS would discuss topics such as, e.g., incidental beneficiary, repudiation, creditor's contribution ... Contrariwise, ET would rather pin point its concept of - negative and positive - externalities.

### 3.7.5 Kinetics vs. dynamics of a system/process

In what follows we shall differentiate between two kinds of inter-temporality:
Kinetics will focus on the interdependency between operational units such as that represented by the logic gate JOIN in Fig. 7. In some sense events (both internal and external) before the gate appear to be the "causes" of the developments behind the gate. Dynamics seeks to disclose the genuine cause of an observed behavior/ /actions of a system/process. Apparently, it is John-the Designer whose choice determines under what conditions and how a Designee may operate. Hence, it is $M A X_{J}$ and John's present
choice $s t r^{*}$ that must be taken as the genuine driving force of the future kinetics of the system/process concerned.

### 3.7.6 Pseudo-condition

We have already proposed to differentiate between must and "must" so as to avoid misunderstandings about a task and a condition, respectively.
A similar problem arising from the imperfection of a natural language can be exemplified by Mary-the Manufacturer and her two employees Miss Weak or Mr. Strong. Let then, John designs Mary so that when Miss Weak is the Executor the magnitude of her task will be proportionally smaller.
One may then recklessly conclude that a higher magnitude of a delivery can be prescribed "if only" Mr. Strong will be available as the Executor. The confusion rests in the ambiguity about whether "Mr. Strong" is an element of the IF- or THEN-component of the respective $M(1)$. In other words, the ordinary language is, again, often unclear about whether a given event is a condition or a particular element of a multi-dimensional task.

For illustration it may be enough to remind the reader of our earlier example where Miss Weak was prescribed to deliver 15 pairs of shoes "only because" Mr. Strong was not available so as to deliver "his usual" 17 pairs.

### 3.7.7 Task vs. plan of action

Our choice to use the term task as a universal representative of the THEN--component of an OP-UN was rather arbitrary. It was also stressed, that the term may appear more or less appropriate depending on the social context, e.g. on whether or not a Designer and Designee are two different persons.
Apparently, all five roles (a Designer, Designee and the triad of Nominees) can be performed by the same person, e.g. the poor Robinson Crusoe analyzed later in this BOOK. Apparently, then, for this kind of a personal union it would seem to be more appropriated to interpret Robinson's task as his plan, objective, or goal. If he fails to fulfill the plan it will be often inappropriate to qualify the outcome as a delict or a wrongful behavior/action.

However, our point here is that a nicer terminology will most probably missed our target to disclose that also the terms as diverse as task/obligation and objective/ plan are in fact of the same nature. In our ugly IT-parlance they are all, in fact, nothing more or less than a THEN-component of some IF-THEN rule.

## 4. SELECTED IMPLICATIONS

Should we promote this PART I as an ordinary self-contained article-size treatise, we should conclude it by a few proposals for academic community and-or professionals in the practical areas of social life.

### 4.1 Economic theory

By allowing LS and BPM enter the sacred universe of ET, what comes to surface in the first place is the notorious complexity of the real-world social choice and behavior.

We may illustrate this rather trivial observation again on alpha-numeric vector strings by which only the society can be realistically described and due to which are immediately disqualified methods of infinitesimal calculus and hence also a great part of the tool-kit supporting the scientific statute of ET. Similarly, it is hard to imagine how ET could efficiently capture the multidimensionality of an agent's task and-or the multi-unit structure of a real-world production process.

And so on, and so forth.
The constructive getaway from this agnostic depression must be primarily based on the thesis that science - just like art - is not there to be useful but beautiful.

Secondly, and more seriously, books like ours may, hopefully, improve our understanding of the factual capacity of science and its genuine sphere of influence. Unfortunately, it is not a commonly accepted practice of scientists to provide the general public with a clear delineation of the range where their know-how can be realistically applied and, hence, seriously, sensibly and honestly recommended for a practical use. In other words, books like ours should encourage the academic community to fully concentrate on express disclosures of what it is incapable of delivering - in the short and-or long run.

In the above sense, the author is hopeful that the societal analysis will - one day - convincingly separate thought experiments from real-life phenomena. Once this border line is established with a reasonable precision, scientists may hope to regain their trustworthiness. Once they determine areas beyond their genuine capacity, they will leave them to consultants in "best practice" or experts on trial and error. Thus retrieved space, time and energy can then be devoted to critical evaluations of the experts' attempts to present their services as outcomes of a scientific method of thought.

Thirdly, and perhaps most importantly, ET has developed its superior methodology that may provide us with well formulated questions about social choice and behavior. And this is no small contribution to our way of thinking, however unattainable my now appear the answers to most of these questions.

### 4.2 Legal scholarship and BPM

Rather paradoxically, it is the intellectually superior ET that has been exposed to stimuli from applied, relatively inferior, branches of research. Still more confusing may then appear that the BOOK's impact could be of a greater import to LS and BPM.

The following recommendations may illustrate this conclusion:

1) It is the author's experience that documents (legal or other) by which systems/ /processes are designed can and should be translated into our IT-parlance. In particular, any system/process can and should be expressed in the universal IF-THEN structure of operational units and only them. Specifically, in legal documents the concept of an agent's right can and should be mercilessly converted into its mirror image - a task/obligation of the respective counteragent.
2) Every operational unit can and should be ascribed its particular stage selected from a finite number of discrete and mutually exclusive stages. Similarly, within an operational unit a task can and should be always expressed in terms of a delivery of "something", be it a delivery of shoes, or peace and quiet.
3) The existence of the notorious infinite recursions can and should be openly admitted. Then, the infinite cause-effect chains can and should be "strangulated" in an easily recognizable form.
These and other recommendations can be best tested by the efficiency with which the respective system/process can be supported by information technology. From among the author's highly immodest ambitions probably the highest one concerns the present "state of art" in IT. His claim or complaint will be that the fascinating capacity of the technology has been so far applied in a surprisingly limited, almost negligible scale. Smart and ever smarter machinery is literarily everywhere, but its meaningful usage is in reality fatally constrained by the medieval level of our understanding the nature of social choice and behavior that the machinery is supposed to support.
This BOOK would like to substantiate this harsh accusation and - hopefully join those who seek to somewhat improve this state of affairs.

## PART II.

## COMMENTARY

Subjects of the following Comments 1-11 can be well classified according to the following three criteria:

1) Firstly we will differentiate problems of dynamics or kinetics or, in other words, whether they focus on Designers' choices or Designees' behavior/ /actions, respectively.
Decision-making of a Designer will be mainly covered by the following Comment 1 and then again in Comments 10 and 11 where, based on an extensive preparatory work, a Designer's choice will be finally established by an express formula for his utility maximization $M A X_{J}$.
Designees' behavior/action will be - to a large extent - assumed in the form of a "lower case maximization problems", such as $\max J_{M}$. The topic of "two kinds of maximization" - such as $M A X_{J}$ and $\max _{M}$ - will thus become one of the key subjects of the overall analysis.
2) Comments 1-11 can be also seen as subsequent applications of morphological patterns (a)-(f) from Fig. 8. We shall attempt to apply each of them in a different social context. Or, conversely, each of the Comments can be taken as a search for a pattern that will fit best the social context under study. Every time our aim will be to associate the pattern to the appropriate context so as to disclose those general phenomena that would not be that well observed elsewhere.
3) Another criterion how to classify subjects of this PART II may be derived from the nature of the social inter-actions under study. With respect to the roles of the inter-acting agents we shall proceed as follows:

- Participants in a collective choice will be under study in Comment 1, where co-Designers will seek to design their collective strategy and only then Nominees will jointly determine whether and in what form a particular task will be prescribed.
- In Comment 11 we shall conclude the analysis with a game-type interaction between two Designers, where a choice of one Designer will depend on how some other Designer will select his-her optimal strategy.
- In between Comments 1 and 11, most of the analysis will be devoted to designs of inter-acting Designees as members of multi-unit multi-phase systems/processes.


## COMMENT 1.

## Designer's choices; the case of a contract

The first morphological patterns under study will be the two two-unit system/ /processes shown in sections (d) or (e) of Fig. 8. Apart from the number of units their key characteristic is that they are both designed by a collective Designer, namely a 2-member organization. In particular the 2 co-Designers will have the empirical meaning of contracting parties whose aim is to establish themselves into the roles of
contract parties
They will be personified by Richard and Mary and hence, when taken together, referred to as RM-the Designer. The strategy determined by this collective 2-member Designer will be denoted as

$$
s t r^{*} / R M
$$

In LS-parlance the strategy's content will be called a contract. In our IT-parlance we will rather speak about
a contractual (contract-designed) system/process
and deal with it - in principle - as with any other set of operational units. We will consider its two particular settings characterized in LS as a contract for life insurance and sale and purchase, respectively. These two social contexts will then allow for different general phenomena to be closely analyzed:

Life insurance contract

Contract for sale and purchase
will clearly expose the roles of Nominees, namely those of a (third-party) Beneficiary and Manager/ /Assessor. Contrariwise, hardly visible will be parameters of the election/negotiation through which the contract is to be formed. is essentially "completely" incomplete with respect to the Nominees. On the other hand it will open way to our concept of a so-called Market-organizer, e.g. a Stock-exchange on whose floor sales contracts are expressly formed and executed.

Both contracts will be analyzed with respect to the voting powers of the participants of the respective collective choices. We shall differentiate:

- a symmetric Negotiation 1 between contracting parties seeking to "form a contract",
- an asymmetric Negotiation 2 between a Beneficiary and Defendant.

The comparison of the "visibility" of these two kinds of elections is the major subject of this Comment 1 . Hence the hundreds of other highly important problems are left aside or postponed to the our later Comments.

## 5. LIFE INSURANCE

There are at least two reasons why insurance (here and in Comment 7) has fallen into the focus of this BOOK:

- the author's experience with designing IT-systems can be traced to this particular area,
- insurance has a strong linkage to the problems of uncertainty discussed throughout this BOOK so as to, finally, become one of the major topics in Comments 10 and 11.


### 5.1 Assumptions, simplifications and notations

The two-unit contractual system/process under study is in Fig. 12 denoted as

$$
\operatorname{SIN}(1)=\{I N(1), C L(1)\}
$$

Its content should be self-explanatory, due to its highly simplistic IE-representation.


Fig. 12

For simplicity we assume in Fig. 12 that:

- the two OP-UNs are designed as independent, where the payment of the prospective recovery/benefit does not depend on whether and how the premium has been paid,
- Both OP-UNs are designed so that each of them can be prescribed only once during the "life-span" of the system/process.


### 5.2 Task prescription: Negotiation 2

For the ease of the discussion we will begin, somewhat unsystemically, with Negotiation 2, i.e. the election whose two participants are already "well defined" as the "products" of the preceding Negotiation 1.

### 5.2.1 Insurer's task

In the social context of insurance the major properties of Negotiation 2 can be best demonstrated on

$$
I N(1) \equiv\left[i n:\langle\overrightarrow{i n}\rangle \rightarrow\left\langle\vec{T}_{i n}\right\rangle\right]
$$

by which Richard-the Insurer is designed and where, let us recall:

- the IF-component contains an "insured event",
- the THEN-component represents the Insurer's task to pay a recovery/ /benefit.

In Negotiation 1 Richard's counter-party is Mary-the Client. Here, in Negotiation 2 it will be somehow nominated Beneficiary who only, by definition, has the exclusive right to enforce the prescription

$$
I N(1) \Rightarrow I N(2)
$$

of the task of Richard-the Insurer.

### 5.2.2 A third-party Beneficiary

By assumption the life insurance from Fig. 12 covers a harm caused by Mary's death and only this kind of a harm. Hence, trivially, the looked-for Beneficiary must be a third party with respect to the two contract parties Richard and Mary. For concreteness, let the Beneficiary be Mary's only son Benjamin.

It may be of value to summarize that:

- it is not Mary but Benjamin who suffers a harm when Mary dies,
- the aggrieved (insured) Benjamin need not know that the contract exists and-or has been formed in his favor,
- Benjamin may decide not to submit his BEN-order, even if he may be rather certain about its correctness and justifiability.


### 5.2.3 Other Nominees

What remains are the roles of a Defendant and Manager.
In order to demonstrate the peculiarity of the insurance context let us assume that Richard is not only designed as an Insurer but also nominated into the two "adjacent" roles of a Defendant and Manager.

Manager is often called an Assessor in the case of insurance. It is noteworthy that Richard-the Assessor will serve in this BOOK as a highly transparent example of an express, perfectly visible Manager.

### 5.2.4 Justifiability

Conditions under which the Insurer can only be prescribed to pay the respective recovery/benefit are the true essence of the contract. Hence, the context of insurance offers a sometimes frustrating opportunity to dwell deeper into the phenomenon of a complexity of an IF-component of the operational unit.

An ordinary real-life insured event is infamous for its chaotic and often inconsistent content. It may become a genuine analytical adventure to get through the Insurer's language flooded by multiple exclusions, not to mention exceptions to exclusions and exceptions to these exceptions.
For the sake of this analysis $I N(1)$ will be taken in its SP-representation, namely in the simplified structure shown in Fig. 13. For dramatic effect the same events seem to be designed- as if paradoxically - as conditions of both the BEN- and DEF-order.


### 5.2.5 Justifiability: example 1

Let $\overrightarrow{i n}_{\text {ben }} \quad$ require that BEN-order is justified only if the number of Benjamin's visits to Mary's hospice is at least 20. As a result, the BEN-order:

- will not be justified if Assessor finds out that there were only 10 such visits,
- will be justified if the actual number of Benjamin's visits is 23; as a result Assessor will deliver the BEN-order to Richard-the Defendant with the information that the justifiably demanded recovery/benefit is, e.g., USD 600000.

Let $\overrightarrow{i n}_{\text {def }} \quad$ state that if the number of Benjamin's visits is smaller than 25, Richard may justifiably demand by his DEF-order a reduction of the recovery/benefit by, say one half.

### 5.2.6 Justifiability: example 2

Arrangement A: let drunkenness be a part of $\overrightarrow{i n}_{\text {ben }}$ and only of $\overrightarrow{i n}_{\text {ben }}$, with the meaning that if Benjamin was visiting Mary intoxicated, his BEN-order cannot be taken as justified and hence, Richard-the Insurer cannot pay the recovery/benefit (is in fact prohibited to pay the recovery),
Arrangement B: let, contrariwise, drunkenness is a part of only $\overrightarrow{i n}_{\text {def }}$ with the meaning that for the sake of justifiability of his BEN-order Benjamin did not have to be sober during his visits. If Benjamin's claim is delivered to Richard, he as a Defendant has an exclusive right to decide whether and in what form he will raise his "defense" in the form of the "statute of drunkenness".

### 5.2.7 Natural language

The main reason why bother the reader with the above technicalities is to demonstrate, again, what a mission impossible it is to express in plain terms the content of a life insurance contract or any other contract indeed. Even in cases where the contract is trivialized to the degree shown in Fig. 12.

Again we may only warn that it has been the author's experience that it is essentially beyond the capacity of natural language to clearly differentiate, e.g., the two above arrangements A and B, namely when the roles of a Defendant and Assessor are performed by the same person - as in the case under study.

### 5.3 Contract formation: Negotiation 1

### 5.3.1 Optimization

Purely formally, the system/process in Fig. 12 represents a collective strategy $s t r^{*} / R M$ obtained as a solution to the maximization problem
$\max U^{\mathrm{RM}}(S I N)$
s.t.: $\operatorname{SIN} \in\left[\left[\operatorname{SIN}(0)=\left\{\operatorname{SIN}(1)_{1}, \operatorname{SIN}(1)_{2}, \ldots, \operatorname{SIN}(1)_{N}\right\}\right]\right]$
$M A X_{R M}$
where:
$U^{\mathrm{RM}}(s t r) \quad$ represents preferences of a two-member organization denoted as RM-the Designer,
$\operatorname{SIN}(0) \quad$ is a set of variant strategies, e.g. $N$ variant configurations ("re-organizations") of a life insurance of a specific category.

As already stressed, the analytical complexity of $M A X_{R M}$ is way off the reach of our contemporary science. All we may offer here is a brief sketch of a "mechanics and logistics" of the underlying Negotiation 1 between Richard and Mary.

### 5.3.2 Ill-defined participants

For the case of Negotiation 2 its counter-parties are well known as they are nominated by the already existing contract. By contrast, in Negotiation 1 the question who is the Negotiator may easily obtain a mysterious character. Loosely said, he-she is a contracting party, i.e., as said, an agent who wants to become a contract party.
Or, we can say that in the role of a Negotiator is a would-be Insurer and a would-be Client and that they express their will by "votes" that we have already called DESIGN-orders, denoted in this particular case as

## IN-order and CL-order

Which opens a still more mysterious question who is there to collect the orders, validate them and - finally - calculate out the "voting result" - in the form of a verdict whether and in what form $\operatorname{SIN}(1)$ will be designed.

For the moment we shall forward these rather depressing questions towards the subsequent chapter on a sales contract. There, as said, we will approach the above questions by our concept of a so-called Market-organizer, e.g. a Stock--exchange on whose floor contracts are expressly formed.

### 5.4 Associated problems, topics and notes

### 5.4.1 Contract of adhesion

LS speaks about Negotiation 1 in terms of an exchange of communications often called an offer and acceptance. In general anybody can initiate the negotiation by his-her offer.

Insurance contracts are traditionally classified as contracts of adhesion, or, equivalently, standard form contracts. Hence, in practice:

- it is Richard-the Insurer who initiates the negotiation by his IN-order,
- the IN-order is submitted in the form of a non-negotiable proposal thus leaving the order's Addressee in a "take-it-or-leave-it" ("yes-or-no") position.

What Richard submits is in fact a pre-printed form that needs only signing. The pre-printed form thus represents what is often called a "policy" and seen as a standardized "product" offered for sale in an marked.

### 5.4.2 Voting powers

The implied statutes of the two-member organizations may vary across jurisdictions but certain "rules of the game" may be taken as universally accepted, namely the rule that the two counter-voters are free to choose whether and what DESIGN-orders they will submit. Moreover, their voting powers are assumed to be equal. As a result:

- Any of them can initiate the election by submitting his-her vote as the first one. However, by nature, the initiating order must contain specification of the subject of the elections - the proposal of the particular form of the two operational units. At the same time the firstly submitted order is also taken as the vote in the affirmative.
- If the second voter abstains, the equality of the voters' powers leads to that the lack of the second vote is taken as a rejection of the proposed concept of the two operational units.


### 5.4.3 Order correctness; zero justifiability

As already stated, a DESIGN-order, as any order indeed, must be somehow processed, e.g. collected, validated and matched with a counter-order.

The essential difference from a BEN- and DEF-order rests in that for a DESIGN--order the notion of justifiability has no meaning. By definition, let us recall, a justifiability condition can "emerge" only as an element of a completed design, as an outcome of a "match" of the respective pair of DESIGN-orders the IN- and CL-order.

The two counter-orders can thus be - in the case of Negotiation 1 - validated only as to their trivial conditions and the conditions of their correctness. As said, by the latter conditions are understood technicalities often only implied in law or fact. It may be useful to differentiate them with respect to:

- a required form of an order, e.g. a usage of a pre-printed form that must be filled in a required language, e.g. English or Czech,
- a required "mental state of mind" the order's submitter (cf., e.g. his-her mental incompetence etc.) or his-her social situation (cf., e.g. undue influence, duress etc.).


### 5.4.4 Hierarchy

Recall that, in general, a Designer may realistically design someone as a Designee only on the proviso that the "someone" is subordinated to the Designer. In the specific contractual setting the required hierarchy can be guaranteed only on the basis of the memberships of the two members of the organization concerned, namely, then, for Mary and Richard.

Put differently, Richard and Mary can jointly design as a contractual Designee $I N(1)$ or $C L(1)$ only themselves.

### 5.4.5 A task to contract

In the usual language of LS a contract would be seen as an exchange of obligations and it will "emerge" (be created) by its formation, which is usually assumed to be an outcome of a so-called agreement of the respective contracting parties.
It has been explained that RM-the Designer seeks to design his-her optimal strategy to fulfill $R M(2)$, if prescribed. Methodological complications and confusions then arises, e.g. when:

- Richard and Mary are designed by their individual $R(1)$ and $M(1)$ and the insurance contract is simply an optimal strategy how to fulfill these tasks. If $M(1)$ represents Mary's task to secure Benjamin's survival, her other strategy may be to buy a shoe-making Firm that Benjamin will inherit after her death.
- Richard and Mary are designed so as to have a some kind of an obligation to submit their IN- and-or CL-orders - under designed conditions, needless to stress.


### 5.4.6 Default rules

With respect to voting powers Negotiation 1 was characterized as symmetric, whereas Negotiation 2 was called asymmetric.

Using a somewhat different terminology Negotiation 1 can be taken as consensual, whereas Negotiation 2 will be enforceable. The most accurate criterion of differentiation is provided by the following default rules:

- If an Addressee of a consensual order, e.g. of a DESIGN-order, does not respond (within a given time limit), he-she is taken as if he-she has rejected the order's content.
- Contrariwise, if an Addressee of an enforceable order, e.g. a Defendant who receives a BEN-order, does not respond, he-she is taken as if he-she has accepted the order's content.


### 5.4.7 Lower-level designs

Given that Fig. 12 depicts the joint strategy of Richard and Mary, each of them - individually - seek to support fulfillments of their higher level tasks (to pay recovery/benefit and premium, respectively) by their lower level strategies

$$
s t r^{*} I N \text { and } s t r^{* / C L}
$$

Hence, again, the Designees concerned, in this case $I N(1)$ and $C L(1)$, may turn into lower level Designers so as to solve their maximization problems

```
\(\max U^{I N}\left(s t r^{I N}\right)\)
s.t.: \(s t r^{I N} \in \operatorname{str}{ }^{I N}(0)\)
\(M A X_{I N}\)
```

and

```
max}\mp@subsup{U}{}{CL}(str\mp@subsup{r}{}{CL}
s.t.: str'CL}\instr\mp@subsup{r}{}{CL}(0
```

$M A X_{C L}$

These lower-level strategies are designed, as explained, with the aim to secure the two respective payments. Needless to stress, that both strategies may be designed as by far more complex systems/processes than the two-unit two-phase system/process the units of which are to be supported.

### 5.4.8 Further problems

The social context of insurance will be also discussed in Comment 7 where some specifics of a compulsory insurance of vehicles will be concerned, in the full analogy with the discussion in this Comment 1. In particular, the following problems will become under study:

1) Various interpretations of Benjamin's choice NOT to submit his BEN-order should be of a deeper interest of analysts and governmental bodies, e.g. tax collectors or even crime prosecutors. The same can be said, mutatis mutandis, for Richard's choice not to submit DEF-order, e.g., not to impose the statute of limitation upon a Beneficiary's claim.
2) In this Comment 1 we have assumed that a Beneficiary is nominated onymously - by his particular name Benjamin. However, a Beneficiary may be nominated also anonymously and collectively, e.g., so that it will be "any child of Mary-the Client living as of the day of Mary's death". The notion of a Nominee's anonymity and collectiveness will be also in some more detail discussed in Comment 7 where into the role of a Beneficiary will be nominated "anybody hit by the insured vehicle and also any Driver of the vehicle". Moreover, rather unexpectedly, the same kind of an anonymity and collectiveness will be in Comment 3 associated to a sanctionative operational unit designed by a Legislator in the social contexts of a tort and crime.
3) Beneficiary - as a third-party Beneficiary - must be strictly differentiated from other types of "third persons", namely agents who are "only" external recipients of the recovery/benefit. Various examples will be offered further in the BOOK of external recipient who a no more than only an "address" demanded by a Beneficiary's BEN-order. Put differently, an external recipient is a parameter of Richard's task such as, e.g., its magnitude and time of delivery.
4) Serious confusions - legal, technical and organizational - are associated with cases when the same person is nominated both as a Defendant and a Manager/Assessor, as above in the case of Richard-the Insurer. Apart from the obvious conflict of interests, the two roles may lead to a conflict between a liability of the two agents vis a vis a Beneficiary who may be "dissatisfied" with what has be the actual outcome of his-her BEN-order..
5) Apparently, unlike in the case of, e.g., shoe-making, the nomination of a Manager/Assessors - as a rule - will not allow for an extensive ex post regulation. In the above example, the Manager/Assessors, as a rule, will not have the right to, e.g., "excuse" the required number of visits to Mary's hospice.
6) We will show later that a so-called order-routing executed by the Manager/ /Assessor may constitute a system/process by far more complex than the system/process "under management". As a result, the factual importance of substantive conditions may be strongly mitigated by the particular form in which a Manager/Assessor is nominated.
7) Civil procedures are traditionally divided into inquisitorial and adversarial. The former type seems to apply to the above discussed Manager/Assessor.

## 6. CONTRACT FOR SALE AND PURCHASE

That LS and ET have a common denominator can be best illustrated by the essentially identical content of a sales contract on the one hand and a supply--demand equilibrium on the other.

### 6.1 Example

### 6.1.1 Two-unit two-phase system/process

Hopefully, the system/process in Fig. 14 is self-explanatory due to its immense trivialization and the simplistic IE-representation. Within SE(1) the zero internal event indicates that the operational unit is the START-point of the system/ process.

In contrast, the flat black arrow aiming at Mary-the Purchaser states that Mary's task may be prescribed only IF the leather is duly delivered and the stage

## SE(+3)-fulfilled occurs

Invoking Richard-the Supplier in Fig. 7, the inter-temporal (processual, two--phase) nature of the system/process in Fig. 14 rests in - what LS would call order of performances. Let it be stressed that Richard and Mary from Fig. 14 jointly decided that it will be Richard who will have to deliver first.


### 6.1.2 Comparisons with life insurance

Our major aim is to show properties that contractual systems/processes have in common regardless whether they are "about" insurance or sales.

At the same time it may be of value to pin point the following few differences between Fig. 14 and Fig. 12:

- The real-world complexity of an insurance contract is often hidden inside the insured (external) event. By contrast, in a sales contract the complications may mainly arise from how the internal events are designed. The major problem thus will often rest in determining whether or not the Seller's task has been fulfilled by his behavior/action $S E(2) \Rightarrow S E(3)$, whether or not the stage $S E(3)$ can be taken as a proper tender. Recall that the complexity of such verdict is given by the multi-dimensionality of every task. Hence, in reality, the task must be fulfilled not only as to its kind and magnitude but also
with respect to its other parameters such as those by which is prescribed the Executor and-or the place and time of delivery.
- A sales contract will rarely include an express nomination of a Manager who would - if only remotely - resemble the Assessor discussed in our previous chapter. As a rule and unlike in the case of insurance a sales contract will say nothing about Nominees, namely about the Manager who would "referee" prospective disputes over the stage $S E(3)$.
- As said insurance contracts are often of the "standard" - "take it or leave it" - form. This kind of an adhesion is rather rare within sales and purchases. The two co-Designers (Seller and Buyer) will as a rule come up with their own mutually different concepts of the contract.


### 6.2 Contract formation: Negotiation 1

### 6.2.1 Design-orders

The analysis of the collective design of the contractual system/process shown in Fig. 14 can proceed in the full analogy with the foregoing discussion for IN-order and CL-order, on the proviso that the two respective DESIGN-orders will now be called

## a SELL-order and BUY-order

As in the case of insurance, also here, the major mystery concerns the question how the two contracting parties search for and finally select each other. Similarly, we have to ask who is there to collect the orders, validate them and - finally calculate out the "voting result" - in the form of a verdict whether and in what form the system/process will be designed

As said, in this BOOK we will - to some degree - go around the core of the problem by our concept of a so-called Market-organizer, or "MO" for short.

### 6.2.2 An organized market place

It would require a separate book if we wanted to go into details on how Sellers and Buyers search for and select each other. Rather we will - as if - simply assume that they will use intermediary services of an agent whom we will refer to as a Market-organizer (" $\mathrm{MO}^{\prime}$ ) - implied or express. The obvious example of an express MO is a Stock-exchange where, in plain language:

- a SELL-order and BUY-order are "simultaneously put on the table in sealed envelopes". Hence none the two orders can be taken as a proposal/offer to which its Addressee will or will not submit his-her acceptance.
- the Exchange "opens the envelopes", validates the two DESIGN-orders and issues the verdict whether and how they will be "matched".

Every MO has its own matching algorithm with the help of which it can "satisfy" also counter-orders that do not demand perfectly identical content of the contract. Hence, by the algorithm's application MO in fact "simulates" the negotiation executed by a sequence of proposals and counter-proposals. By definition, then, every algorithm necessarily contains a specific assumption about voting powers of the Seller and Buyer.
As Market-participants ("MP" for short), they are informed about the "technology" of the intermediary service and use it on the proviso that it will save them all kinds of transaction costs associated to the search of a counter--party, formation of the contract and execution of its content.

DifferentMOs, in principle, compete as to which of them offers to the participants a more efficient and fair service, including the particular matching algorithm.

### 6.2.3 Horizontal vs. vertical relationship

In our IT-parlance the role of an MO established here for the sake of a Negotiation 2 is of the same kind as the role of a Manager nominated so as to support Negotiation 2. Hence, for simplicity we will personify MO, again, by the young lady named Manuela.

The main institutional difference between the two kinds of negotiations is depicted in Fig. 15 where we strictly differentiate between:

- the horizontal relationship between Richard and Mary in their roles of horizontal contracting parties (would-be contract parties),
- and two vertical contracts - one between Richard and Manuela-the MO and the other between Mary and Manuela-the MO.


The vertical contract is likely to be formed in the above discussed "take-it--or-leave-it" (standard form) mode through which Manuela supplies her pre-determined intermediary services in return for pre-determined "fees".

### 6.3 Task prescription: Negotiation 2

### 6.3.1 Nominees

The foregoing analysis of Richard-the Insurer's $\left.I N(1) \equiv[i n:\langle\overrightarrow{i n}\rangle) \rightarrow\left\langle\vec{T}_{i n}\right\rangle\right]$ and the prescription of his task to pay recovery/benefit will now be - essentially replicated for

$$
\left.\operatorname{PU}(1) \equiv[p u:\langle\overrightarrow{p u}\rangle) \rightarrow\left\langle\vec{T}_{p u}\right\rangle\right] \equiv\left[p u:\left\langle\overrightarrow{p u}_{\text {ben }}, \vec{p}_{\text {def }}, \overrightarrow{p u}_{\text {man }}\right\rangle \rightarrow\left\langle\vec{T}_{p u}\right\rangle\right.
$$

i.e. prescription of Mary's task to pay the purchase price of the delivered leather.

However, as already noted, less clear will be the Nominees. Leaving aside this unpleasant problem for the moment, let:
by $\overrightarrow{p u}_{\text {ben }}$ be nominated Richard into the role of a Beneficiary and the justifiability of his BEN-order will thus be designed by his own behavior/action $S E(2) \Rightarrow S E(3)$.
by $\overrightarrow{p u}_{\text {def }} \quad$ be nominated Mary into the role of a Defendant and the justifiability of her DEF-order be derived from her right to estop Richard on the grounds - for illustration - of the statute of limitation of his BEN-order.

Whereas the above Beneficiary and Defendant could be disclosed as "implied in fact" or in line with a common sense, the role of the Manager is intuitively by far less clear. At the same time the problem remains that there simply must be someone who will be the only one in charge to process the BEN- and DEF-orders, namely validate their correctness and justifiability.

### 6.3.2 Correctness

As said on the occasion of insurance, the notion of orders' correctness is being largely left aside our interest.

All that we can do at this point is to stress that what seems to be a "mere technical" requirement on the "carrier of the order", may have immense impact upon the voting power of the respective agent.

The simplest example can be provided by the "compulsory" language in which this or that formulary "must" be filled in - should the order be taken as correct. Still more restrictive may be the "compulsory" place where the order can be only submitted.

## 7. ASSOCIATED NOTES

### 7.1 Market for contractual shares

### 7.1.1 Content of the share

A contractual strategy such as $(S E(1), P U(1))$ in Fig. 14 can be seen as an undertaking into which Richard and Mary invest their "money and time". What each of them obtains can then be interpreted as a share in the undertaking. Let the share have a form of a written confirmation that a given person has become a party on this or that side of the contract.
Apparently, the share has a particular monetary value and as a commercial paper can become a "tradable" in the prospective market for contractual shares.
Leaving aside for simplicity the usual legal restrictions on this kind of transactions, let Mary-the Purchaser, decide to sell her share to a gentleman named Victor.

LS would say that what Victor is buying from Mary is:

- his "right" to demand from Richard a delivery of leather,
- his "obligation" to pay the respective price of leather.

In our IT-parlance we would express the same so that Victor buys from Mary two sets of conditions under which his and Richard's tasks, respectively, can be prescribed.

### 7.1.2 Developments of the share's value

Trivially, the monetary value of the above share, as of any other commodity indeed, depends on many factors each of which may change over time. Of our interest will now be the so-called depreciation or the extent into which the commodity had been used or even consumed until the moment of its sale and purchase.

Let us assume that
Mary-the Purchaser of leather turned Mary-the Seller of the share
decides to sell her share only after she received from Richard the leather. Put formally, the contract between Mary and Victor will be formed only after the original contractual system/process has undertaken the following development

$$
(S E(1), P U(1)) \Rightarrow(S E(2), P U(1)) \Rightarrow(S E(+3), P U(1))
$$

What is on sale is thus the residual portion of the contract. Hence, Victor buys from Mary only her "indebtness".

At this point we can only sketch how the above - relatively trivial - observations can become an analytical nightmare. It will be enough to assume that share offered for sale will represent the stage

$$
(S E(2), P U(1))
$$

and at play will be the "objective" probability that Richard may fail to duly deliver. In Comment 11 we will further enrich the problem by a "subjective" probability by which will be expressed Mary's and-or Victor's aversion to risk.

### 7.2 The role of a Manager

### 7.2.1 A collective Manager

The managerial role may be even performed jointly by the persons who are at the same time in the roles of a Beneficiary and Defendant. In the case like this, Richard and Mary will seek to establish an agreement about whether or not the leather has been duly delivered - i.e. delivered by, let us repeat, a prescribed Executor, in a prescribed kind and magnitude, to a prescribed place and in a prescribed time

Suppose then, that the agreement between the two co-Managers cannot be reached and hence the (ill-designed) Manager simply fails to deliver a well defined verdict.

This lack of a verdict is likely to raise discontent of with the Manager and this discontent is no different, in principle, than may be a discontent with a submitted MAN-order - regardless of how well- or ill-nominate the Manager may be.

As a result Richard and-or Mary may "bring a civil law-suit" and let the respective Court decide instead of the Manager.

Confusions caused by Designers who do not nominate Managers expressly are extensive and ever-present. At least, technically speaking, they can be remedied relatively easily, if the true nature of a Manager is fully understood. For example, a default rule can be accepted that - if not otherwise stated - it is always, e.g., a recipient of the delivery who has the last world as to its true parameters.

### 7.2.2 Managerial failure

Let us assume now that the vertical contracts between Manuela and Richard and Mary, respectively comprise not only Negotiation 1 but also - as shown in Fig. 16 - the services for Negotiation 2.

Somewhat prematurely we will already here make a brief note on what we shall in Comment 3 discuss in depth as a delict. Hopefully it will illuminate MO's role if we differentiate two kinds of delict:

- within the vertical contract Manuela's wrongful delivery of the verdict over Mary's task and
- within the horizontal contract Mary's wrongful delivery of the purchase price.

Apparently, the third possibility we could Mary's wrongful fulfillment of a wrongful verdict. For dramatic effect we can even think about the case where the wrongful fulfillment will be - accidently or intentionally - consistent with what should have been the rightful verdict.


### 7.2.3 Competence of the Court

If Richard brings his complaint about Mary's behavior to a Court, there are in principle two ways how the Court may resolve it:

- the Court will have to proceed as if it was the Manager proper, i.e., it will be obliged to strictly follow the rules agreed upon within the vertical relationship between Richard and the respective Manager,
- the Court will have the right to select its own method how to, e.g., measure magnitudes of leather.

For dramatic effect, let us assume that Richard and Manager have agreed that the magnitude of a factually delivered leather will be always obtained as an outcome of a given generator of random numbers.

### 7.3 Oligopoly and game

The position of Mary-the Purchaser in the work-flow chart in Fig. 14 suggests that she can be taken as a so-called Follower, namely in the sense of the so-called Stackelberg oligopoly or a game.
Put in the respective terminology, Mary's choice depends on what Richard will do. In a somewhat more sophisticated language, it could be said that Mary behaves according to her specific Stackelberg reaction function.

The Stackelberg concept brings forward the question about the level of information - uncertainty - that Richard and Mary may have about each other's - present and future - behavior/action.

Later, e.g. in Fig. 53, we shall generalize the problem so that some data/facts about Mary's environment cannot be available at the time of the task prescription and hence Manuela has to derive her verdict from only somehow conceptualized representative or expected state.
Hence, what Manuela substitutes into the mapping $\left.P U(1) \equiv[p u:\langle\overrightarrow{p u}\rangle) \rightarrow\left\langle\vec{T}_{p u}\right\rangle\right]$ is not $S E(+3)$ but only her belief $S E(3)^{\text {REP }}$ about whether and how Richard has completed his task.

As a result, Mary will not substitute $S E(+3)$ into her reaction function but only $S E(3)^{\text {REP. In words, Mary will not react to Richard's genuine but "only" expected }}$ behavior/action - essentially as it is established by the text-book concept of a Cournot duopoly.

Correspondingly we will also later address a problem of Manuela's "accountability" for what is ex post determined as an incorrectly prescribed PU(2).

## COMMENT 2.

## Multi-unit strategies; the case of a JOIN

Using the classification of Fig. 8 we shall begin with the two-unit pattern (f), continue will the JOIN-type three-unit two-phase system/process (b) and, finally, "fall down" to the simplest pattern (a).

All three patterns will be analyzed within the social context of a production, namely a production of shoes. The analysis will differ from the text-book economics mainly in the following ways:

- the shoemaking Firm will be primarily taken as a multi-unit system/process, including the sequence of operations in the sense of the work-flow chart in Fig. 7,
- internal vs. external events/conditions will be consistently differentiated as concerns their relevance for the Firm's overall behavior/action,
- internal and external events affecting a given operational unit will be allowed to occur in different times,
- some of the variables used to describe the Firm will be of a rather non-standard character.

Given the above major concerns, we, like the standard ET, fully ignore the roles of all three Nominees.

## 8. TWO-UNIT TWO-PHASE PRODUCTION

### 8.1 Introduction

### 8.1.1 A sole internal Supplier

In Fig. 9 all that Mary-the Manufacture $Q(1)$ needed was labor. Now we shall modify her "one dimensional" internal input so that it will be capital represented by leather.


### 8.1.2 Simplifications and notations

Within the two-unit two-phase shown in Fig. 17 we will confine to Phase 2, where, for the sake of the analysis and notational convenience, Mary-the Manufacturer $\left.Q(1) \equiv[q:\langle\vec{q}\rangle) \rightarrow\left\langle\vec{T}_{q}\right\rangle\right]$ is concretized as

$$
Q(1) \equiv[q:\langle K, d ; \gamma\rangle \rightarrow\langle Q\rangle]
$$

where:
Mary's task $\vec{T}_{q} \quad$ has been simplified into a scalar $Q$ representing the only endogenous variable - the magnitude of shoes.

Mary's environment is assumed to be $\langle\vec{q}\rangle=\langle K, \gamma, d\rangle$, where:
$K$ and $d$ are parameters of an internal event, designed by the magnitude $K$ of leather delivered by the respective department of the Firm and $d$ is the time of the delivery,
$\gamma \quad$ is an external event defined as a current rate of competiveness (denominated in percentage points).

As already suggested in Fig. 9, the variable $\gamma$ can represent Richard-the Interventionist whose behavior/action may be the factual cause of the changes in prices $\vec{p}$ or even efficiency $a_{\mathrm{f}}$.

### 8.1.3 Non-standard environment

To illustrate the extent of such simplification, Mary is not only assumed to need no labor but, e.g., also prices are taken as irrelevant. However, apart from kinetics of production, our major interest will rest in the following:

- time $d$ of delivery will be taken as relevant,
- the two-unit two-phase production from (f) of Fig. 8 will be confronted with the formally identical morphology (e) of a sales contract.


### 8.2 Contours of prescribed output

### 8.2.1 Conceivable states and domains

For obvious reasons we will confine to the graphical representation of the mapping $Q(1) \equiv[q:\langle K, \gamma, d\rangle) \rightarrow\langle Q\rangle]$ - mainly in the form of the contours of the outcome $Q$.

Invoking our distinction between a technological vs. behavioral IF-THEN rule, we should firstly stress that the outcome $Q$ is not what the Firm can (technologically) produce but what it has to deliver given her design and the state of its environment - leaving aside the trivial condition that the respective Beneficiary will initiate the manufacturing by his-her BEN-order.

In what follows we shall - so far mostly for graphical convenience - enrich the text-book analysis by considering "technical" conceivability of the input variables of the respective mapping (IF-THEN rule). By " $\Delta$ ", in the particular form of $\Delta K, \Delta \gamma$ and $\Delta d$, we will thus denote intervals of conceivable values, i.e. values that may occur in technical sense. Only in these intervals can be found states that are feasible in the sense that the respective mapping (IF-THEN rule) is defined - exists.

### 8.2.2 Magnitude vs. time of delivery

In Fig. 18 are depicted contours of the mapping

$$
Q(1) \equiv\left[q:\left\langle K, \gamma^{+}, d\right\rangle \rightarrow\langle Q\rangle\right]
$$

on the proviso that variable $\gamma$ is held constant $\gamma=\gamma^{+}$.
magnitude $K$

The red arrow in Fig. 19 indicates, that John has designed, for the case of illustration, Mary so that - at a given level of $K$ - the prescribed magnitude of shoes will decrease with the "delay" in the delivery of leather.

### 8.2.3 Time of delivery vs. competitiveness

Analogously, in Fig. 19

$$
Q(1) \equiv\left[q:\left\langle K^{+}, \gamma, d\right\rangle \rightarrow\langle Q\rangle\right]
$$

represents the case when the variables $K$ is held constant.


The red arrow now indicates, that John, for illustration, has designed Mary so that for a given time of delivery, the bigger is competition the smaller will be the prescribed magnitude of shoes.

### 8.2.4 Magnitude vs. competiveness

Assuming $d=d^{+}, \tau=\tau^{+}$, the mapping concerned will be

$$
Q(1) \equiv\left[q:\left\langle K, \gamma, d^{+}, \tau^{+}\right\rangle \rightarrow\langle Q\rangle\right]
$$

and its contours can take the form shown in Fig. 20.


Apparently, at a given level of $K$, the output of shoes decreases with the rate of competitiveness.

### 8.3 Rationing vs. outsourcing

### 8.3.1 Ex ante pre-determined Supplier

Unless we say otherwise the design

$$
Q(1) \equiv[q:\langle K, \gamma, d\rangle \rightarrow\langle Q\rangle]
$$

must be interpreted so that John-the Designer in fact:

- prohibits Mary from processing other leather than that delivered internally by the "monopolistic" internal Charles-the Supplier,
- makes Mary process all the leather delivered to her by the "monopolistic" internal Charles-the Supplier.


### 8.3.2 Contractual outsourcing

As said we will compare:
Mary-the Manufacturer $Q(1) \equiv[q:\langle K, \gamma, d\rangle \rightarrow\langle Q\rangle]$ from (f) of Fig. 8 designed by Iohn-the Designer and
Mary-the Purchaser $\operatorname{PU}(1) \equiv\left[p u:\langle\overrightarrow{p u}\rangle \rightarrow\left\langle\vec{T}_{p u}\right\rangle\right] \quad$ from Fig. 14 or (e) of Fig. 8 designed by the collective contractual RM-the Designer

Apparently, one interpretation of Mary's "interest" in the sales contract discussed in Fig. 14 will be that through the contract with Richard-the Seller she will seek to replace the internal Supplier, e.g. should he fail to deliver - as further discussed in Fig. 27.

### 8.4 Whose optimum?

Recall that the prescribed magnitude $Q(2)$ is, by definition, calculated out by Manuela-the Manger by substituting the respective data into the above mapping $Q(1) \equiv[q:\langle K, \gamma, d\rangle \rightarrow\langle Q\rangle]$.

Let us stress that the variable $Q$ on the right hand side of the mapping represents values of the optimal output, subject to a given state of Mary's environment. Hence the output can be often seen as a solution to the respective maximization problem $\max _{M}$ designed by John as a part of the mapping $[q:\langle K, \gamma, d\rangle \rightarrow\langle Q\rangle]$.
It will be repeatedly stressed that confusions often arise by not recognizing that any such $\max J_{M}$ represents preferences of John rather than Mary's, leave alone Manuela's.

## 9. THREE-UNIT TWO-PHASE PRODUCTION; THE CASE OF A JOIN

### 9.1 Introduction

Let Phase 2 be designed by Mary-the Manufacturer in the form of

$$
Q(1) \equiv\left[q:\left\langle K, d_{K} ; L, d_{L}\right\rangle \rightarrow\langle Q\rangle\right]
$$

where external events/conditions of manufacturing are neglected for simplicity, and the two pairs

$$
K, d_{K} \text { and } L, d_{L}
$$

of internal events representing magnitudes and times of deliveries of the two production factors.

It is noteworthy that deliveries of $K$ and $L$ are associated to different times $d_{K}$ and $d_{L}$. The actual state of Mary's environment will thus be given by two events each of which - in general - may occur at different times $d_{K} \neq d_{L}$.


Fig. 21

### 9.2 Time vs. time

For concreteness and in order to further illustrate the real-world complexity of conditions, let the times of the two deliveries $d_{K}$ and $d_{L}$ be - as shown in Fig. 22 - constrained so that:

- they both "must" fall into a given $\Delta d=\left(d^{\min }, d^{\max }\right)$ and
- the difference between the first and second delivery "must not" exceed the limit $\Delta d^{+}$.

To illustrate:

- the state of Mary's environment $\left(d_{K}^{0}, d_{L}^{0}\right)$ is feasible within respect to both time limits $-\Delta d$ and $\Delta d^{+}$.
- the inequality $d_{K}^{0}>d_{L}^{0}$ means that, George delivered the labor $L^{0}$ before Richard delivered leather $K^{0}$.


Fig. 22

As to output development, no contours are depicted in Fig. 22. By that we seek to illustrate the fact that John has designed Mary so that the magnitude $Q$ will be the same for all feasible time vectors $\vec{d}=\left(d_{K}, d_{L}\right)$, i.e. for all $\left(d_{K}, d_{L}\right) \in E_{1}$.

### 9.3 Magnitude vs. magnitude

The fact that the grey areas in Fig. 22 and Fig. 23 are both compact, has the meaning that there are no exclusions to the condition that both deliveries must be completed - should Mary's task be prescribed.


The contours in Fig. 23 are, as said, the outcomes of Manuela-the Manager her solving of the ex ante designed optimization model - obtained prior to her prospective ex post regulatory interventions.

### 9.4 Two kinds of maximization "II"

### 9.4.1 Complexity of a design

As already noted, the mapping $Q(1) \equiv\left[q:\langle\vec{q}\rangle \rightarrow\left\langle\vec{T}_{q}\right\rangle\right]$ can be designed on various levels of complexity. The obviously simplest case is a constant mapping where Manuela will in fact only validate whether the state of Mary's environment is feasible. To whatever feasible state is then associated the same magnitude of production.

By far more complex case was shown in Fig. 6 where the mapping is expressed as a maximization problem. Hence, the role of Manuela rests in collecting the respective data/facts, substitute them into the maximization formula and calculate out the respective maximizer.
Invoking Fig. 7, the following multi-unit production strategy str ${ }^{*} / J$ in Fig. 24 involves OP-UNs each of which has been designed by John in a particular form of $\max J_{K}, \max J_{L}$ and $\max J_{Q^{\prime}}$, respectively. Again, it may be well to recall that all three maximizations are outcomes of John's choice, and hence are elements of an overall solution to the maximization problem $M A X_{J}$.

Summarizing, then:
On LEVEL ( $i$ ) John resolves $M A X_{J}$. Manuela resolves $\max _{K^{\prime}}, m a x J_{L}$ and $\max J_{Q}$ each of which represents interests and preferences of

John-the Designer rather than those of the three Designees - Richard, George or Mary, let alone Manuela-the Manager. every Designee is likely to become a Designer of his-her own strategy by solving his-her own $M A X_{R}, M A X_{G}$ and $M A X_{Q}$.


Fig. 24

### 9.4.2 Aggregate demand-supply function

In a highly intuitive and mostly confusing sense the triad $\max J_{Q^{\prime}}, m a x J_{K}$ and $\max J_{L}$ could be seen by some analysts as a des-aggregation of interests and preferences of John-the Designer.
Still more confusing can then be any attempt to aggregate the triad into a single "composite function" denoted in Fig. 24 as an aggregate demand-supply function

$$
K L Q(1) \equiv\left[d s:\langle\overrightarrow{k l q}\rangle \rightarrow\left\langle\vec{T}_{k l q}\right\rangle\right]
$$

Recall that in elemental text-books $K L Q(1)$ is simplified into

$$
\left[d s:\left\langle p_{\mathrm{K}}, p_{\mathrm{L}} p_{\mathrm{Q}}, a_{\mathrm{f}}\right\rangle \rightarrow\left\langle K^{*}, L^{*}, Q^{*}\right\rangle\right]
$$

denoted in PART I of this BOOK as $d \boldsymbol{d} 1$. Put in IT-parlance any such mapping in facts seeks to "contract" a multi-unit multi-phase Firm into one meta-firm who instantaneously responds to the existing state of its meta-environment.

In any case it seems to the author of this BOOK that even if any such contraction could make sense, there is no reason why the interests and preferences represented by $K L Q(1) \equiv\left[d s:\langle\overrightarrow{k l q}\rangle \rightarrow\left\langle\vec{T}_{k l q}\right\rangle\right]$ should have anything in common with the preferences of John-the Designer be it those embodied in $M A X_{J}$, or those des-aggregated into the triad $\max J_{Q}, \max J_{K}$ and $\max J_{L}$.

## 10. ONE-UNIT PRODUCTION; OUTSOURCING

### 10.1 One-unit one-phase production

In Fig. 25 we in fact return to the one-unit pattern (a) shown in Fig. 8. Hence, Mary-the Manufacturer has no internal Supplier. As a result she has to procure leather from outside.

Yet, for the sake of the analysis we will assume that John-the Designer will ex ante regulate Mary so that only Richard and-or George can become her external Suppliers of leather.


### 10.2 Rivalry between external Suppliers

The following examples are to illustrate the almost infinite variety of further specifications of the system/process in Fig. 25.

The first pattern to illustrate the variety will assume that Mary is designed so that she has to:

- wait for a given time period $\Delta d$ until both deliveries arrive and only then,
- determine which of them - if not both - will be utilized for shoe--manufacturing.

Another illustrative example is in Fig. 26, where:

- the supply $K(3)_{\mathrm{CH}}^{0}$ from Charles is bigger and earlier than the supply $K(3)_{\mathrm{G}}^{0}$ from George,
- each of the two deliveries satisfy the Mary's condition - both deliveries fall into the green area in Fig. 26.


Thirdly, we may think of the sum $\left(K_{\mathrm{CH}}^{0}+K_{\mathrm{G}}^{0}\right)$ of the two deliveries and, e.g., consider Mary's re-actions to the case when one or both of the deliveries, by themselves, do not satisfy the condition designed.

### 10.3 External substitution for an internal Supplier

### 10.3.1 JOIN

In Fig. 27 we return to the pattern (b) of Fig. 8, i.e. the JOIN-type three-unit two-phase system/process depicted in detail in Fig. 7. Then, we will assume that John-the Designer will design JOIN in the form of a logic gate "AND", i.e. in the form

$$
K(+3) \text { AND } L(+3)
$$

by which he requires that two deliveries must be not only completed but also fulfilled. In word, leather and labor must be delivered duly as a proper tender.

Put differently, the final manufacturing $Q(1)$ cannot be prescribed until both Suppliers fulfill their tasks.

However, as already noted, whatever stage of a whatever Supplier can be designed as an internal event/condition. To illustrate, let the gate JOIN be designed as

$$
K(2) \text { AND } L(+3)
$$

with the meaning that John-the Designer has designed Mary so that her manufacturing must be prescribed despite the fact that the leather is not available yet. It is enough that the leather is "on the way" - in the stage $K(2)$ prescribed.

### 10.3.2 Failing Supplier

Apparently, Charles-the Supplier may always breach his $K(2)$ after Mary's task has been prescribe with the outcome

$$
[K(-3), Q(2)] \Rightarrow[K(-3), Q(-3)]
$$

The rationale for John's risky design may be that Charles has been always reliable and $K(2)$ can thus be conceived of as if it is actually fulfilled.

Section (a) of the following Fig. 27 essentially replicates the system/process from Fig. 14 designed jointly by Richard and Mary with the aim to sell and purchase leather. The only difference is in that the stage $K(-3)$ is an external event/condition under which Richard's task may be (justifiably) demanded.


Fig. 27
For dramatic effect we could imagine the arrangement when in the role of a "reliable" Seller of leather in section (a) is the same person as in the role of an "unreliable" Supplier of leather in section (b). of the above picture.

## COMMENT 3.

## Stages of a task; the case of a breach

Thesis $B$ introduced in PART I argues that a development of a task (operational unit) can be represented by a finite number of discrete stages. Of them the stages (1)-designed and (2)-prescribed, have been discussed in some more detail in Comment 1, namely as outcomes of Negotiation 1 and 2, respectively. Now, we will focus on
a stage (3)-completed
and its particular instance ( -3 )-breached.
As any stage indeed, also (-3) will become of our interest mainly when relevant for a development of at least one other operational unit in our interest. Put differently, a stage ( -3 )-breached will become a subject of our analysis when designed as an IF-component of some IF-THEN rule.

Apparently, in itself the term breach carries a "negative" connotation. The stage appears to involve a social behavior/action that should be seen as a "social evil" or a social wrong, i.e. somebody's wrongful conduct. Understandably, then, we would expect that the stage ( -3 ) will or should be "awarded" by a sanction due to a liability of the wrong-doer

Using this kind of LS-parlance the same normative intuition leads to the term delict.

As always, our IT-parlance will attempt to avoid any such value-loaded terminology. Hence we will stick to a wholly wertfrei concept of an inter--dependency between OP-UNs and deal with a stage ( -3 ) as with any other event - any other condition of a task.

Yet, again, with the aim to preserve the already built bridges towards LS also we will in this Comment 3 mostly confine to social contexts within which it will make sense to speak about sanctions. Of our interest will be an

$$
\text { operational unit } \operatorname{sn}(1)
$$

whose IF-component will include the stage (-3)-breached of some other operational unit and whose content will have a sanctionative character.

The analysis will then be organized according to who is the Designer of $\operatorname{sn}(1)$. Specifically, we shall differentiate whether $\operatorname{sn}(1)$ is designed by:

- a contractual RM-the Designer or
- a so-called Legislator.

In this sense also we, likewise LS, will discuss firstly a contractual delict and then the so-called tort and crime.

## 11. CONTRACTUAL DELICT AND SANCTION

Using the classification of Fig. 8, we will in this particular chapter deal with a combination of patterns (c) and (e). In words a collective RM-the Designer will design a SPLIT -type three-unit two-phase system/process.

### 11.1 Example

### 11.1.1 SPLIT

Fig. 28 should be self-explanatory as it is in fact no more than a simple expansion of the system/process from Fig. 14. It is obtained so that RM-the Designer adds one more branch to Phase 2 of the system/process by designing
operational unit $\operatorname{sn}(1)$
representing Richard in his role of an Offender.


### 11.1.2 Present vs. future agent

Methodologically, of our primal interest should become the fact that it is the first time in this BOOK that the same person is designed as two agents in the same system-process. In words, the same Richard who is a Seller before the logic gate SPLIT is designed - behind the gate - as an Offender $\operatorname{sn}(1)$.
Let us emphasize, then, that in Fig. 28 - by design - Richard may inter-act with himself.

Unfortunately, we will have to leave this prominent topic for later. It will be no sooner than in Comments 8 and 9 that our preparatory work will enable us to explain in what sense the present Richard may affects behavior/action of the future Richard.

### 11.1.3 Offender

We can read from Fig. 28 that the operational unit $s n(1)$ can be prescribed only as a sanction for Richard's failure to satisfy the internal condition/event $S E(+3)$ of Mary's payment. As a result:

- prescriptions of the OP-UNs behind the gate are mutually exclusive due to the mutual exclusivity of their interior conditions $S E(+3)$ vs. $S E(-3)$,
- even though $S E(3)$ occurs, none of the OP-UNs behind the gate need be prescribed depending on the respective external events/conditions.


### 11.1.4 Multidimensionality of a breach

Only for completion of our debate we will note that the already discussed multidimensionality of a task's design brings forward multiple ways how the task can be breached. To illustrate:

- Richard delivers leather two minutes too early, at 04.28 p.m. instead of 04.30 ,
- the Executor (Miss Weak) uses a lorry of Mr. Black instead of the prescribed lorry of Mr. White,
- Richard has not even attempted to fulfill; no Executor showed up at all, or, put equivalently, Richard delivered "peace and quiet" instead of the prescribed leather,
- Richard delivered, instead of leather, an aggression and malice; he showed up and physically and verbally attacked Mary-the Purchaser.

In our dogmatic IT-parlance all these inconsistencies $S E(3) \neq S E(2)$ will be taken as a a behavior/action that is completely irrelevant with respect to the purchase price that Richard may demand - as irrelevant as yesterday's weather in Rhodes, Greece.

### 11.1.5 Legal norm

As said, notation $\operatorname{sn}(1)$ suggests that the breach $S E(-3)$ is followed by an operational unit that - from the point of view of RM-the Designer - often has a sanctionative character. This kind of an interpretation corresponds to what has been said about the classic three-unit structure of a legal norm

## hypothesis, disposition and sanction

However, in our IT-parlance, as explained, the notion of a norm or business rule has been established so as to consist in only two elements, namely the hypothesis (IF) and the disposition (THEN). Our concept of an sanctionative relationship thus consists in two inter-dependent OP-UN within which the sanction is constituted by an IF-THEN rule (a "norm or business rule") of its own.

Let us also note that in the LS-parlance, another value judgment, namely

## lex imperfecta

is imposed upon an operational unit to which a respective sanctionative operational unit is not associated. If taken seriously both ENDs $P U(1)$ and $\operatorname{sn}(1)$ of the system/process in Fig. 28 would be lex imperfecta.

### 11.1.6 Nominees

Focusing on the "negative" branch of Phase 2, the environment of Richard-the Offender will be in its SP-representation put as

$$
\overrightarrow{s n}=\left(\overrightarrow{s n}_{\text {ben }} \overrightarrow{s n}_{\text {def } ;} \overrightarrow{s n}_{\text {man }}\right)
$$

Also in the case of an Offender the real-life designs are often unclear about the respective Nominees. Leaving aside this unpleasant problem for the moment, let:
by $\left\langle\overrightarrow{s n}_{\text {ben }}\right\rangle \quad$ be nominated Mary into the role of a Beneficiary as she is the "victim", "aggrieved party",
by $\left\langle\overrightarrow{s n}_{\text {def }}\right\rangle \quad$ be nominate Richard-the Offender as a Defendant.
Again, it remains somewhat ambiguous how well is by $\left\langle\overrightarrow{s n}_{\text {man }}\right\rangle$ nominated a Manager who only is entitled to resolve highly probable "discrepancy" between BEN- ad DEF-orders, their correctness and justifiability.

### 11.2 Manager

Using the specific social context of a sanction, the following notes may further clarify some of our foregoing general observations.

### 11.2.1 Collective Manager

The managerial role is often implied in fact so that it is performed jointly by Richard and Mary. As a result, also here, the probability is rather high that the two co-Managers will not reach an agreement as to the two counter-orders validation.

Between Richard and Mary should thus be differentiated:

- their dispute in their roles of a Beneficiary and Defendant, i.e. the dispute manifested by the above mentioned discrepancy between BEN- ad DEF--order,
- their dispute in their roles of co-Managers.

As a result, both Richard and Mary, may decide, individually or collectively, to bring a civil law-suit and let the respective Court replace the incapable Manager.

### 11.2.2 Decision "not to claim"

By definition, neither Mary nor Richard are, in their roles of a Beneficiary and Defendant, in the position to state that their orders are justified. Hence, if e.g. Mary decides to submit her BEN-order all that she may only have is her "suspicion" that Richard failed to duly deliver. Of our interest then could become the following two situations:

- Mary, who has no idea whatsoever about Richard's behavior, submits a "testing" BEN-order for the sake that the respective Manager's verdict will be "in her favor",
- Mary is in addition to her role of a Beneficiary also the respective "full scale" Manager entitled to process her BEN-order.

The arrangement when Mary is both a Beneficiary and Manager brings to fore again the rather peculiar case when Mary decides not to submit her BEN-order even though can be perfectly certain about its justifiability.

Apparently, the foregoing analysis will - mutatis mutandis - hold for the arrangement where it will be Richard who will be in both roles - of a Defendant and Manager.

### 11.2.3 Necessary condition

As said, a SPLIT states that the stages $S E(+3)$ and $S E(-3)$ may open "gates" to two alternative branches of Phase 2.

At the same time, the graphical representation may be misleading if read so that one of the two variants must always occur. Apparently, none of the two variants need be prescribed even though $S E(+3)$ and $S E(-3)$ - trivially - constitute a set of exhaustive and mutually exclusive "states of the world". The red arrows and the two JOINs in Fig. 28 clearly show that each of the two stages $S E(+3)$ and $S E(-3)$ must be aggregated with a particular external events whose particular
forms may modify or even nullify effect of the stages. The environment of Richard-the Offender ${ }^{\text {gsn }}$ may include requirements that, e.g., the children are alive, remain under Mary's custody and are taken care of properly.
Hence, the two stages $S E(+3)$ and $S E(-3)$ represent only one element of the IF-components of $P U(1)$ and $\operatorname{sn}(1)$ - as conditions they are only necessary but not sufficient.

### 11.3 Soft (pseudo-)conditions

### 11.3.1 Excuses and cures I

For concreteness, let us assume that $S E(2)$ has been prescribed so that Miss Weak (as an Executor) is to hire a cargo train and deliver 115 kg of leather to Mary's warehouse in London, on June $15^{\text {th }} 2015$, at 04.30 p.m. sharp.
The dogmatic position of our IT-parlance will then be that none of the above prescribed parameters of the delivery can be "excused" should the prescription be taken seriously. Put differently, the IT-parlance will not allow for "soft conditions" and whenever a condition will appear to be "soft" our analytical responsibility will make us search for a genuine condition that will be "hard" by definition.
As said, - in compliance of law and the contract - whenever the stage $\operatorname{SE}(-3)$ occurs, Richard cannot be paid the purchase price due to the mere fact that one of the necessary conditions of his BEN-order's justifiability is not satisfied. Put still more accurately, Mary-the Purchaser is prohibited by law and the contract to pay the purchase price. If she does pay, the payment will become a so-called unjust enrichment (condictio indebiti).

Hence, if the above "imperfect" delivery can be "excused", then the prescribed time cannot be, by definition, 04.30 p.m. sharp, but, e.g., the time interval

$$
\left\langle t^{\min }, t^{\text {max }}\right\rangle=[04.30 \pm 7 \mathrm{~min}]
$$

Only then the factual time 04.28 p.m. of delivery need not necessarily lead to $S E(-3)$ and can be seen as $S E(+3)$.
However, the underlying problem of "severity" cannot be avoided by principle. Even the latter more "tolerant" OP-UN will come to the point where it must be taken as breached - e.g. when the time of delivery is "only" 04.38.

### 11.3.2 Variant vs. unique prescriptions

As said, any design of an OP-UN represents a set of prospective prescriptions each of which is associated to a particular state of the respective environment. To illustrate, an infinite number of purchase prices from the interval $\left\langle p^{\text {min }}, \mathrm{p}^{\text {max }}\right\rangle$ corresponds to the infinite number of "feasible" times $\left\langle t^{\min }, t^{\text {max }}\right\rangle$.

Contrariwise, $P U(1)$ could be also designed so that:

- the purchase price is always USD 10 000, regardless of the particular form of the fulfillment $S E(+3)$ and
- the contractual penalty is always USD 100 000, regardless of the particular breach $S E(-3)$.


### 11.3.3 Soft vs. hard prescription

As said, with respect to the prescription, it makes no difference whether Richard delivers only two minutes late or fully forgets to deliver. Let us repeat the thesis that it is by definition that even the most "negligible" inconsistency $S E(3) \neq S E(2)$ necessarily leads to the stage $S E(-3)$

$$
\text { should the stage } S E(2) \text { be taken seriously }
$$

It has been also noted that the particular form of the breach matters as it can be seen as a bigger or smaller distance between $S E(3)$ and $S E(2)$. And, depending on this multidimensional distance some other tasks, including those "implied in law", may be prescribed. The following two extreme cases may be of interest:

- $s n(1)$ may be - as if - equivalent to $S E(1)$. The immensely confusingly consequence of $S E(-3)$ can then appear to be that the original task has not extinct due to Richard's failure and that he thus still must fulfill $S E(2)$ "as soon as possible".
- On the other side of the scale of severity will be cases when the behavior/ /action $S E(2) \Rightarrow S E(-3)$ will be seen not only as a contractual wrong but also as a tort or even crime and these three kinds of a wrongful conduct will be sanctioned separately.


### 11.4 Discrete set of states

### 11.4.1 Example

Fig. 28 is somewhat generalized into Fig. 29 where a finite number of mutually different $m$ conceivable completions $S E(3)$ of Richard's $S E(2)$ are depicted. Let, for concreteness and ease of explication, $m=5$, of which, as shown in Fig. 29, two states will represent fulfillment and the remaining three will be a breach. For graphical convenience external events/conditions are omitted.

Fig. 29 seeks to differentiate two kinds of splits one established - as before according to the environment of Mary and Richard-the Offender respectively, the other (the so-called pseudo-split) according to the final outcomes of the system/process as a whole. This kind of a differentiation is in detail discussed later in our Comment 8, where to Fig. 29 corresponds, e.g., Fig. 93.


Fig. 29

### 11.4.2 Foreseen risk

As said, the notion of uncertainty will be continuously mentioned throughout the BOOK. The reason why John designed the system/process in Fig. 29 may rest exactly in his uncertainty about whether and how will Richard fulfill his task.

For the sake of the later discussion of, e.g. the five states in Fig. 53 of Comment 5 and in particular the overall analysis of Comment 11, we will denote already here as

$$
\pi^{i}, i=1,2, \ldots, 5
$$

"objective" probabilities of the above internal events

$$
S E(3)^{1}, S E(3)^{2}, \ldots, S E(3)^{5}
$$

Invoking our brief notes on the logic gates, we should also add that on the proviso that the five events are exhaustive and mutually exclusive. Put formally

$$
\sum_{i=1}^{5} \pi^{i}=1
$$

states that one and only one of the events must occur and hence the sum of the five probabilities is equal to one.

### 11.5 Pseudo-sanction

In this section we will want to show that the concept of a sanction is often mistakenly associated to the mere fact that there may be different instances of Richard's fulfillment, e.g. different forms of his rightful conduct (behavior/action)

$$
(S E(2)) \Rightarrow(S E(+3))
$$

and that to every such instance, in general, a different $P U(2)$ of Mary's task may be prescribed.

In order to illustrate, we will assume in Fig. 30 for simplicity that the only reason why a breach $S E(-3)$ may occur is that Richard fails to deliver within the prescribed time interval, e.g.

$$
\left\langle t^{\min }, t^{\max }\right\rangle=(\text { June } 15, \text { June } 18)
$$



To illustrate the confusion concerned, the function $p=p(t)$ is designed by RM-the Designer so as to have a - as if - sanctionative content

$$
\left(t^{2}>t^{1}\right) \Rightarrow\left(\left(p^{2}<p^{1}\right)\right)
$$

In words, the function states that the later Richard delivers the leather, the smaller will be his revenue - the smaller will be the unit price $p$ that Mary will have to pay for the delivery.

Our main point apparently is that because

$$
t^{2}, t^{1} \in\left\langle t^{\min }, t^{\max }\right\rangle
$$

both states $S(+3)^{1}$ and $S(+3)^{2}$ represent a perfect tender and hence no sanction, in the above established sense, can be prescribed.

### 11.6 LS-parlance

Our notes on an excuse and cure have suggested that LS jargon is exceptionally rich in the context of a breach, delict and sanction. Hence it may be of value to illustrate the language by a few examples - mostly based on our referential text-books or, simply taken from Wikipedia.

### 11.6.1 Repudiation and revocation

A nice example of how lawyers define Richard's stage $S E(-3)$ is that he: ... fails to make delivery or repudiates or Mary rightfully rejects or justifiably revokes acceptance of the leather.

Alternatively, a breach would be a state: ... in which the seller has repudiated, or has otherwise wrongfully failed to deliver, or has delivered nonconforming goods which the buyer has properly refused to keep.
The definitions certainly need further qualification of seller's repudiation and buyer's rejection and revocation. As to this:

- From among many kinds of repudiations of our linguistic interest could be mainly that when Richard simply announces his intention not to perform.
- Similarly interesting could be attempts to define - in natural language Mary's acceptance of goods, rejection of acceptance and revocation of the acceptance, e.g. her refusal to keep delivered leather and her notification to Richard.


### 11.6.2 Excuse, waiver and estoppel

Somewhat surprisingly the terms excuse, waiver or estoppel are often traditionally associated to a modification of a contract, i.e. the phenomenon consistently treated further in Comment 8 under the category of an OP-UN's transformation, its validity and effectiveness.

In contrast, we offer a different interpretation based on the already discussed Nominee's choice not to submit his-her order, namely the choice of:

- of a Beneficiary not to submit a BEN-order, i.e. not to demand fulfillment of the respective task,
- of a Defendant not to submit a DEF-order, e.g., not to raise the statute of limitation against Beneficiary's fully justifiable BEN-order.

Needless to stress again, that neither Beneficiary nor Defendant are - by definition - entitled to determine justifiability of their orders. The question, posed from time to time in this BOOK, thus is to what extent a Nominee can be taken accountable for not submitting an "apparently justifiable order".

### 11.6.3 Lack of action; omission

Invoking our notion of a distance between the breach $S E(-3)$ and prescription $S E(2)$, it is certainly tempting to differentiate breaches according to whether the distance is big or small, where the smaller is the distance the less "ugly" will be the breach.

Intuitively, we should differentiate a wrongful delivery of "some leather" from a "zero delivery" (lack of action) due to the fact that Richard simply forgot, or intentionally repudiated.
In our dogmatic IT-parlance:

- Once $S E(2)$ is prescribed, Richard will be always taken as an agent who will perform some non-empty behavior/action $S E(2) \Rightarrow S E(3)$ or deliver "something, be it his comfort before TV, aggression, malice etc.
- If the delivery "something" is not consistent with $S E(2)$, the stage $S E(-3)$ will occur regardless of the distance between $S E(-3)$ and $S E(2)$.


### 11.6.4 Plaintiff's contribution

As said, the internal event/condition $S E(-3)$ may be combined with a variety of external events/conditions of the sanction $\operatorname{sn}(1)$. One of them can be based on the requirement that Mary herself must not contribute to the breach $S E(-3)$.
Returning to the above example with the prescribed time of delivery June $15^{\text {th }} 2015,04.30$ p.m. - we may assume that the leather proved to be in Mary's warehouse somewhat later at only 04.33 but the reason was that she could not find the proper keys.

Hence the so-called contributory behavior/action of the aggrieved agent is in our IT-parlance nothing more or less than another example of the fact that the inequality $S E(3) \neq S E(2)$ is only a necessary condition under which a sanction may be prescribed. In the same IT-parlance we will also stress that Mary without the proper keys is an external event, however "internal" she may appear - given her role of a contract party.

Recall that the roles of a Plaintiff is to us of the same kind as that of a Beneficiary and that the latter role need not be performed by the person who is designed as a Designee. The question then arises how to compare the above Plaintiff's contribution with a prospective contribution of a Designee.

### 11.6.5 Compensatory damages

As a step toward the following chapter where Legislator-the Designer will be discussed we may already here assume that in addition to $s n(1)$ a task

$$
\operatorname{gsn}(1)
$$

will be designed within a civil law, e.g., in the form of the so-called compensatory damages.

Here and in the following chapter we will simply take for granted that $g s n(1)$ can be seen as a sanction supplied into the contract as a so-called gap-filler simultaneously with the contractual design of other operational units. Hence, once $\operatorname{gsn}(1)$ is accepted, the system/process in Fig. 28 will be automatically expanded into a four-unit two-phase

$$
[S E(1), P U(1), \operatorname{sn}(1), g s n(1)]
$$

where of our interest should be:

- how well informed can be Richard and Mary the existence of $\operatorname{gsn}(1)$, given that the sanction is "only" implied in law,
- how mutually different are the two sanctions $s n(1)$ and supplied $g s n(1)$, including mutually different Nominees,
- to what extent may the respective Beneficiary demand prescription of only one of the two $s n(1)$ and $g s n(1)$ or both of them.


## 12. TORT AND CRIME

The social contexts of a tort and crime will return us to the simplest one-unit one phase system/process (a) of the classification in Fig. 8. However, instead of John, the Designer will now have the empirical meaning of a Legislator.

### 12.1 Legislator as a pseudo-Designer

As a preparatory step towards the genuine subject of this Chapter we will firstly explain how a Legislator may, confusingly enough, become a pseudo-Designer of an implied in law contractual sanction.

### 12.1.1 Contractual gap filler

Let the section (a) of Fig. 31 have the meaning that Richard and Mary - as a RM-the Designer - on the occasion of their divorce - jointly design a contractual three-unit two-phase system/process where:

FT(1) involves Father's Task to pay monthly alimony,
$D C(1)$ involves Mother's Duty of Care,
$L / \operatorname{sn}(1) \quad$ is a so-called gap filler, i.e. an implied in law sanctionative unit:

- whose particular wording is provided by a Legislator but,
- that is included into the contractual system/process as if designed by RM-the Designer.


Fig. 31

### 12.1.2 Comparison

A system/process in (a) of Fig. 31 is essentially of the same contractual nature as that in Fig. 28. Yet, a closer look at the two pictures leads to that unlike in Fig. 28,

- Phase 2 now consists in OP-UNs that are not mutually exclusive: Mary may be prescribed her task even if Richard fails to deliver; her task depends only upon how exactly Richard completes the task.
- Richard and Mary have now decided to leave it fully to the Legislator what will happen to Richard if he breaches his task.

As said, by definition, the sanction $L / s n(1)$ must be taken as if designed by Richard and Mary themselves. Put differently, L/sn(1) has the same "status" as $s n(1)$ in Fig. 28 - both sanctionative units are "ordinary elements" of the contractual system/process. In this sense Richard and Mary are genuine co-Designers of $L / \operatorname{sn}(1)$, whereas the Legislator is only a "ghost-writer" of the respective formula.

### 12.1.3 Associated notes

1) In LS-parlance: $s n(1)$ in Fig. 28 is a contractual sanction belonging to the body of a contract law, whereas $L / \operatorname{sn}(1)$ is a civil sanction constituted within a civil law.
2) By nature, $s n(1)$ and $L / s n(1)$ are mutually exclusive. The wording of the respective legislation may be that "unless the contracting parties state otherwise, they will be taken as if they agreed upon a sanction $L / \operatorname{sn}(1)$.
3) Richard is always designed into the role of an Offender either an Offender ${ }^{\text {sn }}$ in Fig. 28 or Offender ${ }^{\mathrm{L} / \mathrm{sn}}$ in (a) of Fig. 31.
4) A sanction of the kind of $L / \operatorname{sn}(1)$ was already noted in the context of a so-called compensatory damage.
5) Invoking the concept of production cycles discussed in Fig. 4, also $F T(1)$ in fact involves a set of "identical" operational units $F T(1)^{1}, F T(1)^{2}, \ldots$, each of which may be re-designed, at the start of every month.

### 12.2 Legislator as a genuine Designer

### 12.2.1 Introduction

As a Designer the Legislator will decide about sanctionative units presented:

- in (b1) of Fig. 31 as a tort sanction $\operatorname{trt}(1)$ and
- in (b2) of Fig. 31 as a crime sanction crm(1).

Social contexts of a tort and crime in fact return the analysis to the simplest pattern (a) of the classification in Fig. 8, on the proviso that the individual Designer John has the social standing of a Legislator. The respective one-unit one phase system/process then consists in only Richard-the Designee, in his respective roles of a tort or crime

$$
\text { Offender }{ }^{\text {trt }} \text { and Offender }{ }^{\text {crm }} \text {, respectively }
$$

Only as a concluding note we will add that the same wrongful conduct can then be "processed" and sanctioned separately and "awarded" by several separate sanctions. Contrariwise, put in LS-parlance "... a plaintiff will sometimes be able to state a cause of action on the basis of the same facts under either express tort liability or implied warranty liability".

### 12.2.2 Nomination vs. design of a Beneficiary

For the sake of the analysis we will focus on a Beneficiary in whose favor is designed Richard-the Offender from (b) of Fig. 31, namely Richard-the Criminal/ /Offender ${ }^{\text {crm }}$.

Given our methodological objectives Richard will be designed in the SP-representation or the mapping $\operatorname{crm}(1) \equiv\left[\operatorname{crm}:\left\langle\overrightarrow{c r m}_{\text {ben }}, \overrightarrow{c r m}_{\text {deff }} ; \overrightarrow{c r m}_{\text {man }}\right\rangle \rightarrow\right.$ $\left.\left\langle\vec{T}_{c r m}\right\rangle\right]$ where - to make the following debate somewhat less cumbersome - the mapping will be simplified into

$$
\operatorname{crm}(1) \equiv[\mathrm{crm}:\langle\overrightarrow{b e n}, \overrightarrow{d e f}, \overrightarrow{\mathrm{man}}\rangle \rightarrow\langle\vec{t}\rangle]
$$

Moreover, we shall use the opportunity of the social context and focus on one particular problem of a general nature, namely that the Beneficiary concerned can be:

- not only nominated by the sub-domain $\left\langle\overrightarrow{c r m}_{\text {ben }}\right\rangle$ but also
- designed as a separate operational unit (OP-UN) ben.

Recall that the domain $\langle\overrightarrow{b e n}, \overrightarrow{d e f ;} \overrightarrow{m a n}\rangle$ represent conditions under which Richard--the Criminal may be "sent to prison". Our major question will then be in whose favor is $\operatorname{crm}(1)$ designed, who will have the exclusive right to demand that Richard will spend a prescribed some time in prison.

The looked for role of a Beneficiary will be in the case of a crime performed again by the aggrieved party, namely the respective state represented, for concreteness, by a Prosecutor.
Hence, by the sub-domain $\langle\overrightarrow{b e n}\rangle$ are designed conditions under which the Prosecutor's demand will be justified, in particular what kind of Richard's behavior/action will suffice to satisfy the respective condition of justifiability. Let a lady named Priscilla personify the Beneficiary. In her role of a nominated prosecutor she will be referred to as priscilla. Summarizing, then, priscilla will personify the only agent with the right to demand prescription of $\mathrm{crm}(1)-$ demand that Richard-the Criminal will be sent to prison.

### 12.2.3 Design of a Prosecutor

Now we shall assume that apart from $\operatorname{crm}(1)$ one more operational unit $(\mathrm{OP}-\mathrm{UN})_{\text {ben }}$ is designed and that it will be again the lady named Priscilla who will - as PRISCILLA - personify it this second unit.

The reason for designing PRISCILLA will be that by design, Priscilla, will not only have a right to demand the prescription of $\operatorname{crm}(1)$, but - under specific conditions - also a task to do so, in a particular way.
Let $(\mathrm{OP}-\mathrm{UN})_{\text {ben }}$ be designed in the form $P R O(1) \equiv[P R O:\langle\overrightarrow{\mathrm{BEN}}, \overrightarrow{\mathrm{DEF}} ; \overrightarrow{\mathrm{MAN}}\rangle$ $\rightarrow\langle\vec{T}\rangle]$ and stress that $\operatorname{crm}(1)$ and $P R O(1)$ are in general designed by different Designers, e.g., different ministries of the government.


The arrangement in Fig. 32 can be summarized as follows:
On level 1 (represented in italic and lower-case characters):
by $\underset{\rightarrow r m}{\operatorname{crm}}(1)$ is designed Richard-the Criminal/Offender ${ }^{\text {crm }}$, by $\overrightarrow{b e n} \quad$ is nominated pricilla into the role of a beneficiary, i.e. the agent entitled to demand, by her ben-order that Richard be prescribed his sanctionative task.

On LEVEL 2 (represented by upper-case characters) is by $P R O(1)$ designed "the same" PRISCILLA by her task to submit the ben-order.

To keep the complexity of the arrangement under control, we will leave aside the three Nominees on LEVEL 2 and their BEN-, DEF- and MAN-ORDERs.

### 12.2.4 Levels vs. phases

There is another danger of becoming confused by the two-level diagram in Fig. 32. To disclose it, in (a) of Fig. 33 we depict a work-flow chart whose two phases state the following:

- firstly PRISCILLA is prescribed to launch her PROSECUTOR's behavior/ /action and
- only then may be Richard prescribed to perform his behavior/action.


Fig. 33
The confusion may rest in that the two OP-UNs in (a) of Fig. 33 do not constitute a system/process in the sense established in this BOOK. They constitute a "mere" set of OP-UNs namely because they are designed by two different Designers for two different purposes.

Hence the affinity between (a) of Fig. 33 and the two-unit two-phase system process in, e.g. Fig. 14 is purely formal and largely misleading. And hence, only (b) of Fig. 33 is correct; fulfillment $P R O(+3)$ of PRISCILLA's task is taken as an external event/condition whose specific kind may remind the reader of Richard--the Interventionist discussed in Fig. 9.

### 12.2.5 Anonymity and collectiveness of a Designee

In the context of life insurance discussed in Comment 1 we have assumed that a Beneficiary is nominated onymously - by his particular name Benjamin. However, a warning was issued that the contractual RM-the Designer may decide to nominate a Beneficiary anonymously and collectively, e.g., so that it will be "any child of Mary-the Client living as of the day of Mary's death".

In the case of a tort and crime, the Legislator-the Designer in fact, by definition, has no choice but to design the respective Designee as an anonymous and collective operational unit, e.g., by the IF-THEN rule according to which

IF: anybody unlawfully kills another human without justification and valid excuse,
THEN: the killer will be prescribed a task to "deliver" $x$ years in prison.

Also the nomination of a Beneficiary is necessarily anonymous and collective. To illustrate, the Beneficiary may be designed in a semi-anonymous form so that it will be who sill be prescribed to fulfill the role of a Prosecutor by BRITNEY-the super-Prosecutor. Apparently, BRITNEY's BEN-ORDER must be justified with respect to the kind, place and time of the would-be murder.

## COMMENT 4.

## Nominees; the case of a Manager

Throughout the BOOK we "complain" that Designers of real-world systems/ /processes by and large neglect the roles of Nominees.
However, there are exceptions: In Comment 1 the context of a life insurance let us clearly observe the roles a (third-party) Beneficiary and Assessor. In Comment 2 we could introduce our own construction of a so-called Market--organizer who - as a Manager - provided Market-participants with a floor where sales contracts were formed. In the preceding Comment 3 PRISCILLA--the PROSECUTOR was dealt with as a kind of Beneficiary.

Now we will rather extensively focus on the role a Manager. The Comment will be organized as follows:

- its first chapter will deal with a Manager of Richard-the Seller from the contractual two-unit two-phase system/process depicted in Fig. 14,
- then we shall focus on a Manager of Mary-the Manufacturer from the profit--making three-unit two-phase system/process in Fig. 7,
- the remaining chapters will be devoted to miscellaneous topics of a essentially - general nature.

Nevertheless, if the author should ever accept that there will exist readers who would progress as far as this point of the BOOK, the advice to them will be to only skim the following Comment 4 . Its apparent problem rests in that it is overly affected by the author's adventurous experience with his own design of a real-world Manager, namely the above mentioned Assessor.

Hence, it should be more than enough for a reader to glance down the text so as to get a reliable picture about the complexity of the inter-dependence of the horizontal and vertical relationships depicted already in Fig. 15 and Fig. 16.

## 13. MANAGEMENT OF A SALE AND PURCHASE

Invoking the general morphological classification from Fig. 8 this chapter will focus on its pattern (e).

### 13.1 Introduction

### 13.1.1 Negotiation 2

As explained, in the social context under study Manuela-the Manager should be differentiated according to whether she "referees":

Negotiation 1 between Richard and Mary in their roles of contracting parties seeking to become contract parties and
Negotiation 2 between Richard and Mary in their roles of already designed contract parties, namely the roles of a Creditor and Debtor.

For the sake of this analysis, we shall focus on Negotiation 2 through which a Beneficiary and Defendant jointly decide about whether and in what form a task under study will be prescribed. In other words, in what follows Manuela will be to issue a verdict over a dispute between a Creditor and Debtor.

For concreteness we will confine to Richard's task, i.e. to the transition $S E(1) \Rightarrow S E(2)$. For simplicity we will assume that in the role of the respective Beneficiary is Mary. Hence, there is no "third person" in whose $S E(1)$ would be designed. Mary and only Mary is thus entitled to enforce prescription of Richard's task to deliver the leather.

Similarly, we will assume that in the role of the respective Defendant is Richard.

### 13.1.2 Change of notation

For the sake of notational convenience, the notation used in Fig. 14 will now be changed as shown in Fig. 34.


Hence, a transition $S E(1) \Rightarrow S E(2)$ under study will now be - purely formally rewritten as $s e(1) \Rightarrow s e(2)$.

The reason for the change is that in the full analogy with Fig. 32 where we differentiated for the lady named Priscilla her roles of priscilla and PRISCILLA we will now for Manuela consider her in the roles:
manuela-the manager nominated by the sub-domain $\langle\overrightarrow{m a n}\rangle$ of the operational unit $\operatorname{crm}(1) \equiv[c r m:\langle\overrightarrow{b e n}, \overrightarrow{d e f} ; \overrightarrow{m a n}\rangle \rightarrow\langle\vec{t}\rangle]$ and
MANUELA-the MO where "MO" stands for a MARKET-ORGANIZER designed by a separate operational unit $(\mathrm{OP}-\mathrm{UN})_{\text {man }} \equiv M O(1) \equiv[M O:\langle\overrightarrow{B E N}, \overrightarrow{D E F} ; \overrightarrow{M A N}\rangle \rightarrow\langle\vec{T}\rangle]$
Like in the case of Priscilla, the reason for designing MANUELA-the MO is that by the design $M O(1)$, Manuela, will have to - under specific conditions resolve conflicts between Mary and Richard over Richard's delivery.


Apparently, Fig. 35 is of the same kind as earlier discussed Fig. 32. Only, now, our focus is on a different kind of a Nominee - the focus on a Beneficiary is no replaced by that on a Manager.

Let us summarize that Mary and Richard on level 1:

- are not only in the horizontal roles of a Purchaser and Seller, respectively,
- but also in their vertical roles of a beneficiary and defendant, respectively.

As in Fig. 32, we again leave aside the question about the Nominees on LEVEL 2.

### 13.2 Contractual Market-Organizer

### 13.2.1 Market-Participant

Using the terminology applied on stock-exchanges we will now conceive of Mary and Richard as Market-Participants ("MP"). Hence, apart from their Seller-Purchaser and Creditor-Debtor relationships we will consider each of them in his-her separate relationship with MANUELA-the MO.

For concreteness and simplicity, our focus will be on the relationship between
Mary-the MP and MANUELA-the MO

In this additional role of an MP, Mary will be entitled to demand that Manuela will process her ben-order by which she demands that Richard will deliver leather.

Trivially, then, Mary should know who is in charge to receive and validate her ben-order and match it with Richard's prospective def-order. In short, mary-the beneficiary should somehow learn whether and how exactly is designed the Market-organizer

$$
(\mathrm{OP}-\mathrm{UN})_{\operatorname{man}} \equiv M O(1)
$$

One way how Mary may obtain the information is by becoming a contractual co-Designer of , e.g. by entering into a managerial contract with Manuela.

### 13.2.2 Managerial contract

Using our IT-parlance, we will assume that Mary and Manuela will constitute a collective contractual

## MO/MP-the Designer

and jointly design a two-unit two-phase system/process represented by a workflow chart in Fig. 36.


Fig. 36
In plain language, Mary and Manuela will be assumed to form a contract according to which:
Manuela will have a task to "deliver" particular managerial services, e.g. those described in detail further in Fig. 38,
Mary will have a task to pay to Manuela the agreed upon price for the services.

### 13.2.3 Horizontal vs. vertical systems/processes

Recall that it is only our (intuitively well acceptable) simplification that Mary is nominated into the role of a Beneficiary vis a vis Richard. As a result of this particular nomination, then, the "same" Mary becomes "simultaneously" designed as two mutually different operational units

$$
p u(1) \text { and } M P(1)
$$

in two mutually different contractual systems/processes - one shown in Fig. 34 the other in Fig. 36. Apparently, they are of the same morphological pattern, i.e. in the form of a two-unit two-phase system/process shown in section (e) of Fig. 8.

Invoking the discussion of a stock-exchange in Fig. 15 and with the aim to differentiate the two systems/processes in which Mary is an operational unit:

- the "underlying" system/process shown in Fig. 34 will be characterized as horizontal and associated to level 1 in Fig. 35,
- the "adjacent" system/process shown in Fig. 36 will be characterized as vertical and associated to LEVEL 2 in Fig. 35.


### 13.2.4 Inter-dependent systems/processes

Fig. 37 is only a somewhat deeper corroboration of the arrangement in Fig. 33. The outcome of the VERTICAL communication between MARY and MANUELA is taken as an external event/condition on the horizontal level. Again, as in Fig. 33, a behavior/action of an external agent may remind us of Richard-the Interventionist discussed in Fig. 9.


### 13.2.5 Infinite recursion

The merciless logic of our argument makes us raise again the unavoidable fact that the vertical BEN-ORDER is as any other order and hence must be also somehow processed, namely validated. Hence, what is to be validated is a demand for validation. By definition, then, there must exist someone with an exclusive right to issue a verdict about BEN-ORDER's correctness and justifiability.

As already noted, the author of this BOOK believes that the only realistic method how to strangulate infinite chains of the above kind must be based on a selection of the critical level from which on:

- conditions of justifiability will be no longer permitted to be designed,
- conditions of correctness (such as the implied in law conditions of a mental competence, duress etc.) will be simply assumed to be satisfied, i.e. will not be factually validated.


### 13.3 Multi-order

### 13.3.1 Double-Beneficiary

As stressed it is only by our simplifying assumption that Mary is a Beneficiary vis a vis Richard. Similarly, in general, whoever can be - in principle - nominated (by Mary and Manuela) to be a Beneficiary within (OP-UN) man $\equiv M O(1)$.

Given that the arrangement under study is already complicated enough, we will assume that Mary will be nominated:

- not only as a beneficiary entitled to submit a ben-order demanding prescription of $s e(1)$ but also
- as a BENEFICIARY entitled to submit a BEN-ORDER demanding prescription of $M O(1)$.

Then, for obvious practical reasons, Mary-the Double-Beneficiary will often "fill-in" her ben-order and BEN-ORDER into one single legal document. By calling the document a multi-order our IT-parlance only seeks to express that the demand involves more than only one OP-UN and, moreover, the OP-UNs belong to two systems/processes.

Yet, the efficiency of a multi-order is paid for by the loss of a clear frontier between the two fundamentally different demands, the ben-order and BEN-ORDER. We will show shortly that the loss will prove particularly costly, should we need to separate liabilities and sanctions for two kinds of breaches

$$
s e(-3) \text { and } M O(-3)
$$

### 13.3.2 Insurance claim

A nice example of a multi-order can be provided by a damage claim which in fact demands two things:

- not only that the Insurer will pay a recovery/benefit,
- but also that the Assessor will process the claim, namely validate it as to its justifiability.

The case of insurance is worth mentioning also because, as explained, the roles of an Assessor and Insurer are often performed by the same person, albeit a legal person.

### 13.4 Order routing

### 13.4.1 Lower-level managerial strategy

For the sake of the analysis we will assume that Manuela is into the role of an MO designed in the simplest morphological pattern of a one-unit one phase system/process $M O$ (1). All that she will have to do is to deliver "one piece of a verdict" - somewhere sometime.

The apparent incompleteness of such design necessitates, as explained in Fig. 5, that the lady named Manuela "must" design her own multi-unit multi-phase strategy. Put formally the incompleteness makes Manuela solve the following lower-level maximization problem

$$
\begin{aligned}
& \max U^{M A N}(s t r) \\
& \text { s.t.: } \operatorname{str}_{\mathrm{MAN}} \in\left[\operatorname{str} \mathrm{MAN}(0)=\left\{s \operatorname{tr}_{1 \mathrm{MAN}}, \operatorname{str}_{2 \mathrm{MAN}}, \ldots, \operatorname{str}_{\mathrm{NMAN}}\right\}\right] \quad \quad M A X_{M A N}
\end{aligned}
$$

The solution $s t r^{* M A N}$ to $M A X_{M A N}$ is then the optimal strategy how Manuela may best fulfill her task if $M O(1)$ is factually prescribed.

### 13.4.2 Phases of managerial operations

For the sake of the analysis we will summarize the following major characteristics and simplifications of $\operatorname{str}{ }^{*}$ MAN as follows:

- Manuela - as a lower-level Designer - will design only herself as a lower--level Designee or Designees,
- within every Designee/OP-UN Manuela herself will perform all the respective roles of an Executor, Beneficiary, Defendant and Manager.
As to the content, variant strategies $\left\{s t r_{1 \text { MAN }}, \operatorname{str}_{2 \text { MAN }} \ldots, \operatorname{str}_{\text {NMAN }}\right\}$ may be, in general, "whatever". For dramatic effect, they may involve corruption or random generation of names and numbers. Leaving aside such dramatic examples, we will assume that the managerial operations will involve a successive validation of mary's ben-order and, then, depending on the outcome, richard's def-order.
In some more detail $s t r^{* M A N}$ will be now illustrated by a "flow" of operational units or phases $P h_{1}(1) \ldots P h_{8}(1)$ shown in Fig. 38.
This kind of a "linear" pattern of an eight-phase eight-unit system/process corresponds to what is on stock-exchanges often called an order routing. Its trivialization rests mainly in that it contains none of the usual complications, namely those represented by a variety of JOINs and SPLITs.

Moreover, in reality, a designer has to cope with a rather mysterious phenomenon of a cycle, where, e.g., depending upon the outcome of $\mathrm{Ph}_{6}(1)$ the system/process must return to $P h_{2}(1)$. In words, if a def-order of a particular kind is factually submitted, the correctness of the ben-order must be "checked again".


Fig. 38

### 13.4.3 Who is the Designer

Recall that MO/MP-the Designer has been so far assumed to have designed MANUELA as a one-unit one phase system/process $M O$ (1). Hence, the order routing in Fig. 38 has been taken as a lower-level strategy of Manuela how to fulfill $M O(1)$.

By contrast, morphologically the same order routing could have been as well designed "directly" by the MO/MP-the Designer. Again, then, a warning is in order against confusions over who in fact is the genuine Designer of otherwise identical systems/processes.

### 13.5 Monetary value of an order's stage

In Fig. 39 we seek to demonstrate that:
on the horizontal level Mary's ben-order demands a transition $s e(1) \Rightarrow s e(2)$, on the vertical level the respective order routing passes through eight stages (+3)-fulfilled - namely $P h_{1}(+3) \ldots P h_{8}(+3)$.

Every $\boldsymbol{i}$-the stage $P h_{i}(3)$ can thus be interpreted as
a stage of the ben-order
Then, every $\boldsymbol{i}^{\text {th }}$ stage of the ben-order can be formally projected into a particular $i^{\text {th }}$ "point" se(1) in the "segment" constituted by the border "points" se(1) and $s e(2)$.


Fig. 39

It more than obvious that for Mary, Richard and Manuela it will matter where the actual point se(1) act is - how far it is from the upper border "point" se(2). In words, and still only intuitively, the nearer is $s e(1)_{a c t}$ to its "destination" $s e(2)$ the better for Mary and worse for Richard.

For example, let $\operatorname{se}(1)_{a c t}=s e(1)_{3}$ and $P h_{3}(+3)$ contains a verdict that the ben-order is justified. Under these circumstances, it is fair to expect that Mary will be better off than was after $\mathrm{Ph}_{2}(+3)$. In the accounting sense, the above verdict will make, in principle, Mary wealthier compared to $s e(1)_{2}$ when the order was only correct.

Hence, in the case of a tradable order, we may expect that the market price of a correct order will be lower than that of a justified order.

## 14. MANAGEMENT OF A PRODUCTION

Now, our focus will fall on the pattern (b) of the classification summarized in Fig. 8 according to which an individual Designer John designs a JOIN-type three-unit two-phase production system/process of the kind shown in Fig. 7.

### 14.1 IE-representation

In what follows we will:

1) assume that the system/process under study can be conceived of as a production or, for simplicity a shoe-making Firm,
2) focus on the Phase 2 of the system/process, namely the IE-reprepresentation $Q(1) \equiv\left[q:\left\langle\left\langle\vec{q}^{\text {int }}\right\rangle,\left\langle\vec{q}^{\text {ext }}\right\rangle\right\rangle \rightarrow\left\langle\vec{T}_{q}\right\rangle\right]$ of Mary-the Manufacturer from Fig. 7.

As explained, the IE-representation of $Q(1)$ is completely silent about who is a Beneficiary and Defendant and hence also nothing about a "referee" of their ben- and def-orders. Exactly this kind of an incompleteness leads to essential emptyness of our observations about the managerial role. All that we can say is that, by definition of the managerial role, the respective Manager will "always organize his-her work" into the following "flow of operations":

Phase A: he-she firstly collects the relevant data - the values of exogenous variables $\vec{q}^{\text {int }}$ and $\vec{q}^{\text {ext }}$ by which the domain of the mapping (the IF-component of $Q(1)$ ) is designed,
Phase B: he-she substitutes the collected data into the mapping and finds out whether Mary's task can be prescribed - whether Mary's state is feasible, falls into the domain $\langle\vec{q}\rangle$,
Phase C: he-she calculates out the resultant prescription of Mary's task.

Who is be substituted to the above "he-she" is, unfortunately, often left depending on jurisdiction - to this or that "legal dispute".

This said, we will in fact abandon the subject under study and only use the opportunity of the social context to digress towards a few essentially general problems.

### 14.2 Stability of a validation outcome

As said the problems of uncertainty is discussed throughout this BOOK and will be established as one of the key subjects of Comment 10. Before this, the following notes may be of interest.

### 14.2.1 Example

Given the IE-representation of Mary-the Manufacturer $Q(1) \equiv\left[q:\left\langle\left\langle\vec{q} \overrightarrow{\mathrm{int}}^{\text {in }}\right\rangle,\langle\vec{q}\right.\right.$ ext $\left.\rangle\right\rangle \rightarrow$ $\left.\left\langle\vec{T}_{q}\right\rangle\right]$ we may, again, for concreteness and simplicity, take its THEN-component $\vec{T}_{q}$ as a scalar $Q$ representing the only endogenous variable - the magnitude of shoes.

As to the IF-component of $Q(1)$, it will be concretized as follows:
the internal conditions will be taken in a rather non-standard form $\vec{q}^{\text {int }}=\left(K, L, d_{K}, d_{L}\right)$ discussed already in Fig. 22 and Fig. 23,
the external conditions will be assume in their text-book form $\vec{q}{ }^{\text {ext }}=\left(\vec{p} ; a_{f}\right)$ representing prices and technological efficiency.

In sum, Mary-the Manufacturer will be analyzed in the form

$$
Q(1) \equiv[q:\langle K, \gamma, d\rangle \rightarrow\langle\mathrm{Q}\rangle]
$$

where, let us recall:
$K$ and $d$ are parameters of an internal event, designed by the magnitude $K$ of leather and time $d$ of its delivery,
$\gamma \quad$ is an external event designed by a rate of competiveness in the industry (denominated in percentage points) and further exemplified by choices of Richard - Mary's only competitor.

### 14.2.2 Time of validation

Further, we shall denote as $\tau$ the time when Manuela-the Manager completes her tasks and submits her verdict - her managerial MAN-order.

It seems to be perfectly acceptable to assume the following: Having learned that at $d^{0}$ Richard delivered the leather Manuela-the Manager will - "without delay" - at the time $\tau^{0}$

$$
\tau^{0} \geq d^{0}
$$

- confirm the values $K^{0}$ and $d^{0}$ of the two relevant parameters of the delivery,
- attest the parameters' feasibility $K^{0} \in \Delta K$ and $d^{0} \in \Delta d$ and, if they are feasible,
- find out at the time $\tau^{00}$ the percentage value $\gamma^{0}$, i.e. the value of $\gamma$ at the time $\tau^{00} \geq \tau^{0}$.

If we assume, highly unrealistically that at $\tau^{00}$ can be submitted the resultant Manuela's MAN-order, the problem under study rests, obviously, in the generally non-zero time intervals

$$
\begin{aligned}
\Delta \tau^{0} & =\left(\tau^{0}-d^{0}\right) \\
\Delta \tau^{00} & =\left(\tau^{00}-\tau^{0}\right)
\end{aligned}
$$

during which whatever may happen both to the leather stored in Mary's warehouse and Richard's firm, as illustrates in (a) and (b) of Fig. 40, respectively.


### 14.2.3 Practical solutions

At this stage of our analysis we could only attempt to illustrate the infinite variety of mostly unsolvable problems that Manuela may face in her role of a real--life Manager. As a highly imperfect rescue, she thus may apply the following procedures how to address the above instability of validation outcomes:

- where possible, as in the case of money on an account Manuela immobilizes, blocks respective transactions,
- in order to save time and other costs, Manuela may decide to trust Charles--the Supplier and take for granted whatever he declares as to his delivery,
- the hard data by which, e.g., the competitiveness is designed as a conditions may be softened by somewhat softer data, such as, e.g., a monthly average of the variable $\gamma$.


### 14.2.4 Efficiency of validation

In Fig. 40 are depicted two times $\tau^{0}$ and $\tau^{00}$ at which Manuela-the Manager could attempt to validate whether and how the deliveries of leather and labor have been completed. We may only note that:
$\tau^{0}$ is too early because the leather may be still on the way,
$\tau^{00}$ may seem to be too late because the time $\Delta \tau^{00}=\left(\tau^{00}-d_{K}^{0}\right)$ may be taken as wasted; Mary could have been prescribed her task already at $d_{\mathrm{K}}^{0}$.

We should also note that:

- by denoting $\tau_{p}$ the time when Mary-the Manufacturer (her respective Manuela) validates the prices, it is apparent that $\tau_{p} \leq d_{c}$, the external event $p_{Q / c}$ is validated before it actually occurs,
- as a result of $\tau_{p} \leq d_{c}$ a specific kind of uncertainty arises - it may easily happen that Mary's expectation about her future output prices will prove incorrect, $p_{Q / p} \neq p_{Q / p}$.


### 14.2.5 Expected vs. factual data

The rationale of the following Fig. 41 is twofold. In the first place it introduces the case of a doubleSupplier who is to deliver two kinds of resources - generally - to two different places at two different times.

Secondly, Fig. 41 continues with the topic of the inter-temporal nature of production by differentiating

- the present and future times $T^{\alpha}$ and $T^{\beta}$, respectively and
- prices of shoes according to whether they are "only" expected (as of $T^{\alpha}$ ) or factual (as of $T^{\beta}$ ).



## 15. MANAGERIAL FAILURE

As said, the remainder of this Comment 4 will focus on a contractual two-unit two-phase system/process, namely the SP-reprepresentation of Richard-the Seller. In the attempt to get under control the complexity of the problem we will keep to the relatively transparent arrangement of a Manager designed by MO/MP-the Designer and Mary-the double-Beneficiary.

### 15.1 Manager's delict and sanction

### 15.1.1 Sources of Beneficiary's frustration

Let us begin with the already stressed fact that any managerial verdict/man--order can be of two kinds, in the affirmative or negative, depending on whether or not the respective conditions have been satisfied. Intuitively, then, every time the verdict is in the negative we can expect that the Beneficiary may be dissatisfied and ready to complain.

However, our genuine concern will be in only those cases when the frustration is justified, i.e. the cases when the stage (3)-completed occurs in its "negative" instance

$$
s e(-3) \text { and-or } M O(-3)
$$

By assumption, Mary is exposed to the danger of both such failures of her counter-parties. Her actual concern may thus be "whom to blame" of whose behavior/action, Richard's and-or MANUELA's, she can complain - who
of them is accountable for her disappointment with the factual delivery or non-delivery of leather.

Among the most peculiar cases then will be Richard's breach of a wrongfully prescribed task, e.g., when:

- Richard ignores $s e(2)$ that should not have been prescribed,
- Richard's behavior/action leads (intentionally or accidentally) to an outcome that is fully consistent with what should have been rightfully prescribed.


### 15.1.2 Multidimensionality of a breach

In the full analogy to what was said about an "ordinary" delivery of leather we should note also here that a "delivery" of a verdict/ben-order must be consistent in every component of its multi-dimensional prescription $M O(2)$. Hence, the stage

$$
M O(-3) \text {-breached }
$$

will occur whenever the verdict is delivered:

- by a wrong Executor (a person different from manuela), and-or,
- to a wrong place, e.g. by a wrong communication channel, and-or,
- at a wrong time (too early or too late).


### 15.2 Management of Manager's sanction

### 15.2.1 Additional sanctionative branch

As said, the "same" Mary is designed as $p u(1)$ and $M P(1)$ in two contractual systems/processes in Fig. 34 and Fig. 36, respectively, who share the identical morphological pattern of a two-unit two-phase system/process shown in section (e) of Fig. 8.

The following Fig. 42 is then established simply by a mere addition of a sanctionative branch to the two systems/process. Apparently this expansion is of the same kind as the one imposed upon Fig. 14 with the aim to obtain Fig. 28, in which a contractual delict and sanction was introduced to our analysis. In sum:
section (b) of Fig. 42 is in fact (except for a minor change in the topology and notation) identical to Fig. 28 and
section (a) of Fig. 42 is a graphical representation of the same kind of sanctionative implications for the vertical layer of the analysis.

It would require a separate book to deal in a sufficient detail with the relationship between the two layers (a) and (b). So only to illustrate the character of the problems and our approach to them we will devote only a few brief and highly intuitive notes to the vertical sanction $S N(1)$.

### 15.2.2 Sanctionative Manager

To begin with:

- we will recall that by definition there must exist an agent with an exclusive right to process a BEN-ORDER, i.e. Mary's demand to prescribe MANUELA'S managerial task $M O(1)$,
- we will, then, add to the analysis a fully analogous agent with an exclusive right to process Mary's demand to prescribe MANUELA's sanctionative task $S N(1)$.

Let this sanctionative Manager of a Manager be implied in law as a respective Court of justice.

### 15.2.3 Sanctionative order routing

As to the Court, in its role of a sanctionative Manager of a Manager, we will only note that there are, essentially, two options how it may proceed:

- the Court will launch its own "investigation" of the case,
- the Court will strictly limit itself to the validation that $M O(-3)$ occurred, e.g. to the validation that MANUELA's behavior was consistent with her $M O(2)$.

The latter option means that if by $M O(2)$ is prescribed so that a generator of random addresses and numbers must be used, the Court's verdict will be $M O(-3)$ should MANUELA have applied an imperfect generator.


Fig. 42

### 15.3 Partial fulfillment and breach

It will be reasonable to assume in Fig. 39 that the order-routing is designed so that whatever breach $P h_{i}(-3)$ makes it impossible for Manuela to continue. In other words, it is natural to take the stage $P h_{i}(+3)$-fulfilled as the internal condition of the subsequent phase.

To illustrate, let the order-routing arrive at the stage

$$
P h_{1}(+3), P h_{2}(+3), P h_{3}(-3), P h_{4}(1), \ldots, P h_{8}(1)
$$

From the many fundamental question associated to this stage we will only note here that it is often interpreted - intuitively and mostly confusingly - so that

Manuela completed her work so that she has fulfilled her task partially - up to $P h_{2}(+3)$. Put reversely, Manuela breached the task partially - from $P h_{3}(-3)$ on.

Hence, as explained the same stage $M O(-3)$ may obtain all kinds of realizations to which will then - in principle - different sanctions. In the case concerned the stage $M O(-3)$ may be be associated to a different degrees of partiality of the fulfillment or breach. The corresponding sanction may then reflect that, e.g., it matters to Mary at what point $s e(1)_{i}$ the order-routing has collapsed or what is
the last rightful "stage" in the segment $\langle s e(1)$, se(2) $\rangle$
For example, the damage caused to Mary need not be that big if Manuela managed to rightfully validate that Mary's ben-order is justified.

## 16. DIGRESSION: TOKENS

From among the tools of BPM (work-flow analysis, operations research) we have so far mentioned logical gates, name JOINs and SPLITs. Similarly frequent in BPM models are the so-called tokens.

### 16.1 Example

### 16.1.1 Order routing

For illustrations we will return to the management of a contract and the order-routing shown in Fig. 38. In the very brief, tokens "move" within a work--flow chart so as to indicate "where" the system/process currently is and how it got to the point. Tokens are often colored in order to indicate that the same element of the system/process (OP-UN in our IT-parlance) may obtain different stages of its development. Different stages of Richard-the Seller - S(0), S(1), S(2), $S(3)$ will thus be indicated by different colors or even sizes of the respective tokens. Differently colored tokens can also differentiate different steps towards a task completion.

One of our aims here is to show that any of position of a token may represent a condition under which this or that OP-UN of the system/process may be activated.

### 16.1.2 Subsequent phases

Sections A) through G) of Fig. 43 represent various phases of the order routing of the BEN-order. They should be self-explanatory and deserve only the following brief notes:
A) $P h_{1}(+3)$ and $P h_{2}(+3)$ : The big green token determines that Mary decided to submit her BEN-order, Manuela-the Manger successfully accepted the order and duly found it correct - found that its trivial condition $\left\langle\vec{S}_{\text {ben }}^{\text {triv }}\right\rangle$ is satisfied.
B) The three tokens have the following meaning:

- the small empty token carries the information about the outcome of $P h_{2}(+3)-$ about who demand what,
- the small green token states that the outcome of $P h_{3}(+3)$ is that the non-trivial condition of justifiability has been duly validated and its outcome is that Mary's BEN-order is justified - that all substantive conditions of justifiability are satisfied,
- the big green token formally confirms that both conditions entering the JOIN concerned are satisfied - it contains the information in what particular specification is Mary's BEN-order correct and justified, what Mary correctly and justifiably demands.
C) $P h_{4}(+3), P h_{5}(+3), P h_{6}(+3), P h_{7}(+3)$ : The big green token states that:
- Richard did not submit DEF-order at all, or the order was found as incorrect and-or unjustified,
- Richard submitted a correct and justified DEF-order that may "extinguish/ /nullify or reduce/modify" Richard's task; that - as to Manuela - there are two correct and justified counter-orders "on the floor of the two-member organized market".
D) The two green tokens have the following meaning:
- the small green token states that the outcome of $\mathrm{Ph}_{8}(+3)$ is Manuela duly delivered the verdict/MAN-order,
- the big green token states that the MAN-order is in the affirmative - it contains the particular specification of Richard's $S E(2)$.
E) The big green token states that the first step towards completions of Richard's task has been made, e.g. that $15 \%$ of the prescribed leather has been factually delivered.
F) The big red token states the ultimate outcome of the delivery is a failure.



### 16.1.3 Other cases

Hopefully, the diagrams in Fig. 44 are self-explanatory.


### 16.2 Associated notes

### 16.2.1 Trivial condition

As already said, by splitting $\left\langle\vec{e}_{\text {ben }}\right\rangle$ into $\left\langle\overrightarrow{s e}_{\text {ben }}^{\text {friv }}\right\rangle$ and $\left\langle\vec{e}_{b e n}^{\mu s t}\right\rangle$ we take hold of the mere fact that should BEN-order be validated as to its justifiability it must be correctly submitted. By definition, only correctly submitted order can be recognized by Manuela as an order. Only from a correctly submitted order Manuela can deduce who demand what.

This is exactly what is required by the sub-partial condition $\left\langle\overrightarrow{s e}_{b e n}^{t r i v}\right\rangle$ of a sub-condition $\overrightarrow{S e}_{b e n}$.

Recall that - put purely formally - this kind of disintegration of the overall condition $\langle\overrightarrow{s e}\rangle$ is only a technical consequence of our decision to establish substantive and procedural layers of Richard's environment. And that this decision was made with the aim to approach the terms substantive and procedural as they are used in LS.

However, two notes may shed more light on the problem:

- A submission of a BEN-order became - in itself - a substantive condition, whereas - intuitively, at least - it would be rather seen as procedural.
- Given that BEN-order "must" be submitted raises the "chicken-and-egg" question about how can one recognize that "a document" submitted, is not, e.g., a completely irrelevant "weather report", moreover written in - say Arabic.

While the former problem is mostly terminological, the latter on is not that easy to resolve in practice. In our real-life IT-systems we have resolved it by converting order-submission into order-reception. As submitted were then taken orders that the Manager was capable - technically speaking - of receiving. Technical capacity to receive an order was then defined by the Manager's capacity to provide to the respective submitter a copy of his-her order.

The green tokens in Fig. 43 represent the state of the order "partially valid", with the meaning that the respective partial condition is satisfied. The red tokens then represent the opposite and also carries information about the exact form of this state.

Let us recall that $S(1)$ under study is designed by the contractual system/ /process in Fig. 14 and hence Mary is the Beneficiary and Richard the Defendant. Moreover, as before Mary will be designed by separate (OP-UN) ben, and hence, her BEN-order will be submitted as a fulfillment of her respective task.

### 16.2.2 Macro-task - parallel setting

We have already noted a delivery within which two kinds of good are to be delivered to two different places at two different times. Now we will add a few notes to the topic.

Only to continue with our ambition to demonstrate the infinite variety of real-life tasks, section (a) of Fig. 45 is to show a prescription

$$
s e(2)=\left(s e(2)^{1} \cup s e(2)^{2}\right)
$$

where:

$$
\begin{array}{ll}
s e(2)^{1} & \begin{array}{l}
\text { states that two thirds of the overall amount of leather is } \\
\text { to be delivered by Mr. Strong to London in June, }
\end{array} \\
s e(2)^{2} & \begin{array}{l}
\text { states that the remaining one third will be delivered } \\
\text { Miss Weak will be delivered to Prague in September. }
\end{array}
\end{array}
$$

The big red token in (a) of Fig. 45 states that only the delivery to London has been fulfilled which can be, intuitively, interpreted as "partial" fulfillment of the overall macro-task $S E(2)$.


### 16.2.3 Macro-task - serial setting

For dramatic effect section (b) of Fig. 45 contains an almost equivalent arrangement as section (a). Apparently, a detailed discussion of similarities and differences of the two cases would exhaust a separate book. In this BOOK we have no other ambition than merely attract readers' attention upon the technique with which the topic can be presented.

However, two notes may be in order:

- System/process in the lower section (b) of Fig. 45 is of the same kind as the one depicted in Fig. 38 where - in the position of Richard's leather-to-be--delivered was Manuela's verdict (her MAN-order). The stage se(-3) thus stops the process in the same sense as did the negative outcome of $\mathrm{Ph}_{3}(+3)$ claiming that Mary's BEN-order was unjustified.
- It was noted already in the context of Fig. 38 that it is not purely obvious to interpret the setting in (b) of Fig. 45 as partial fulfillment of some overall serial macro-task.


### 16.2.4 Infinitesimal conditions

The concept of gates and bullets often, misleadingly, suggests that the conditions (the IF-component of the OP-UN) and tasks (the THEN-component of the OP-UN) are both of the binary ("digital", yes-or-no, true-or-false, left-or-right) nature.

It was shown that within the prescribed time period $\Delta t$ it matters when exactly the delivery occur as, to every second of duly a different purchase price will be associated. Analogously, to every time outside the time period $\Delta t$, different sanction can correspond.
This is the true meaning of the sentence by which we ask "whether and how the condition has been satisfied", "whether and how the task will be prescribed". The question is not only whether Richard will have to deliver leather but also who will be the Executor, what kind of leather, how much, where and when. Not only in pure theory but.
Section B) of Fig. 43 can be interpreted so that Richard's task is partially activated due to the fact that Mary's BEN-order has been found correct and justified. Equivalently, then, the same section has the meaning that Manuela's respective of task order routing is partially fulfilled.

Quantitatively, we could even estimate that this particular partiality amounts to, e.g., $20 \%$ of the overall scope of Manuela's overall task and hence that Richard's task is activated up to $20 \%$.

Analogously, section E) of Fig. 43 could be interpreted so that:

- Richard's task is $100 \%$ activated and consequently and Manuela's task $100 \%$ fulfilled,
- Richard's fully activated task is fulfilled up to $15 \%$, given that the first step towards se(+3) can be estimated as $15 \%$ of the Executor's overall effort or costs.

Apparently, then, in general, a continuous (infinitesimal) range of thus defined partiality can be established and, what is our point here, applied for designing the environment of Mary-the Purchaser, or conditions under which her $P(1)$ can be activated.

To illustrate, the big green token in section B) of Fig. 43 was associated to a particular state of Mary's BEN-order, or, more accurately, to its concrete correct and justified content. Already this "state of affairs" can be designed as a condition of justifiability of Richard's BEN-order demanding purchase price from Mary-the Purchaser.

Analogously, Fig. 46 depicts a somewhat less extravagant condition when Richard may justifiably enforce Mary's payment even before his delivery has been completed, e.g. when it is enough when $15 \%$ of the prescribed leather is factually delivered.


For dramatic effect Richard is shown to fail - finally - and along with the $P U(1)$, also the sanctionative $s n(1)$ can be activated.

## COMMENT 5.

## One-unit strategies; the case of the Firm

The general subject of our analysis has been conceptualized as a relationship between two kinds of agents - a Designer and Designee. Of them, the first was formalized by a maximization problem and the second by a mapping representing conditions under which his-her task can only be prescribed.

Naturally, what we seek for are operational formulas in which the maximization problem and the mapping will be expressed. One way how to do it is to assume, as any science in fact does, that the societal universe can be divided into a finite number of

## self-contained systems

each of which can be ascribed a specific set of behavioral characteristics.
Leaving aside all kinds of Godelian paradoxes, ET would claim that one of such self-contained systems is its so-called economy, where, moreover, the looked-for behavioral characteristics of two and only two kinds called a production and consumption. On the proviso that this concept makes sense, economists then hope to develop, as said, an operational or even scientific representation of the "upper-case" maximization problem by which a Designer is defined and the mapping by which a Designee is designed.

To begin with, a Producer (Firm) will be dealt with in this Comment 5. The case of a Consumer (Households) will be discussed in the subsequent Comment 6.

The following additional introductory notes may then be of value:

- A production was in Comment 2 represented mainly by the JOIN-type three-unit two-phase system/process shown in (b) of Fig. 8. In what follows, for simplicity and with the aim to keep in touch with text-book ET, both production and consumption will be often in the form of a one-unit one-phase system/process shown in (a) of Fig. 8.
- The same ambition will lead to a Designee designed in the form of a "lower--case" maximization problem. Hence, again, two kinds of maximizations will be strictly differentiated in the respective sections - marked "III/F" in Comment 5 and "III/C" in the subsequent Comment 6.
- Apart from the obvious methodological - bridge-building - objectives it should be also noted that both Comments 5 and 6 should be also taken as preparatory steps towards later more advanced treatises on the intertemporal choice and uncertainty. In particular, Comment 9 will deal with systems/processes designed so as to re-allocate some present wealth between future agents including future Firms and-or Consumers.


## 17. ELEMENTAL TOOL-KIT OF MICROECONOMICS

### 17.1 Two kinds of maximization "III/F"

### 17.1.1 Major contribution to the theory

With the aim to fully concentrate on our methodological objectives, we will return to PART I where John-the Designer was defined by $M A X_{J}$ and his optimal one-unit one-phase strategy

$$
s t r^{* / J}=M(1) \equiv\left[m:\langle\vec{m}\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]
$$

was exemplified by Mary-the Shoe-maker.
In Fig. 5 we then explained that the general mapping $\left[m:\langle\vec{m}\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right.$ ] can obtain all kinds of forms, including that of a maximization problem $\max _{M}$.

Given the specific environment of production or Firm, let $M A X_{J}$ be now denoted as $M A X_{J / F}$ and $s t r^{* / J}$ be designed as

```
\(\boldsymbol{\operatorname { m a x }} \psi(K, L, Q)\)
s. t.:
\(\max J_{M / F}\)
\(Q \leq a_{f}^{* / J} . \mathrm{f}^{0}(K, L)\)
\(\psi \geq b n^{* / J}\)
```

where the sub-scripts " $M / F$ " in $\max J_{M / F}$ states that what is being designed is Mary and that Mary's role has the empirical meaning of the be the Firm.

As before:
$\psi(K, L, Q) \quad$ is profit,
$a_{f}^{* / J} \cdot \mathrm{f}^{0}(K, L)$ is the Firm's production function.
Moreover:
$b n^{* / J} \quad$ represents John's "budgetary needs" or the minimal profit that must be generated - should Mary be prescribed to launch her shoe-making operations.

The asterisk "*" in the superscript "*/J" is to state expressly the particular values $a_{f}^{* / J}$ and $b n^{* / J}$ are outcomes of John's choice - parts of the solution $s t r^{* / J}$ to $M A X_{J / F}$. Hence, as before we strictly differentiate between

$$
M A X_{J / F} \text { and } m a x J_{M / F}
$$

and this differentiation can be in fact taken as our key contribution to the "general theory of the Firm".

The following remarks may put some additional light on this contribution:

1) Formula $\max J_{M / F} \equiv s t r^{* / J}$ embodies John's decision through which he equips Mary with a technology $a_{f}^{* / J}$ and expects from her a delivery of a profit on the level of at least $b n^{* / J}$.
2) Whereas $a_{f}^{\star / / J}$ and $b n^{* / J}$ are endogenous variables with respect to John's $M A X_{J / F}$, the same parameters are exogenous with respect to Mary's max $J_{M / F}$. Prices $p_{\mathrm{K}}, p_{\mathrm{L}} p_{\mathrm{Q}}$ are then exogenous for both agents concerned - John and Mary.
3) If $b n^{* / J}<0$, the same inequality determines the highest tolerable loss of the Firm.

### 17.1.2 Demand and an indirect utility (profit) function

As explained, $\max J_{M / F}$ represents nobody's choice. Repeatedly we emphasize that for us it is "only" a formula into which Manuela-the Manager substitutes the collected data and from which she calculates out her verdict about whether and in what form Mary's task will be prescribed.

It has been also stressed that it is more accurate to express the above strategy

$$
\max J_{M / F} \equiv s t r^{* / J}=M(1)
$$

in the form of an express IF-THEN rule, namely as

- a demand-supply function $\left[d s:\left\langle p_{\mathrm{K}}, p_{\mathrm{L}}, p_{\mathrm{Q}} ; a_{f}^{* / J}, b n^{* / /}\right\rangle \rightarrow\left\langle K^{*}, L^{*}, Q^{*}\right\rangle\right]$ or
- a profit function $\left[\left[\vartheta:\left\langle p_{\mathrm{K}^{\prime}} p_{\mathrm{L}} p_{\mathrm{Q}} ; a_{f}^{* / J}, b n^{* / J}\right\rangle \rightarrow\left\langle\psi^{*}\right\rangle\right]\right]$.

Unfortunately, thus expressed IF-THEN rule suffers - again - from all the shortcomings of an IE-representation of an operational unit. In particular, unlike SP-representation the mapping [ds: $\left\langle p_{\mathrm{K}}, p_{\mathrm{L}}, p_{\mathrm{Q}} ; a_{f}^{* / /}, b n^{* / J}\right\rangle \rightarrow\left\langle K^{*}, L^{*}, Q^{*}\right\rangle$ ] contains no information about the respective Nominees.
Again our warning must then be that their invisibility should not be taken as their non-existence. For example, again, it goes by definition that there simply must exist "some Manuela" who only will be in charge to determine the particular values for the exogenous variables, e.g. the prices $p_{\mathrm{K}^{\prime}} p_{\mathrm{L}}$ and $p_{\mathrm{Q}}$.

### 17.2 Marginal product and costs

### 17.2.1 Iso-profit lines

For the sake of this chapter we shall leave aside the constraint $\psi \geq b n^{* / J}$. In Fig. 47 and Fig. 48 we then show two ways how elemental text-books represent a solution to the simplified $\max J_{M / F}$.

Apparently, the profits of Mary's firm increase in the north-west direction as shown by the green arrow in Fig. 47. For given $a_{f}^{+}, K^{+}$and prices $p_{K^{\prime}}^{+} p_{L^{\prime}}^{+} p_{Q^{\prime}}^{+}$ the prescribed task $\vec{T}_{m}^{*}$ is constituted by the tangent point between the set of iso-profit lines, namely
and the graph of a short-term production function $a_{f}^{+} . \mathrm{f}^{0}\left(K^{+}, L\right)$.
To derive the solution $\vec{T}_{m}^{*}$ analytically, Manuela will have to use the text-book Kuhn-Tucker procedure and obtain the first order conditions of the optimum

$$
p_{L}^{+}=p_{Q}^{+} \cdot \frac{\partial\left(a_{f}^{+} \cdot \mathrm{f}^{0}\left(K^{+}, L^{*}\right)\right)}{\partial L}
$$

In words, in the Firm's optimum the so-called monetary value of marginal product of the respective input must be equal to its exogenously given price.


Fig. 47

### 17.2.2 Cost function

In Fig. 48 we express the above phenomena under study by variables $T R(Q)$ and $T C(Q)$ representing the Firm's total revenue and total minimal costs, respectively. The Firm's profit

$$
\psi(Q)=[T R(Q)-T C(Q)]
$$

then depends only on the level of output $Q^{*}$, on the proviso that the costs have been minimized. The optimal level $Q^{*}$ of the output is thus where the profit $\psi(Q)$ obtains its maximal level $\psi^{*}$. The text-book condition of this level $Q^{*}$ requires that

$$
\left[\frac{d T R\left(Q^{*}\right)}{d Q}=p_{Q}^{+}\right]=\frac{d T C\left(Q^{*}\right)}{d Q}
$$

In words, in the optimum the Firm's marginal costs must be equal to the exogenously given output price $p_{Q}^{+}$.


## 18. REGULATION OF A PRODUCER

In this chapter we shall focus on the conditions under which a Producer may be prescribed or prohibited to launch the production - through which a Designer regulates Producer's behavior/action.

### 18.1 Existence problem

### 18.1.1 Optimum not existing

Returning to $\max J_{M / F}$ that includes the constraint $\psi \geq \mathrm{bn}^{*}{ }^{*}$, the following Fig. 49 shows three text-book cases when Mary will be at a loss as to what to do, what is her task. Put differently, in the following three cases Manuela will not be able to issue her verdict about whether and in what form Mary's task will be prescribed.

The obvious two arrangements under study are the following:
a) In (a) of Fig. 49 the two tangent points $\vec{T}_{m}^{t / 1}$ and $\vec{T}_{m}^{t / 2}$ impose on Manuela the well known unsolvable dilemma of the legendary Buridan's ass: she is to choose from two identically best options. The looked-for optimum seems to exist but is not unique.
b) The opposite case is depicted in (b) of Fig. 49. While in section (a) Mary was in danger to receive two conflicting tasks, in (b) the model provides no verdict whatsoever as the looked-for optimum does not exist.

The remaining (c) of Fig. 49 is the rationale for bothering the reader with the above text-book essentials. The tangent point $\vec{T}_{m}^{t}$ exists and is unique, the respective profit is non-negative $\psi^{t} \geq 0$ but smaller than John's budgetary needs $\psi^{\mathrm{t}}<b n^{* / J}$.


Fig. 49

### 18.1.2 Domain of a mapping

With the aim to approach the language of LS and BPM let us recall that $M(1)$ is an IF-THEN rule and that by the variables in the IF-component $\left\langle p_{\mathrm{K}}, p_{\mathrm{L}}, p_{\mathrm{Q}} ; a_{f}^{* / /}, b n^{*}{ }^{*}\right\rangle$ is designed Mary's environment or - equivalently conditions by which John-the Designer decided to regulate Mary's future behavior. An apparent question then is, in what states of Mary's environment are the regulatory conditions satisfied.

In order to re-formulate this key question, we may recall that, formally speaking, the IF-component is a domain of the respective mapping. Hence what we look for are states that fall into the domain - knowing that for such and only such states the mapping is defined and, consequently, the corresponding tasks exist.

Such states have been called feasible, in contrasts to the states outside the domain $\left\langle p_{\mathrm{K}}, p_{\mathrm{L}}, p_{\mathrm{Q}} ; a_{f}^{* / J}, b n^{*} J\right\rangle$, which will be taken as unfeasible.

One of our aims will then be to show that, in general, to every feasible state of Mary's environment a set of her feasible tasks corresponds.

### 18.2 Feasibility vs. conceivability

### 18.2.1 Profit function; domain and conceivability

Let us recall that the text-book tool of a profit function determines how the optimal profit $\psi^{*}$ changes with the states of Mary's environment. By definition, $\psi^{*}$ is obtained by substituting values ( $K^{*}, L^{*}, Q^{*}$ ) - if they exist - into the formula for ordinary (direct) profit

$$
\psi^{*}=\left(p_{\mathrm{Q}} \cdot Q^{*}-\left(p_{\mathrm{K}} \cdot K^{*}+\mathrm{p}_{\mathrm{L}} \cdot L^{*}\right)\right)
$$

As a result the profit function can be put as

$$
\left[\left(\psi:\left\langle p_{\mathrm{K}}, p_{\mathrm{L}}, p_{\mathrm{Q}} ; a_{f}^{* / I}, b n^{*} \mathrm{~J}\right\rangle \rightarrow\left\langle\psi^{*}\right\rangle\right]\right.
$$

with the meaning that it is not defined for profits $\psi^{*} \leq b n^{*}$.
For graphical convenience and not only for that matter we have enriched the text-book analysis by considering "technical" conceivability vs. inconceivability of the inputs of the profit function. Put formally, we will further focus on states constrained by the following limits:

$$
\begin{aligned}
\Delta a_{\mathrm{f}} & =\left(a_{f}^{b}, a_{f}^{\mathrm{q}}\right) \\
\Delta \vec{p}=\left(\vec{p}^{\mathrm{b}}, \vec{p}^{\mathrm{g}}\right) & =\left(\Delta p_{\mathrm{K}}, \Delta p_{\mathrm{L}} \Delta p_{\mathrm{Q}}\right)
\end{aligned}
$$

where superscripts $b$ and $g$, have the meaning bad and good, respectively. For example, if $K$ represents leather, as in our many examples so far, the worst conceivable price $p_{K}^{b}$ can be USD 500000 per 1 kg and the best conceivable price $p_{K}^{g}$ will be, say, zero.

### 18.2.2 Example

For graphical convenience we shall hold constant - e.g. on their best conceivable $g$-levels - the following variables

$$
\begin{gathered}
p_{K}=p_{K}^{g} \\
a_{f}^{* / J}=a_{f}^{g} \\
b n^{* / J}=b n^{g}
\end{gathered}
$$

and confine the analysis to the two-dimensional rectangle ( $\Delta p_{\mathrm{L}}, \Delta p_{\mathrm{Q}}$ ) containing all conceivable price combinations. Excluding setting shown in (a) and (b) of Fig. 49 the two-dimensional profit function

$$
\left[\psi: p_{\mathrm{K}}^{\mathrm{g}}, p_{\mathrm{L}}, p_{\mathrm{Q}}, a_{\left.\left.\underset{,}{\mathrm{o}}, b n^{\mathrm{g}}\right) \rightarrow\left\langle\psi^{*}\right\rangle\right]}\right.
$$

can be represented - as shown in Fig. 50 - by its upward sloping contours (the so-called price indifference curves). Apparently, the optimal profit $\psi^{*}$ increases in the north-west direction - as shown by the red arrow.

In words, the arrangement in Fig. 50 is to demonstrate that the optimal profit $\psi^{*}$ increases with $p_{\mathrm{Q}}$ and decreases with $p_{\mathrm{L}}$.

### 18.2.3 IF-THEN representation

The rectangle of conceivable prices in Fig. 50 is divided into green and red areas denoted $E_{1}$ and $E_{2}$ respectively. For example the state $\vec{p}^{0}$ from (b) of Fig. 50 is in $E_{1}$

$$
\vec{p}^{0} \in E_{1}
$$

and hence the IF-THEN rule is defined and the looked-for task exists.
Contrariwise, in the state $\vec{p}^{+}$shown in (b) of Fig. 50 we have

$$
\vec{p}^{+} \in E_{2}
$$

and hence:

- the state $\vec{p}^{+}$is infeasible, or, put equivalently,
- the condition is not satisfied in $\vec{p}^{+}$, or, put equivalently,
- the instruction for Mary's action - the Manuela's verdict - is not available in $\vec{p}^{+}$.


Fig. 50

In brief:

IF: Manuela-the Manager ascertains that the state of the Mary's environment is
$\left(\ldots, p_{\mathrm{L}} p_{\mathrm{Q}} ; \ldots\right) \in E_{1}$
THEN: the conditions of Mary's task are satisfied and Manuela will only calculate its specification by simply

IT 3a substituting the respective data into
$M(1) \equiv\left[m:\left\langle\ldots, p_{\mathrm{L}} p_{\mathrm{Q}} ; \ldots\right\rangle \rightarrow\left\langle M(2) \equiv \vec{T}_{m}^{*}=\left(K_{m}^{*}, L_{m}^{*}, Q_{m}^{*}\right)\right\rangle\right]$
ELSE: $\quad \operatorname{IF}\left(\ldots, p_{\mathrm{L}}, p_{\mathrm{Q}} ; \ldots\right) \notin \mathrm{E}_{1}$, THEN $[M(1) \Rightarrow M(1)]$ with the meaning of Mary's non-action (empty behavior)

### 18.2.4 Distance from satisfaction

With respect to the looked-for inter-disciplinary bridges, let us compare the following:

- If the condition of Mary's activity is satisfied as $\left(\ldots, p_{\mathrm{L}}, p_{\mathrm{Q}} ; \ldots\right) \in E_{1}$, it matters how concretely it is satisfied, whether it is satisfied as $\vec{p}^{g g}=\left(p_{L}^{g} p_{Q}^{g}\right)$ or $\vec{p}^{0}=\left(p_{L}^{0} p_{\mathrm{Q}}^{0}\right)$. Two different prescriptions $\vec{T}_{m}^{g g}$ and $\vec{T}_{m}^{0}$, respectively will be prescribed in these two feasible states.
- Contrariwise, if $\left(\ldots, p_{\mathrm{L}}, p_{\mathrm{Q}}, \ldots\right) \in E_{2}$ it is irrelevant "how far" from the "feasibility border" $\psi=b n^{g}$ the infeasible state is. With respect to $M(1)$, it is by definition, that every state that does not fall into $E_{1}$ is as irrelevant, e.g., yesterday's temperature in Rhodes, Greece or the bark of Mary's neighbor's dog.

Needless to add that the distance from satisfaction may be relevant for some other OP-UNs, e.g. those characterized as a sanction.

### 18.3 Forbearance to act; shut-down conditions

### 18.3.1 Confusion I: a zero-action

Let us recall production cycles established in Fig. 4 and the options of John-the Designer to re-design his actual strategy at the start of this or that cycle. To some extent arbitrarily a non-empty behavior/action of a Designer was defined as a re-design of the existing strategy.

Let John decide to replace for the sake of CYCLE ( $i$ ) the existing $M(1)$ with a new strategy $M(1)_{Z R}$ with the meaning of the following IF-THEN rule.

IF: Manuela-the Manager ascertains that the condition of Mary's task is satisfied, i.e. the state of the Mary's environment is
$\left(\ldots, p_{\mathrm{L}} p_{\mathrm{Q}} ; \ldots\right) \in E_{1}$
THEN: Manuela will calculate the prescription by simply substituting the respective data into IT 3b
$M(1)_{Z R} \equiv\left[\left[m:\left\langle\ldots, p_{\mathrm{L}} p_{\mathrm{Q}} ; \ldots\right\rangle \rightarrow\left\langle M(2)_{Z R} \equiv \vec{T}_{m}^{*}=\left(K_{m}^{*}, L_{m}^{*}, Q_{m}^{*}\right)\right\rangle\right]\right]$ where, however, the mapping $M(1)_{\mathrm{ZR}}$ is constant and ascribes $M(2)=\varnothing$ (a zero-action) to every feasible price vector $\left(\ldots, p_{\mathrm{L}} p_{\mathrm{Q}} ; \ldots\right) \in E_{1}$
ELSE: $\quad \operatorname{IF}\left(\ldots, p_{\mathrm{L}} p_{\mathrm{Q}} ; \ldots\right) \in E_{1}, \operatorname{THEN}[M(1) \Rightarrow M(1)]$ with the meaning of Mary's non-action (empty behavior)

The apparent peculiarity of $\boldsymbol{I T} \mathbf{3} \boldsymbol{b}$ is in that the constancy of $M(1)_{\text {ZR }}$ at the ZERO level. Mary, then, may appear to be prescribed to refrain from acting under all conceivable circumstances.


However:

- IF prices $\left(\ldots, p_{\mathrm{L}} p_{\mathrm{Q}} ; \ldots\right) \notin E_{1}$ occur, THEN Mary will be "inactive" because she will not know what to do,
- IF prices $\left(\ldots, p_{\mathrm{L}}, p_{\mathrm{Q}} ; \ldots\right) \in E_{1}$ occur, THEN Mary will know perfectly well that she is to do "nothing".

As a concluding note we may add that the latter outcome is our interpretation of the so-called shut-down conditions. In elemental economic text-books the firm is prescribed to stop current production under prices when average variable costs become higher than marginal costs, or put differently when the Firms loss (negative profit) is higher than its fixed costs.

### 18.3.2 Confusion II: a delivery of peace and quiet

In Fig. 52 we assume that at $t^{(\mathrm{i})}$, John decides to replace for the sake of CYCLE (i) the existing $M(1)$ with a new strategy $M(1)_{P Q}$. The meaning of this re-design is that Mary's "regular" shoe-making is to be intermitted for CYCLE ( $i$ ) in the sense that only peace and quiet can be justifiably demanded by the respective Beneficiary.


Let Manuela-the Manager keep her role for both designs $M(1)$ and $M(1)_{P Q}$, whereas Benjamin-the "original" Beneficiary be replaced with Guido nominated within the new $M(1)_{P Q}$. For dramatic effect, then:

- Let ill-informed Benjamin - at $t^{(i)}$ - decides to demand shoe-making. Due to the redesign, the verdict of Manuela-the Manager will be in the negative and hence a non-action (empty behavior) $M(1) \Rightarrow M(1)$ will be the outcome. As a result peace and quiet may be in fact - as if - delivered by Mary.
- However, also Mary's non-action (empty behavior) $M(1) \Rightarrow M(1)$ may produce noise during CYCLE (i). To make sure that no noise will be produced, let Guido-the Beneficiary "must" demand expressly a delivery of peace and quiet through the prescription of $M(1)_{P G}$.

In sum, both outcomes $M(1)$ and $M(2)_{P Q}$ of the two unjustified vs. justified BEN-orders can be interpreted, albeit confusingly, as Mary's duty to refrain from shoe-making.

## 19. A REPRESENTATIVE STATE

As already noted on various occasions in this BOOK - in Comment 10 in particular - we seek to address the notion of an uncertainty and disclose its inter-temporal nature. A few introductory notes on the problem may be of value already here.

### 19.1 Example

To begin with, invoke the problem of a stability of a validation outcome discussed in Fig. 40 where we showed that by the time Manuela inserts the collected data into $M(1)$ the state of Mary's environment may change and the prescription $M(2)$ may thus be calculated from data that have lost their actuality.

Generalizing, then, the data need not be available at the time of the task prescription and Manuela has to somehow prefabricate her own "vision" about Mary's environment.

Example in Fig. 53 should be self-explanatory as it basically replicates section (b) of Fig. 50, given that the exogenous variable $p_{Q}$ is now replaced with $a_{f}$.


The interpretation of the values $a_{f,}^{1} a_{f, \ldots,}^{2}, a_{f}^{5}$ on the horizontal axis is the following: The level of Mary's technology is by John - at present - determined as $a_{f}^{* / J}$, but, in reality, the future technological efficiency may factually obtain five mutually exclusive levels $a_{f}^{1}, a_{f}^{2}, \ldots, a_{f}^{5}$ depending on phenomena beyond Mary's control.

### 19.2 Present efficiency vs. future production

### 19.2.1 Representative efficiency

For the sake of the analysis, a time interval $\left\langle t^{1}, t^{2}\right\rangle$ will be established as follows:
$t^{1}$ is the time when Manuela collects the data needed for her verdict about the prescription $M(2) \equiv \vec{T}_{m \prime}^{*}$
$t^{2}$ is the time when can be observed the stage $M(3)$-completed and hence also the respective factual efficiency $a_{f}^{f c}$.
Let Manuela - at $t^{1}$ - be certain that the factual price of labor is $p_{L}^{f c}$. However, as to the efficiency let her only know that $a_{f}^{f c}$ will be one of the five variants

$$
a_{f,}^{1}, a_{f}^{2}, \ldots, a_{f}^{5}
$$

Hence, all that Manuela can do - at $t^{1}$ - is to somehow "prefabricate"

$$
\text { a representative level } a_{f}^{R E P} \text { of the efficiency }
$$

The resultant prescription $M(2)$ will be - at $t^{1}$ - calculated by the following substitution:

$$
\left[m:\left\langle p_{K}^{g}, p_{L}^{f c}, p_{Q^{\prime}}^{g}, b n^{g}, a_{f}^{R E P}\right\rangle \rightarrow\left\langle M(2) \equiv \vec{T}_{m}^{*}=\left(K_{m}^{*}, L_{m}^{*}, Q_{m}^{*}\right)\right\rangle\right]
$$

### 19.2.2 "Mistaken" representative efficiency

Based on Fig. 53, the IF-THEN representation of the above procedure can be put as follows:

IF: $\quad\left(\ldots, p_{L}^{f c}, \ldots, a_{f}^{R E P}\right) \in E_{1}$
THEN: the task will be prescribed as $M(2) \equiv \vec{T}_{m}^{*}=$ Mary's non-empty behavior

IT 3c
ELSE: $\quad \operatorname{IF}\left(\ldots, p_{L}^{f c}, \ldots, a_{f}^{R E P}\right) \notin \mathrm{E}_{1}$, THEN $[M(1) \Rightarrow M(1)]$ with the meaning of Mary's non-action (empty behavior)

The obvious direct question is how to interpret the case when at the time $t^{2}$ Manuela - or whoever else - finds out that

$$
a_{f}^{f c} \neq a_{f}^{R E P}
$$

and hence Manuela's verdict is "wrong" - regardless of whether in has been in the affirmative or negative.

Summarizing, then, our questions should be the following:

- by what particular "formula" should Manuela determine the representative value of Mary's efficiency $a_{f}^{R E P}$.
- who is in charge to determine the "formula",
- how to resolve the above noted inequality $a_{f}^{f c} \neq a_{f}^{R E P}$.


### 19.2.3 Weighted averages

As already promised, the notion of a representative state will be later in Comment 10 applied in the context of a so-called uncertainty choice. Here, at first brush, $a_{f}^{R E P}$ can be taken as a "somehow" weighted average of the above five discrete (mutually exclusive) efficiencies.
Then, the elementary text-book approach to the search for reasonable weights would be mostly based upon the concept of a probability distribution

$$
\vec{\pi}=\left(\pi^{1}, \pi^{2}, \ldots, \pi^{5}\right)
$$

where $\pi^{j}, j=1,2, \ldots, 5$ is the probability that $a_{f}^{j}$ will eventually at some future time $t^{2}$ prove to be Mary's actual ("true") efficiency. The simplest ever formula for $a_{f}^{R E P}$ will thus apparently be

$$
a_{f}^{R E P}=\sum_{j=1}^{5} \pi^{j} \cdot a_{f}^{j} \equiv \tilde{a}_{f}
$$

or the "ordinary" mean (expected) value of the respective variable-technological efficiency in this particular case.
To conclude, given that $\left(a_{f}^{1}, a_{f}^{2}, \ldots, a_{f}^{5}\right)$ is, by assumption, an exhaustive list of mutually exclusive states of Mary's environment, it is by definition, that one and only one of the states will certainly occur and hence $\sum_{j=1}^{5} \pi_{i}^{j}=1$.

### 19.2.4 Entitled agent

The second question will ask who is to determine that Manuela is to take the above mean (expected) value $\tilde{\mathrm{a}}_{f}-$ and no other value - to be the looked-for $a_{f}^{R E P}$.
Apparently, it is John-the Designer who only can determine how Mary's future states will be represented at present. It is thus John who has the exclusive right to design Mary by

$$
\left[m:\left\langle p_{\mathrm{L}}^{+} \ldots, \tilde{a}_{f}, \ldots\right\rangle \rightarrow\left\langle M(2) \equiv \vec{T}_{m}^{*}=\left(K_{m}^{*} L_{m}^{*} Q_{m}^{*}\right)\right\rangle\right]
$$

to be read as:
IF: the mean value of Mary's efficiency is $\tilde{a}_{f}$ or "better",
THEN: Mary's task by $M(2) \equiv \vec{T}_{m}^{*}$ will be prescribed so as to generate optimal profit $\psi^{*}=\tilde{\psi}$.

Only if John is silent on this issue, Manuela may be allowed to apply her own methods of "facts finding".

## 19.3 "Wrongful" representative state

The following two sub-sections are to cast some light on the final question raised by the inequality $a_{f}^{f c} \neq a_{f}^{R E P}$.

### 19.3.1 Feasibility of a state vs. that of a behavior/action

We assume in Fig. 54 that $a_{f}^{f c}$ may obtain only two levels of technological efficiency - the bad $a_{f}^{b}$ and the good $a \frac{g}{g}$ - and all other parameters are held fixed on their good levels, including the constraint $b n^{8}$.


Fig. 54

Apparently, under the bad technology $a_{f}^{b}$, the maximally attainable profit $\psi^{b}$ is not sufficient as $\psi^{b}<b n^{g}$. In other words, the state

$$
\left(p_{K}^{g}, p_{L}^{g} p_{Q}^{g} ; a_{f}^{b}, b n^{g}\right)
$$

is not feasible and the same then applies to task $\vec{T}^{b}$ representing behavior/action in which the profit would be maximal under given circumstances. Intuitively speaking, $\vec{T}^{b}$ can be seen as feasible technologically but not economically.

Contrariwise, under the good technology a ${ }^{\circ}$, a whole set $X^{g}$ of tasks will be feasible. In other words, to one single feasible state

$$
\left(p_{K^{\prime}}^{g}, p_{L}^{g}, p_{Q}^{g} ; a, a_{j}^{g}, b n^{g}\right)
$$

corresponds an infinite number of feasible tasks situated below the graph of a production function and above the iso-profit line $\psi=b n^{g}$. In other words, to two mutually different feasible states, in general, two different sets of feasible tasks will correspond.

### 19.3.2 Alternative representation

In Fig. 55 the relationship between states and tasks (or optimal profits) is depicted in the way explained in Fig. 50 and Fig. 53. In particular, the contours represent how the optimal profit will change with the parameters $p_{Q}$ and $a_{f}^{* / / J}$. Apparently, the contours of the mapping

$$
\left[\psi:\left\langle p_{\mathrm{K}}, p_{\mathrm{L}}, p_{\mathrm{Q}} ; a_{f}^{*}, b n^{* / J}\right\rangle \rightarrow\left\langle\psi^{*}\right\rangle\right]
$$

increase towards the north-east as indicates the red arrow. The lowest level contour $\psi=b n^{* / J}$ thus corresponds to the combinations ( $p_{Q} ; a_{f}^{* / J}$ ) where -cf. Fig. 54 - the graph of a production function is tangent to the iso-profit line $\psi=b n^{* / J}$. For this kind of combinations the set of feasible tasks has only one element which is then, "automatically" the optimum.
To any other combination

$$
\left(p_{Q} ; a_{f}^{* / J}\right) \in E_{1}
$$

is associated a "whole set" of feasible tasks from which the optimal task "must be" selected.


### 19.3.3 "Budgetary" constraint

It is noteworthy that, from the point of Mary, the smaller is the profit constraint $b n^{* / I}$, the better as the "shut-down" point becomes "softer". For example:

- if the requirement on the minimal profit is $b n^{+}$shown in Fig. 56, the efficiencies on the green dotted segment $\left\langle a_{f}^{+}, a_{f}^{g}\right\rangle$ will satisfy the condition,
- in turn to the most demanding ("the bad") budgetary needs $b n^{b}$ corresponds - in Fig. 56 - a technological efficiency on the level at least $a_{f}^{x}$ which is, however, beyond the conceivable capacity of the firm.



### 19.3.4 Task breached vs. inefficient

Let $Q_{m}^{*}$ in Fig. 57 be the magnitude of Mary's task calculated by Manuela as of the time $t^{1}$ on the basis of the representative efficiency $a_{f}^{R E P}$. In words, Mary is prescribed to employ $L_{m}^{*}$ and deliver $Q_{m}^{*}$.

Let us then assume, for the sake of the analysis, that the obedient Mary did employ $L_{m}^{*}$ at $t^{1}$ but by the time

$$
t^{2}>t^{1}
$$

of the completion of the production her labor productivity has changed. For illustration, we consider in Fig. 57 two variant factual ("true" at the time $t^{2}>t^{1}$ efficiencies $a_{f}^{f c / b}$ and $a_{f}^{f c / g}$, bad and good, respectively.


It should be intuitively well acceptable to differentiate the two variants as follows:
under $a_{f}^{f c / b}$ the outcome of the production will be, $Q_{m}^{5}<Q_{m}^{*}$ and hence, Mary's task will be $M(-3)$-breached; the prescribed output $Q_{m}^{*}$ is way beyond Mary's "bad" technological capacity $a_{f}^{f c / b}$,
under $a_{f}^{f c / g} \quad$ Mary's task will be $M(+3)$-fulfilled; however, the fulfillment will be - taken by standard measures - technologically inefficient as lying under the graph of the factual production function Mary's production possibility frontier.

## 20. PECULIARITIES OF REAL-WORLD DESIGNS

With the aim to get somewhat nearer to the real-world production we shall now return to the JOIN-type three-unit two-phase system/process such as that in Fig. 7.

### 20.1 Analytically unfriendly variables

### 20.1.1 Example

The analysis of Fig. 7 will concentrate on Mary-the Manufacturer designed by mapping $Q(1) \equiv\left[q:\langle\vec{q}\rangle \rightarrow \vec{T}_{q}\right]$, where:

- $\vec{T}_{q}=\left(K_{q}, L_{q}, \vec{Q}_{q}\right), \vec{Q}_{q}=\left(Q_{a / q}, Q_{b / q}, Q_{c / q}, Q_{d / q}\right)$ is Mary's task and
- $\vec{q}$ represent conditions under which the task can be prescribed.

Under the specific circumstances of Fig. 7, Mary's $Q(1)$ can be prescribed only if $K(+3)$, i.e. only if Richard-the Supplier duly delivers the leather - fulfills his task. For illustration of the topic of complexity, the condition $K(+3)$ may require that Mr. Slow (as Richard's Executor) hired a cargo train and duly delivered 115 kg of leather to Mary's warehouse in London, on June 15 ${ }^{\text {th }} 2015$, at 04.30 p.m.

Let us simplify so that Mary-the Manufacturer will be designed "only" as

$$
Q(1) \equiv\left[q:\langle K, d\rangle \rightarrow\left\langle L, Q_{b}, Q_{d}\right\rangle\right]
$$

where:
$K, d \quad$ represent the conditions (Mary's environment), namely by only two parameters - magnitude $K$ of leather and time $d$ of its delivery,
$L, Q_{b}, Q_{d}$ represent the resultant task, where:
$L$ is an Executor of the shoe-manufacturing,
$Q_{b}$ is magnitude of the shoes to be manufactured,
$Q_{c}$ represents the place where the manufacturing is to be performed.

### 20.1.2 Executors

In Fig. 58 is shown how Mary's states $(K, d)$ affect the optimal value of $L$ representing who will be to execute her task. Put analytically, under our investigation is a mapping

$$
[q:\langle K, d\rangle \rightarrow\langle L, \ldots\rangle]
$$

assuming that $Q_{b}$ and $Q_{c}$ are ex ante given.
However, our point here is that the variable $L$ is - in reality - expressed in the form of an alpha numeric vector string, or a set of data such as name, age, address, experience etc. For obvious reasons, then, different values of $L$ are represented by only the Executors' names.

The highly stylized contours of the above mapping are in Fig. 58 associated to three persons who may become Executors of Mary's task. Their names and the north-east red arrow suggests that the mapping is designed by John so that the bigger and later is the delivery of leather, the stronger must be the Executor.


### 20.1.3 Magnitudes

In Fig. 59 the curves represent contours of the mapping

$$
Q(1) \equiv\left[q:\langle K, d\rangle \rightarrow\left\langle\ldots, Q_{b}, \ldots\right\rangle\right]
$$

In words, John is assumed to design Mary so that the magnitude $Q_{b}$ of shoes is an increasing function in the magnitude $K$ of leather and a decreasing function of the time $d$ of the leather's delivery - the later is the leather delivered, the smaller will be the magnitude of the manufactured shoes.


### 20.1.4 Places of delivery

Finally, in Fig. 60 we can se "contours" of the mapping

$$
Q(1) \equiv\left[q:\langle K, d\rangle \rightarrow\left\langle\ldots, Q_{c}\right\rangle\right]
$$

where, again, variable $Q_{c}$, will most obtain the form of an alpha numeric vector strings. Fig. 60 represents the case when only two places may be prescribed. For illustration they have the meaning of the ultimate place of shoes' delivery and the assumption is that the bigger and later is the delivery of leather, the less prestigious is the address.


### 20.2 Structure of conditions

Conditions under which Mary's OP-UN are in reality mostly designed not only in an analytically unfriendly language but also in immensely complicated structures.

### 20.2.1 Exclusions

The notion of exclusion belongs to the everyday jargon of insurers and will be discussed in more detail in the context of a liability insurance of a vehicle. In this section we will thus firstly stress that Fig. 61 and Fig. 62 representing a noninsurance context are of the same kind as those discussed in Comment 7.

Hence, at this point, we will only briefly invoke the arrangement from Fig. 55 and, in contrast to it, in Fig. 61 assume that Mary:

- is allowed to generate whatever financial loss and
- is prohibited to apply two kinds of technologies characterized by the two intervals of efficiency $\Delta^{1}$ and $\Delta^{2}$.


Fig. 61

The two kinds of technologies may be taken as exclusions in the sense that none of them can be turned feasible by combining with however favorable price $p_{Q}$ or, indeed, whatever value of any other exogenous variable.

### 20.2.2 Exceptions to exclusions

In Fig. 62 is illustrated the infamous phenomenon of an exceptions to exclusions. In particular, John-the Designer allows Mary to apply the "otherwise forbidden" technology if $p_{Q} \geq p_{q}^{0}$. Put differently, Mary is prohibited to apply the two kinds of technologies but the regulation can be "excused" whenever the output price is "high enough".


A real-life "exception from the rule" can be based on whatever. Moreover it can have its own exceptions. For example: $\Delta^{1}, \Delta^{2}$ may be prohibited for all days except Mondays, save for Mondays in June and September, with the exception that the Monday is a nation-wide holiday.

## COMMENT 6.

## One-unit strategies; the case of the Consumer

As said, economists believe that a so-called economy can be conceived of as a self-contained social system that consists in two and only two kinds of agents - a Producer (discussed in Comment 5) and a Consumer to be dealt with now. In the full analogy with the preceding analysis, also a Consumer will be simplified into the pattern shown in (a) of Fig. 8.

In other words, Mary in her role of a Consumer, will be taken as a one-unit one-phase strategy

$$
\operatorname{str}^{*} \| J=M(1) \equiv\left[m:\langle\vec{m}\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]
$$

For completeness, let us summarize that:

- by $\vec{m}=\left(m_{1}, m_{2}, \ldots\right)$ is designed Mary's environment consisting in whatever John-the Designer wishes to be relevant, e.g., prices, budget, crime rate, weather, health, neighbors' plans, ...,
- $\vec{T}_{m}$ has the empirical meaning of a task whose completion will represent Mary's behavior/action.

By definition, $s t r^{*} / J$ is derived as a solution to $M A X_{J / C}$ where the subscript " $\mathrm{J} / \mathrm{C}^{\prime \prime}$ states that John is in the role of a Designer and his strategy can be characterized as Consumption.

Invoking that John - by designing $M(1)$ - seeks to support fulfillment of his own $J(1)$, our first contribution to the text-book theory of a consumer demand will differentiate two ways how the support can be provided:

Strategy No. 1 is based upon the mere fact that fulfillment of - essentially - any task can be supported by money. Hence, John's str ${ }^{*} / J$ can always have the empirical meaning of a profit-making Firm. His strategy can then obtain the form of Mary-the Shoe-maker, if only John imposes upon Mary the constraint $\psi \geq b n^{*} J$, where $b n^{* / J}$ is the lower limit of John's budgetary needs.
Strategy No. 2 will represent John's decision to procure bread and wine "directly" on the grounds that Mary will be constrained by an ex ante given "budget".

Apparently, Strategy No. 1, has been in fact discussed already - in Comment 5 as a solution of $M A X_{J / F}$. Hence, what remains is to analyze the other strategy.

## 21. STRATEGY NO. 2

### 21.1 Two kinds of maximization "III/C"

### 21.1.1 Major contribution to the theory

As said, in the case of Strategy No. 2 the strategy by which Mary-the Consumer is designed

$$
s t r^{*} / J=M(1) \equiv\left[m:\langle\vec{m}\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right]
$$

will be taken, for the sake of easy differentiation, as a solution to $M A X_{J / C}$. Also here we should stress that the general mapping $\left[m:\langle\vec{m}\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right.$ ] can obtain all kinds of forms and that - for the sake of this analysis - we will select the form of the following maximization problem

```
\(\max U^{* / J}\left(x_{1}, x_{2}\right)\)
\(\max J_{M / C}\)
s.t.: \(\left[\left(p_{1} \cdot x_{1}+p_{2} \cdot x_{2}\right)\right] \leq B^{*} / J\)
```

where the sub-script " $M / C$ " in $\max J_{M / C}$ states that what is being designed is Mary and that Mary is to be a Consumer and where, as in many text-books:
$\vec{x}=\left(x_{1}, x_{2}\right) \quad$ represents a combination of bread and wine, $\vec{p}=\left(p_{1}, p_{2}\right) \quad$ are prices of bread and wine.

As before, a superscript "*/J" in $U^{*} J$ and $B^{* / J}$ embodies the fact that the concrete form of $\max _{M / C}$ is a solution to John's $M A X_{J / C}$. Put in more detail:
$U^{* / J}\left(x_{1}, x_{2}\right) \quad$ includes the information about John's inter-good preferences, i.e. about how much more or less he "likes and-or needs" bread than wine,
$B^{*} / J \quad$ represents John's decision about how much Mary may at most spent on bread and wine.

The limit $B^{*} J$ is an endogenous variable with respect to John's $M A X_{J}$, whereas it is exogenous with respect to Mary's problem $\max J_{M / C}$. Prices $p_{1}, p_{2}$ are exogenous for both agents concerned - John and Mary.

### 21.1.2 Consumption cycles

We may again assume that John will be prescribed the "same" task $J(1)$ repeatedly or even regularly. For the sake of every single consumption cycle, John may select - in general - different strategy, based on specific, mutually different inter-good preferences and-or budgetary limit $B^{* / J}$.

However, for the moment we will in Fig. 63 assume that John-the Designer will be in-active in the sense that will keep to his existing strategy throughout all cycles.


Fig. 63

### 21.1.3 Demand and an indirect utility function

To the problem $\max J_{M / C}$ correspond two text-book demand functions

$$
\begin{aligned}
& x_{1}^{*}=x_{1}\left(p_{1}, p_{2} ; B^{* / J}\right) \\
& x_{2}^{*}=x_{2}\left(p_{1}, p_{2} ; B^{* / J}\right)
\end{aligned}
$$

represented in our IT-parlance by a mapping

$$
M(1) \equiv\left[m:\left\langle p_{1}, p_{2} ; B^{* / J}\right\rangle \rightarrow\left\langle x_{1}^{*}, x_{2}^{*}\right\rangle\right]
$$

whose THEN-component $\vec{x}^{*}=\left(x_{1}^{*}, x_{2}^{*}\right)$ is, again, Mary's task. The particular form of the task is calculated out by the respective Manuela-the Manager, subject to the respective state of Mary's environment ( $p_{1}, p_{2} ; B^{* /}$ ).

To complete the introductory recapitulation of our "general theory", let

$$
\left[v:\left\langle p_{1}, p_{2} ; B^{* / J}\right\rangle \rightarrow\left\langle U^{*}\right\rangle\right]
$$

denote a so-called indirect utility function. In the full analogy to the Firm's profit function, the formula represents how the optimal utility changes with the state of Mary's environment.

### 21.2 Existence problem

### 21.2.1 Task existing

What Fig. 47 represented for the Firm will now be shown in Fig. 64, where prices $p_{1}, p_{2}$ are held constant on the levels marked by " + " as

$$
p_{1}^{+}, p_{2}^{+}
$$



Fig. 64
The blue straight line in Fig. 64 is rectangular to the green arrow representing a price vector $\left(p_{1}^{+}, p_{2}^{+}\right)$. As a budget line the line represents a set of variant tasks whose monetary value is equal to the designed budget $B^{* / J}$.

Hence, the yellow shaded area under the budget line represents combinations of bread and wine that Mary is allowed to procure by John's design. Put in our IT-parlance, John has decided to regulate ex ante Mary so that she is prohibited to spend on bread and wine more than $B^{*} J$.

The tasks from the yellow shaded area are thus feasible with respect to Mary's budget.

The black down-ward sloping curves in Fig. 64 are contours of the (direct) utility function $U\left(x_{1}, x_{2}\right)$. They are usually referred to as indifference curves and have the same meaning as the iso-profit lines in Fig. 47. The utility increases in the north-east direction - as shown by the red arrow. The solution $\vec{x}^{*}$ to $m a x J_{M / C}$ is thus represented by the point $\vec{x}^{*}$ at which the highest indifference curve is tangent to the budget line.

Under the circumstances of Fig. 64 such point exists and is unique. Manuela can deliver a well-determined verdict about the prescription $M(2)$ of Mary's task.

### 21.2.2 Task not-existing

In the analogy to (a) of Fig. 49 the following (a) of Fig. 65 represents the same case of a Buridan's ass for Mary-the Consumer.


Fig. 65

In (b) of Fig. 65 we show that due to the change of Mary's environment

$$
p_{1}^{+} \Rightarrow p_{1}^{++}
$$

the un-defined task becomes perfectly defined. Whereas the combination $\left(p_{2}^{+}, B^{* / /}\right)$ is infeasible when combined with $p_{1}^{+}$, the very combination $\left(p_{2}^{+}, B^{* / /}\right)$ becomes feasible due to the above change in the price $p_{1}$.

### 21.3 Feasibility of states

### 21.3.1 Conceivability

The domain $\left\langle p_{1}, p_{2} ; B\right\rangle$ - by definition again - represents all states of Mary's environment that are designed by John as feasible, i.e. the states in which the respective mapping exists. Put differently, if a combination of prices and budget is from the domain, Manuela will be able to calculate a well--defined Mary's task - to deliver her MAN-order with a specification of the task's prescription.

As before, we will focus on only conceivable states of Mary's environment $\left(p_{1}, p_{2} ; B\right)$ :

$$
\begin{gathered}
\Delta B=\left(B^{b}, B^{g}\right) \\
\Delta \vec{p}=\left(\vec{p}^{b}, \vec{p}^{g}\right)=\left(\Delta p_{1}, \Delta p_{2}\right) ; \\
\Delta p_{1}=\left(p_{1}^{b}, p_{1}^{g}\right) ; \Delta p_{2}=\left(p_{2}^{g}, p_{2}^{b}\right)
\end{gathered}
$$

where the superscripts $b$ and $g$, have, again, the meaning bad and good, respectively. For example $p_{2}^{b}$ denotes the worst (the highest) conceivable price of wine.

### 21.3.2 Indirect utility function

As said, the (direct) utility function $U\left(x_{1}, x_{2}\right)$ is an analogue for what was profit $\psi(K, L, Q)$ in the case of Mary-the Producer. Similarly, the analogue to the profit function will be - for Mary-the Consumer - the so-called indirect utility function

$$
\left[v:\left\langle p_{1}, p_{2} ; B\right\rangle \rightarrow\left\langle U^{*}\right\rangle\right]
$$

arrived at by substituting optima $x_{1}^{*}, x_{2}^{*}$ into the direct utility function $U\left(x_{1}, x_{2}\right)$ :

$$
U^{*}=U\left(x_{1}^{*}, x_{2}^{*}\right)=U\left(x_{1}\left(p_{1}, p_{2} ; B\right), x_{2}\left(p_{1}, p_{2} ; B\right)\right) \equiv v\left(p_{1}, p_{2} ; B\right)
$$

Holding Mary's budget constant at $B=B^{+}$, the downward sloping curves in Fig. 66 represent contours of $v\left(p_{1}, p_{2} ; B\right)$ - the so called price indifference curves. Due to the standard text-book assumptions - the indirect (optimal) utility increases in the south-west direction - as shown by the red arrow.


## 22. CONSTRAINTS TO CONSUMER BEHAVIOR/ACTION

Recall that in the social context of the John's budgetary needs s $\psi \geq b n^{* / J}$ have been designed by him as the lowest profit he requires or, still more interestingly, as the highest tolerable loss "delivered" by Mary-the Designee.
In the full analogy a similar constraint will be now imposed upon Mary-the Consumer.

### 22.1 Minimal level of utility

Let at the time $t^{(\mathrm{i})}$ when the $i$-th consumption cycle begins, John's strategy be represented by the following maximization problem

```
max UU/J}(\mp@subsup{x}{1}{},\mp@subsup{x}{2}{}
s.t.: max m/a
[(p)
U*/J}(\mp@subsup{x}{1}{},\mp@subsup{x}{2}{})\gequ\mp@subsup{n}{}{*/J
```

where superscript "*/J", let us recall, is to emphasize that the two parameters $B^{* / J}$ and $u n^{* / J}$ are John's optima derived, for the sake of the $i$-th consumption cycle, from $M A X_{J / C}$. Hence, by $\max J_{M / a}$ John regulates (ex ante, for the sake of coming cycle) Mary's behavior as follows:
by $B^{*} / J$ he imposes an upper limit on Mary's expenditure, by $u n^{*} / J \quad$ he requires the lowest level of his satisfaction.

In words, John designs Mary so that she could be sent to a market with a task to procure bread and wine in the amounts qualified indirectly by the constrains $B^{* / J}$ and $u n^{* / J}$.

To conclude, the operational unit under study will now be expanded in the mapping

$$
M(1) \equiv\left[m:\left\langle p_{1}, p_{2} ; B^{* / J}, u n^{*} / J \rightarrow\left\langle x_{1}^{*}, x_{2}^{*}\right\rangle\right]\right.
$$

where let us recall, $B^{* / J}$ and $u n^{* / J}$ are endogenous variables with respect to $M A X_{J}$ and exogenous variables with respect to $\max J_{M / a}$.

### 22.2 Consumer's behavior/action

### 22.2.1 Optimal task

Let at the time $t^{(i)}$ when CYCLE $(i)$ begins, existing prices $p_{1}$ and $p_{2}$ be such that the two sets

$$
\begin{gathered}
{\left[\left(p_{1} \cdot x_{1}+p_{2} \cdot x_{2}\right)\right] \leq B^{* / J}} \\
U^{* / J}\left(x_{1}, x_{2}\right) \geq u n^{* / J}
\end{gathered}
$$

have a non-empty intersection - as shown in the blue-shaded area in Fig. 67.


Then, under standard text-book assumptions, the solution $\vec{x}^{*}$ to $m a x J_{M / a}$ exists and is unique. For simplicity, let

$$
\vec{x}^{*}=\left(x_{1}^{*}, x_{2}^{*}\right)>0
$$

with the meaning that both bread and wine are prescribed to be procured - by Mary for John - in non-zero amounts.

### 22.2.2 Zero saving

Given that the task $\vec{x}^{*}=\left(x_{1}^{*}, x_{2}^{*}\right)$ shown in Fig. 67 lies on the budget line we may also note that Mary will spend all that she is allowed to spend.

Under given circumstances, we obtain in the optimum $\vec{x}^{*}$ a zero amount of saving

$$
\Delta B^{* / M}=0
$$

where

$$
\Delta B^{* / M}=\left(B^{* / J}-B^{* / M}\right)
$$

represents the part of the budgetary limit $B^{* / J}$ that remains unspent, i.e. saved. In plain words, under the above state of affairs Mary will be prescribed to spend all the monetary limit.
Later, namely in Comment 9, we will analyze cases when the agent may be prescribed not only to save some part of his-her budgetary limit but also exceed it.

### 22.2.3 Non-action vs. zero action

Let now, at the time $t^{(i+1)}$ when the consumption CYCLE ( $i+1$ ) begins, prices $p_{1}$ and $p_{2}$ change so that the two sets become disjunctive as depicted in (a) of Fig. 68.
Invoking the comparison between IT $3 a$ and IT $3 b$ in the analogous context of the Firm, the prospective BEN-order may lead - depending on the particular design of $M(1)$ - to two kinds of outcomes:
$M(1) \Rightarrow M(1) \quad$ representing a non-action (empty behavior), when Mary is prohibited from going to the market impliedly,
$M(1) \Rightarrow[M(2)=\varnothing] \quad$ representing a zero action, when Mary is expressly prohibited to go to the market.


Summarizing then:

- section (a) of Fig. 68 is depicted for a given price vector $\vec{p}^{+}$,
- developments based on changes of the vector are shown in section (b) of Fig. 68,
- in terms of consumption cycles the case of a zero action is in Fig. 69.


Fig. 69

### 22.3 Expenditure function

### 22.3.1 Designer's saving

Let at the time $t^{(i)}$ John-the Designer decide to change his strategy for CYCLE ( $i$ ) so that $\max _{J_{M / a}}$ is re-designed into

```
max UU/J}(\mp@subsup{x}{1}{},\mp@subsup{x}{2}{}
s.t.:
[( }\mp@subsup{p}{1}{},\mp@subsup{x}{1}{}+\mp@subsup{p}{2}{}\cdot\mp@subsup{x}{2}{})]\leq\mp@subsup{B}{}{*/J
UU/M}(\mp@subsup{x}{1}{},\mp@subsup{x}{2}{})\gequ\mp@subsup{n}{}{*//
```

Hence, John re-designs the ex ante regulation of Mary's behavior thus: by $B^{*} / J \quad$ John imposes, as before, an upper limit on Mary's expenditure, by $u n^{*} / J \quad$ John allows himself for only a given highest level of his own satisfaction.


Fig. 70 illustrates that, given $\max _{J_{M / b}}$, the sets $\left[B \leq B^{* / /}\right]$ and [ $U^{\left.J^{/ M} \geq u n^{* / J}\right]}$ have always a non-empty intersection and hence the solution of $\max J_{M / b}$ always exists. At the same time, however, the solution is likely not to be unique.

For example, combinations of bread and wine denoted as (*) and (**) in Fig. 70 are equally "best" as they both provide the maximally allowed level of satisfaction $u n^{*} J$.

It is somewhat peculiar that the amounts of saving are different in these two "allegedly" equivalently best optima

$$
\Delta B^{* / M}>\Delta B^{* / M}
$$

where, let us recall

$$
\begin{aligned}
\Delta B^{* / M} & =\left(B^{* / J}-B^{* / M}\right) \\
\Delta B^{* * / M} & =\left(B^{* * / J}-B^{* / M}\right)
\end{aligned}
$$

### 22.3.2 Expenditure minimization; duality

The peculiarity of the above inequality $\Delta B^{* / M}>\Delta B^{* * / M}$ rests in that it is natural to assume that if a non-zero amount $\Delta B^{* / M}$ remains unspent it is to be returned to John, for who is thus procured a commodity bundle

$$
\left(\left(x_{1}^{*}, x_{2}^{*}\right) ; \Delta B^{* / M}\right)
$$

Let, then at the time $t^{(i+1)}$ John-the Designer decides to correct his existing strategy for the CYCLE $(i+1)$ so that $\max J_{M / b}$ is re-designed into

```
\(\min \left(p_{1} \cdot x_{1}+p_{2} \cdot x_{2}\right)\)
s.t.:
\(\left.\left(p_{1} \cdot x_{1}+p_{2} \cdot x_{2}\right)\right] \leq B^{* / J}\)
\(U^{J / M}\left(x_{1}, x_{2}\right) \geq u n^{* / J}\)
```

by which Mary will be to minimize "costs" of the procurement of John's satisfaction on the level $u n^{* / J}$ at worst. The two sets in Fig. 71 represent that:
by $B^{* / J}$ John imposes, again, an upper limit on Mary's expenditure, by $u n^{*} J$ John requires that his satisfaction must not fall below the level $u n^{* / J}$.


Fig. 71

Under the circumstances depicted in Fig. 71 the "costs" $B^{* / M}$ of "delivering" the optimal combination $\vec{x}^{*}$ is feasible as

$$
B^{* / M}<B^{*} / J
$$

Apparently, given the prices $p_{1}, p_{2}$ and the limit $B^{* / J}$ are fixed, the higher will be the designed level of $u n^{*} / J$, the higher will be the respective "costs" $B^{* / M}$ of achieving it.
Generalizing, then, text-book micro-economics describe the related phenomena by the respective properties of the so-called
expenditure function $e\left(p_{1}, p_{2} ; u n^{*} /\right)$
that ascribes minimal income needed to achieve the level $u n^{*} / J$, at given prices. Minimization problem $\max J_{M / c}$ is then taken as "dual" to $\max J_{M / a}$.

## 23. ENDOWMENT

### 23.1 Maximization problem

Let at the time $t^{(i)}$ John-the Designer decides to change his strategy for CYCLE (i) so that $\max _{J_{M / a}}$ is re-designed into

```
\(\max U\left(x_{1}, x_{2}\right)\)
s.t.:
\(\left[\left(p_{1} \cdot x_{1}+p_{2} \cdot x_{2}\right)\right] \leq\left[\vec{p} \cdot \vec{x}^{* / J}=\left(p_{1} \cdot x_{1}^{* / J}+p_{2} \cdot x_{2}^{* / J}\right) \equiv B^{* / J}\right] \quad \max J_{M / d}\)
\(U^{J / M}\left(x_{1}, x_{2}\right) \geq u n^{* / J}\)
```

where

$$
\vec{x}^{* / J}=\left(x_{1}^{* / J}, x_{2}^{* / J}\right)
$$

is a so-called endowment, namely an endowment expressed in physical units ( kg of bread and liters of wine) and $B^{*} / J$ is its monetary value.
The empirical meaning of $\max _{J_{M / d}}$ is that John designs Mary so as to send her to the market and exchange given amounts of bread $x_{1}^{* / J}$ and wine $x_{2}^{* / J}$ for amounts that would better satisfy him.

As a sort of a digression we may note that, invoking the concept of a "lower--level" strategy, Mary herself may decide to support fulfillment the task by her own strategy, e.g. that she will sell $x_{1}^{* / J}$ and $x_{2}^{* / J}$ and for thus generated income buy bread and wine in some other market.

### 23.2 Endogenous vs. exogenous variables

Recall that the superscript "*/J" indicates the fact that the particular specification of the endowment $\vec{x}^{* / J}$ is a solution to $M A X_{J / C}$. Analogously, the superscript " $/ M$ " will now differentiate

$$
\vec{x}^{\star / M}=\left(x_{1}^{* / M}, x_{2}^{* / M}\right)
$$

as a solution to $\max J_{M / d}$. The design of Mary-the Consumer will thus take the form

$$
M(1) \equiv\left[m:\left\langle p_{1}, p_{2} ;\left(x_{1}^{* / J}, x_{2}^{*} /\right)\right\rangle \rightarrow\left\langle x_{1}^{* / M}, x_{2}^{* / M}\right\rangle\right]
$$

and the respective demand functions will be:

$$
\begin{aligned}
& x_{1}^{* / M}=x_{1}\left(p_{1}, p_{2} ;\left(x_{1}^{* / J}, x_{2}^{* / J}\right)\right) \\
& x_{2}^{* / M}=x_{2}\left(p_{1}, p_{2} ;\left(x_{1}^{* / J}, x_{2}^{* / J}\right)\right)
\end{aligned}
$$

leading to the indirect utility function $v=v\left(p_{1}, p_{2} ;\left(x_{1}^{*} / J, x_{2}^{*} /\right)\right)$.
In sum, whereas $\vec{x}^{* / J}=\left(x_{1}^{* / J}, x_{2}^{* / J}\right)$ are endogenous variables with respect to $M A X_{J}$, the same combination is exogenous with respect to $\max J_{M / d}$. Put differently,
$\vec{x}^{* / J}=\left(x_{1}^{* / J}, x_{2}^{* / J}\right) \quad$ are components of a state of Mary's environment, $\vec{x}^{* / M}=\left(x_{1}^{* / M}, x_{2}^{* / M}\right) \quad$ is Mary's task calculated by Manuela-the Manager.

### 23.3 Budget line

The blue straight line in Fig. 72 passing through $\vec{x}^{* / J}=\left(x_{1}^{* / J}, x_{2}^{* / J}\right)$ and rectangular to the blue price vector represents a set of variant tasks whose monetary value is the same, namely equal to the monetary value $B^{+}$of the physical endowment $\vec{x}^{* / J}=\left(x_{1}^{* / J}, x_{2}^{* / J}\right)$. As seen - for dramatic effect - the endowment $\vec{x}^{* / J}$ is in Fig. 72 chosen so as to be infeasible - with respect to the required minimal satisfaction.


Fig. 72

The broken green line in Fig. 72 passing again through $\vec{x}^{* / J}=\left(x_{1}^{*} / J, x_{2}^{* / J}\right)-$ rectangular to the green price vector demonstrates that the budget line pivots around the endowment due to respective changes in relative prices.

### 23.4 Self-transfer of wealth

### 23.4.1 Action/non-action

The mapping $\left[m:\left\langle p_{1}, p_{2} ;\left(x_{1}^{* / J}, x_{2}^{* / J}\right)\right\rangle \rightarrow\left\langle x_{1}^{* / M}, x_{2}^{* / M}\right\rangle\right]$ can be, for simplicity, expressed as an exchange or even transfer of goods

$$
\vec{x}^{\star / J} \Rightarrow \vec{x}^{* / M}
$$

Later in this BOOK it will be interpreted as a "transfer of wealth". In the case under study here, the transfer can be characterized as "intra-temporal" and as a transfer "from John to John". Its performance has been referred to as Mary's behavior/action, on the proviso that the respective BEN-order lead to the respective prescription

$$
M(1) \Rightarrow M(2)
$$

e.g. on the proviso that the BEN-order is "at least" submitted.

In Fig. 73 is depicted the case when sets $\left[B \leq \vec{p} \cdot \vec{x}^{* / /}\right]$ and $\left[U^{J / M} \geq u n^{* / /}\right]$ are disjunctive.


Fig. 73

Invoking, again, our discussion of IT $3 a$ and IT $3 b$, the outcomes may be the following:

$$
\begin{array}{ll}
M(1) \Rightarrow M(2) & \begin{array}{l}
\text { that represents a non-action (empty behavior), when } \\
\text { Mary is prohibited from going to the market impliedly, }
\end{array} \\
M(1) \Rightarrow[M(2)=\varnothing] \quad \begin{array}{l}
\text { that represents a zero action, when Mary is expressly } \\
\text { prohibited to go to the market. }
\end{array}
\end{array}
$$

### 23.4.2 Pseudo-non-action

Let $\left(\Delta x_{1}^{*}, \Delta x_{2}^{*}\right)$ denote the looked for optimal transfers, where as shown in Fig. 72

$$
\Delta x_{i}^{*}=\left(x_{i}^{\psi / M}-x_{i}^{* / J}\right), \boldsymbol{i}=1,2
$$

For completeness, let us then differentiate the "ordinary" case $\vec{x}^{* / M} \neq \vec{x}^{* / J}$ when a non-zero transfers will be prescribe from a somewhat extraordinary opposite case when

$$
\vec{x}^{\star / M}=\vec{x}^{\star / J}
$$

with the intuitive meaning that Mary is ordered to go to the market just to find out that the best that can be obtained there is exactly what John actually has got.

Her optimal behavior is thus - as if - do nothing. Given our interest of various forms of non-actions, the equality $\vec{x}^{* / M}=\vec{x}^{* / J}$ is as if another interpretation of what the lawyers would call a forbearance to act.

Let us thus stress again that our concept of a non-empty behavior is firmly grounded on the fact that the respective OP-UN has been prescribed, however intuitive may be a prescription that - as if - orders an agent "not to act".

## 24. MISCELANEOUS NOTES

### 24.1 Additional regulation

### 24.1.1 Maximization problem

Let John-the Designer - at time $t^{(i)}$ - decide to change his strategy for CYCLE (i) so that $\max _{J_{M / a}}$ is re-designed into

```
\(\max U^{I / M}\left(x_{1}, x_{2}\right)\)
s.t.:
\(\left[\left(p_{1} \cdot x_{1}+p_{2} \cdot x_{2}\right)\right] \leq \vec{p} \cdot \vec{x}^{* / J}=\left(p_{1} \cdot x_{1}^{* / J}+p_{2} \cdot x_{2}^{* / J}\right) \equiv B^{* / J}\)
\(x_{2} \leq r_{2}^{*} J\)
```

Put differently, let John decide to impose upon Mary one more ex ante regulatory measure $x_{2} \leq r_{2}^{* / J}$. In other words, Mary will be designed by the demand function

$$
M(1) \equiv\left[m:\left\langle p_{1}, p_{2} ;\left(x_{1}^{* / J}, x_{2}^{* / J}\right), r_{2}\right\rangle \rightarrow\left\langle x_{1}^{* / M}, x_{2}^{*} / M\right\rangle\right]
$$

and the indirect utility function

$$
v=v\left(p_{1}, p_{2} ;\left(x_{1}^{*} / J, x_{2}^{* / J}\right), r_{2}^{* / J}\right)
$$

### 24.1.2 Feasible tasks

Due to the additional regulation the original set of feasible tasks (cf. the blue shaded area in Fig. 72) has shrunk into the green shaded area in Fig. 74.


Fig. 74
As a result, given the arrangement in Fig. 74, the tangent point $\vec{x}^{\mathrm{t}}$ can no longer be Mary's optimum and the optimal consumption $x_{2}^{* / M}$ of wine will be equal to the upper limit $r_{2}^{* / J}$ that Mary is allowed - by John - to procure.

### 24.1.3 Feasible states

Let $p_{1}$ be constant and the conceivable range of the regulation be

$$
\Delta r_{2}=\left(r_{2}^{b}, r_{2}^{g}\right)
$$

As shown in Fig. 74 the contours of the indirect utility function will be, under standard text-book conditions, upward-sloping and increase in the north-west direction - as shown by the red arrow.


### 24.2 Multi-unit strategy

Fig. 76 represents a three-unit two phase physical strategy in the form clearly referring to the production system/process from Fig. 7.


Fig. 76
What we aim at here is that a real-world consumption can be designed so as to utilize a particular technology through which some inputs will be transformed in outputs - in prescribed kind, magnitude, place and time of delivery.
The actual difference between the two roles is that the output $Q$ is not for sale but consumption. Hence, as will be applied in more detail later, consumption can be seen as an investment that brings up zero revenue. Mary's overall consumer "profit" thus cannot be other than only negative.

To conclude we should note that the system/process in Fig. 76 does not, in itself, contain the information who is its Designer. Unless stated otherwise we may expect that it is designed on the "basic level of our analysis", i.e. by John-the Designer.

However, as explained, John may have designed his strategy as only a one-unit one phase system process str"/J $=M(1)$ and in Fig. 76 is a "lower-level" system/ /process designed by Mary as a Designee-turned-Designer.

## 24.3 "Partial" conditions

One of our notes on Fig. 54 was that - intuitively speaking - the task $\vec{T}^{b}$ can be seen as feasible technologically but not economically.
As a kind of aggregation of the above analysis of a Producer and a Consumer we may summarize the intuition so that their behavior is regulated by two kinds of conditions, namely:

1) technological conditions:
$Q \leq a_{f} . f(K, L) \quad$ for a Producer and

$$
\vec{p} \cdot \vec{x} \leq B^{* / J} \quad \text { for a Consumer. }
$$

2) economic conditions:
$\psi \geq b n^{* / J}$
$U \geq u n^{* / J}$
The geometrical representation of the fact that both conditions must be satisfied has been shown for a Producer in Fig. 54 by a non-emptiness of the set $X^{g}$. In the full analogy, the condition for a Consumer requires that the areas under the budget line and the area above the minimum-satisfaction curve must have a nonempty intersection.

One such non-empty set is represented by the blue shaded area in Fig. 77. For completeness, we could also recall that the set of feasible tasks corresponds to one single state of Mary's environment constituted by the combination of values

$$
\left(p_{1}, p_{2} ; B^{* / J}, u n^{* / J}\right)
$$

where, however, the values $B^{* / J}, u n^{* / J}$ are under John's control.


In conclusion the following two notes may be of interest:

- None of these two kinds of a condition (technological and economic) can be taken as an exclusion. Each of them can be made satisfied if combined with the appropriate form of the other one. In this sense the elasticity of substitution between, e.g., values $B^{*} / J$ and $B^{*} /$, can be analyzed.
- In ordinary language it is often said, highly misleadingly, that the overall condition of a production or consumption is "only partially" satisfied, or - still more confusingly - "so far only partially" satisfied. It follows from our discussion, that there is, in fact, no such thing as a "partial satisfaction" of an overall condition. As stressed already elsewhere, it is not to say that different forms of the condition's dis-satisfaction, may not have different consequences.


## COMMENT 7.

## Stages of a task; validity and effectiveness

For the sake of this Comment we will return to the social context of a contractual system/process or - invoking the pattern (e) of the classification in Fig. 8 - the two-unit two-phase strategy designed collectively by Richard and Mary.

As in Comment 1 we will consider a system/process $\operatorname{SIN}(1)=\{\operatorname{IN}(1), C L(1)\}$ consisting in

Richard-the Insurer $I N(1)$ and Mary-the Client $C L(1)$
Of the two Designees we will focus, again, on Richard-the Insurer and recall that Thesis $B$ from PART I states that $I N(1)$ - as any operational unit, indeed - may obtain a limited number of discrete stages, namely $\operatorname{IN}(1)$-designed, $I N(2)$-prescribed and $\operatorname{IN}(3)$-completed.

We will firstly further enrich the foregoing analysis of the stage $\operatorname{IN}(2)$. Secondly, we will expand the analysis by a new kind of a stage $I N(1)_{r d}$ established as an outcome of

$$
\text { a transition } I N(1)_{a c} \Rightarrow I N(1)_{r d}
$$

where $I N(1)_{a c}$ is the actual design of Richard-the Insurer and the resultant stage $\operatorname{IN}(1)_{r d}$ is a re-design of the Insurer. With the aim to compare our analysis of life insurance in Comment 1 the major subject in this Comment 7 will be the so-called "vehicle insurance", or somewhat more accurately
a (compulsory) liability insurance of a vehicle
As always, we shall also seek to disclose a miscellaneous selection of general phenomena that may not be that well observable elsewhere. In particular, the aim will be to broaden our understanding of:

- architecture of a contract,
- anonymity and-or collectivity of an agent,
- exclusion from a condition,
- repeated prescription of "the same" task,
- substantive vs. procedural conditions of a task.

However the topic that is to differentiate this Comment from the preceding analyses will rest in the notion of validity vs. effectiveness of a design and redesign of an operational unit.

## 25. UNIVERSAL ARCHITECTURE OF AN INSURANCE CONTRACT

As always, we will seek to uncover a common nature of phenomena that often appear to be mutually different only because different terminologies are applied in different social contexts. For that sake we will begin with an unexpected and rather counter-intuitive affinity between life insurance discussed in Comment 1 and "vehicle insurance" to be discussed here.

### 25.1 Harmed/injured victim ("HIV")

It was already stressed that apart from the five major categories of agents (a Designer, Designee and the triad of Nominees) there are may other agents designed as elements of the IF- and THEN-Component of an operational unit. We have so far mentioned the roles of an Interventionist, Executor and External Recipient. Within the social context of insurance, we will expand the list of such agents by a harmed/injured victim ("HIV") designed as an element of the IF-component representing in $I N(1)$ the so-called "insured event".

### 25.2 Comparison

Similarities and differences between life vs. vehicle insurance can then be clearly characterized in Fig. 78, where:
life insurance in (a) of Fig. 78 involves:

- a harm caused to Vera by an "ordinary" bodily injury as a result of whatever event - including her own reckless behavior, e.g. driving,
- a harm caused to Benjamin by Vera, namely Vera's death,
vehicle insurance in (b) of Fig. 78
involves:
- a harm caused to Vera by the fact that she may become liable for a harm caused to a pedestrian named Benjamin as a result of her driving,
- a harm caused to Benjamin by an "ordinary" bodily injury as a result of Vera's reckless driving.

The following notes may make the arrangement somewhat clearer:

1) For concreteness and intuitively acceptable interpretation we assume in Fig. 78 that Mary is an Employer of Vera. In other words, Vera, as Mary's Employee, is insured by her Employer.

2) It is noteworthy that HIV is - in itself - nothing more or less than one of the parameters by which one particular condition is designed. In other words HIV is a parameter of an external ("insured") event under which only Richard's task can be prescribed.
3) A strictly applied parallel between (a) and (b) uncovers the somewhat counter-intuitive fact that Vera from (a) of Fig. 78 is a "liable culprit" if she causes harm to, e.g., her son Benjamin by making him an orphan. In the full analogy to Vera from (b) of Fig. 78 who causes harm to Benjamin by hitting him on the road.

### 25.3 Anonymous and collective Beneficiary

### 25.3.1 Nominees

Keeping our focus on Richard-the Insurer, we should also ask "Who are there the Nominees?".

As in Comment 1 a Manager will be expressly nominated as the Assessor. Intuitively well acceptable will be to assume that Richard himself will be in the role of "his own" Defendant.

Similarly usual construction is where a Beneficiary is the person who is in the role of the respective HIV. Hence, in the specific case of Fig. 78:

- It is not Mary (not to mention Richard) who can become HIV and hence also the Beneficiary. Apparently, then, the contract is, by definition, formed in favor of some third-party Beneficiary.
- In each of the two sectors (a) and (b) of Fig. 78 are nominated two fundamentally different third-party Beneficiaries - Vera or Benjamin denoted as:
in sector (a): Beneficiary ${ }^{\text {injir }}$ and Beneficiary ${ }^{\text {life }}$,
in sector (b): Beneficiary ${ }^{\text {liab }}$ and Beneficiary ${ }^{\text {vehi }}$.


### 25.3.2 Pseudo-insurance of a non-human object

For illustration, let us consider the above (BEN-order) ${ }^{\text {liab }}$ from (b) of Fig. 78, about which Vera-the Beneficiary may decide - put in LS-parlance - to demand that Richard-the Insurer cover her liability to Benjamin; on the grounds that the insurance protects the vehicle owner and any person who drives the vehicle against claims for liability in respect of the events covered by the insurance, i.e. of property damage or bodily injury caused by the fault of the owner or driver.
In plain terms the collective and anonymous nature of:

$$
\text { Beneficiary }{ }^{\text {liab }} \text { and Beneficiary }{ }^{\text {vehi }}
$$

nominated for the case of a liability insurance of a vehicle can be -- characterized so that two roles may be performed by:

- any person who happens to drive Mary's vehicle and
- any person who happens to become a victim of an accident "caused by Mary's vehicle".

It is the phenomenon of this collectivity and anonymity that gives way to the broadly shared misinterpretation that what is insured is the vehicle rather the respective agent - be it Mary, Vera or Benjamin.

### 25.4 Universal structure

Similarities and differences between the two kinds of insurances may be further illuminated as follows:

- We have fully left aside the peculiarity that in most jurisdictions every owner is obliged to cover his-her vehicle by the liability insurance, which obligation in fact contradicts the very notion of a contract.
- Life insurance is specific in that the Beneficiaries - as a rule - are not informed about their right to submit their BEN-orders. Hence, if Richard-the Insurer has the information that Vera has died and who are her children, the question is to what extent will be Richard obliged to share this information with an uninformed Beneficiary ${ }^{\text {life }}$. In the case of vehicle insurance, this kind of the information asymmetry is in practice often resolved through somehow organized "central register of vehicles".
- Also Beneficiaries in life insurance may be nominated collectively and anonymously. For example, instead of Benjamin-the most favorable son of Vera the RM-the Designer could have nominated as a Beneficiary "Vera's existing children".
Assuming the above established anonymity and collectivity of HIVs we will claim that the architecture of sections (a) and (b) of Fig. 78 may be taken as a design universally applicable upon any insurance relationship. Hence, other kinds of insurance contracts may be taken as a mere simplification of the universal architecture from Fig. 78.


## 26. EXCLUSIONS FROM A CONDITION

In Fig. 61 and Fig. 62 of Comment 5 we showed how the notion of exclusion can be transferred from the insurance jargon to the analysis of the Firm. Now, back in its "home" context of insurance, we shall attempt to somewhat enrich the concept.

### 26.1 Example

As said, in this Comment 7 we shall mostly focus on a system/process designed by a (compulsory) liability insurance of a vehicle, namely the conditions under which the task of an Insurer may be prescribed.

To begin with we will assume Richard-the Insurer in the form:

$$
I N(1) \equiv[i n:\langle a, d\rangle \rightarrow\langle B\rangle]
$$

where:

- the THEN-component $B$ represents a monetary value of the prospective benefit/recovery,
- the IF-component is designed by the following exogenous variables:
$a$ the harm/injury that is covered by the insurance,
$d$ the time at which the harm/injury occurs.
For concreteness, $a$ will be an alpha-numeric variable representing a diagnosis of a bodily injury.

Mapping [in: $\langle a, d\rangle \rightarrow\langle B\rangle$ ] will be, as before, graphically represented by a set of two dimensional contours of the output variable $B$.

### 26.2 Examples

The domain of $[i n:\langle a, d\rangle \rightarrow\langle B\rangle]$ and the contours representing values of $B$ are depicted in Fig. 79. As said, if the external event is a bodily injury, the respective condition is designed by a set $\Delta a$ of diagnoses covered by the insurance under study.

Variables $a, d$ in Fig. 79 are organized along their axes from their "bad values" $a^{b}$ and $d^{b}$, respectively to the "good ones" $-a^{g}$ and $d^{g}$. The value judgments "good" or "bad" are interpreted in the sense of the north-east red arrow. (For illustrative purposes only the arrow indicates that the coverage is the smaller the lighter is the injury and the later it occurs.)

### 26.2.1 Insured period

The time interval $\Delta d=\left\langle d^{8}, d^{b}\right\rangle$ is a so-called insured period that represents the times at which the injury must occur should any coverage be prescribed. We assume in Fig. 79 that the accident "must" occur within an insured period of two years:

- starting on $d^{b}=$ January $1^{\text {st }} 2018$,
- ending on $d^{g}=$ December $31^{\text {st }} 2019$.


In contrast, the insured period in Fig. 80 is not compact. The non-shaded area above the time interval $\Delta d^{e x}$ represents the fact that no Mary's injury will be insured during $\Delta d^{e x}$. For example, Richard and Mary may have agreed on this provision on the grounds, that Mary's vehicle will not be used on public roads during the first half of the year 2019.
Later, with the aim to approach LS-parlance, we will interpret $\Delta d=\left\langle d^{b}, d^{g}\right\rangle$ as a time period within which the operational unit $\operatorname{IN}(1) \equiv[i n:\langle a, d\rangle \rightarrow\langle B\rangle]$ is effective. Hence the lower bound $d^{b}$ of the interval will be seen as the time at which $I N(1)$ becomes effective as compared to the time at which the same $\operatorname{IN}(1)$ becomes valid.

### 26.2.2 Insured diagnoses

As said, the interval $\Delta a=\left\langle a^{b}, a^{g}\right\rangle$ in Fig. 80 consists in all diagnoses that are covered by Richard-the Insurer - except for a broken elbow denoted as $a^{e x}$ in Fig. 80. The rationale for not including $a^{e x}$ may be based on infavourable statistics: Should $a^{e x}$ be also covered, the insurance would become disproportionally more expensive - Mary would have to pay substantially higher premium.


It is noteworthy that the terminology discussed is rather arbitrary, as shown by uncovered injuries $a^{0}$ and $a^{00}$ that are not likely to be called "exclusion" due to the purely formal fact that they do not fall into the insured interval $\Delta a=\left\langle a^{b}, a^{g}\right\rangle$.

### 26.2.3 Domain of justifiability

The domain of the mapping

$$
I N(1) \equiv[\text { in: }\langle a, d\rangle \rightarrow\langle B\rangle]
$$

shown in Fig. 80 is constituted by two unions
$\Delta d^{+} \cup \Delta d^{++}$
$\Delta a^{+} \cup \Delta a^{++}$
For only combinations $(d, a)$ that fall into thus established domain the above mapping is defined, exists. If, e.g., the state $A^{+}=\left(d^{0}, a^{00}\right)$ occurs, it will be - with respect to the insurance concerned - as irrelevant as, e.g., yesterday's temperature in Rhodes, Greece.

## 27. REPEATED PRESCRIPTIONS OF A TASK

### 27.1 Introduction

Let us return to the four production cycles depicted in Fig. 4 where John-the Designer had a choice to selects his strategy for the coming cycle. We assumed, for the sake of that analysis that the set of variant strategies always consists in three mutually different operational units $\left\{M(1)_{1}, M(1)_{2}, V(1)\right\}$. Recall, then, that at the start of Cycle 4 John decided not to change his strategy and keep to $M(1)_{2}$ applied in the preceding Cycle 3. Our interpretation then was that John, as a Designer, decided for a non-action.

Now our interpretation will be somewhat different. We will say that the respective Benjamin-the Beneficiary will for the sake of Cycles 3 and 4 repeatedly demand prescription of "the same task".

Analogously, we will now assume an insurance contract covering two instances of the "same" kind of a bodily injury, e.g. Benjamin's broken arm, elbow, ... Put formally, let RM-the Designer design a three-unit system/process

$$
\operatorname{SIN}(1)=\left(\operatorname{IN}(1)_{1}, \operatorname{IN}(1)_{2} ; C L(1)\right)
$$

where by $\operatorname{IN}(1)_{1}, \operatorname{IN}(1)_{2}$ can be compensated - loosely said - the first and second occurrence of the injury. Apparently, then, if Benjamin suffers three or more injuries of the kind, only two of them can be compensated.

In our IT-parlance we could also say that a given $\operatorname{IN}(1)$ has been designed "for two usages".

### 27.2 Misleading "equivalence"

### 27.2.1 Introduction

Also it may be tempting to interpret the pair $\operatorname{IN}(1)_{1}, \operatorname{IN}(1)_{2}$ so that the RM-the Designer has decided to apply the "same" strategy for every insurance cycle, where the cycle is - loosely said, again - established by repeated instance of a broken arm.

However, the above intuition may be often misleading. The two operational units $I N(1)_{1}, I N(1)_{2}$ may often only appear to be "identical". In reality, as we will show shortly, their actual form will be - almost unnoticeably - different.

Moreover, we will also stress that that the two tasks must be demanded by separately submitted orders, denoted further as BEN-order/1 and BEN-order/2.

### 27.2.2 Domains of time

In (a) of the following Fig. 81 Richard's $I N(1)_{1}$ and $I N(1)_{2}$ are designed as

$$
\begin{aligned}
& I N(1)_{1} \equiv\left[i n:\left\langle a ; \Delta d^{1}\right\rangle \rightarrow\langle B\rangle\right] \\
& I N(1)_{2} \equiv\left[i n:\left\langle a ; \Delta d^{2}\right\rangle \rightarrow\langle B\rangle\right]
\end{aligned}
$$

In words, they are the same" except for their mutually exclusive time intervals $\left\langle\Delta d^{1}\right\rangle$ and $\left\langle\Delta d^{2}\right\rangle$. Put differently, each of the two mappings is defined (exists) for a different - subsequent, mutually exclusive - insurance periods. In this sense the two periods $\left\langle\Delta d^{1}\right\rangle$ and $\left\langle\Delta d^{2}\right\rangle$ are of the same meaning as the above mentioned sequence of a production Cycles 3 and 4 . In other words, within $I N(1)_{1}$ and $\operatorname{IN}(1)_{2}$ the injuries under coverage are the same but the times of their occurrence must fall into two different time intervals - e.g. June $\left\langle\Delta d^{1}\right\rangle$ and July $\left\langle\Delta d^{2}\right\rangle$.

Hence, if Benjamin's elbow $a^{+}$broken by Vera's car is in $\langle a\rangle$, he:

- can be recovered twice if the two car accidents occur on, e.g., June $30^{\text {th }}$ and July $1^{\text {st }}$,
- can be recovered only once if the two accidents occur on, e.g., June $1^{\text {st }}$ and June $30^{\text {th }}$.

We should also note that in the latter case Benjamin will face a question which of the two June-injuries he will claim by his respective BEN-order/1.

### 27.2.3 Different recoveries of the same diagnosis

The two mappings can be different not only with respect to the time of occurrence but also the way how the benefit/recovery is calculated depending on whether it is June-injury or July-injury. Analytically, the difference can be expressed by the following difference between $i n_{1}$ and $i n_{2}$ :

$$
\begin{aligned}
& I N(1)_{1} \equiv\left[i n_{1}:\left\langle a ; \Delta d^{1}\right\rangle \rightarrow\langle B\rangle\right] \\
& I N(1)_{2} \equiv\left[i n_{2}:\left\langle a ; \Delta d^{2}\right\rangle \rightarrow\langle B\rangle\right]
\end{aligned}
$$

For illustration we show in (a) of Fig. 81 the two different mappings so that two different benefits/recoveries $B^{1} \neq B^{2}$ are associated to the "same" broken elbow $a^{+}$.

### 27.2.4 Domains of diagnoses

In (b) of Fig. 81 the pair $\operatorname{IN}(1)_{1}$ and $\operatorname{IN}(1)_{2}$ is designed so that the exogenous variable $a$ is divided into two categories of diagnoses

$$
\left\langle\Delta a^{1}\right\rangle \text { and }\left\langle\Delta a^{2}\right\rangle
$$



Again, only two benefits/recoveries can be awarded. The two respective injuries may occur whenever in June and July but must be of a mutually different category.

### 27.3 Associated notes

### 27.3.1 Mixed cases

To interpret the four designs $I N(1)_{1}, \ldots, I N(1)_{4}$ depicted in (c) of Fig. 81 let us take, for illustration, Richard's $I N(1)_{4}$ that states that only one injury can be covered from those:

- that belong to the diagnoses $\Delta a^{1}$ and
- occur during the first half $\Delta d^{2}$ of the overall insurance period $\Delta d$.

In sum: Only one injury of a given category can be covered from those that occur during a given half of the overall insurance period $\Delta d$.

### 27.3.2 Overlapping domains

Only to illustrate the almost infinite variety of designs, let the following

$$
\operatorname{SIN}(1)=\left(\operatorname{IN}(1)_{1}, I N(1)_{2}, I N(1)_{3} ; C L(1)\right)
$$

consists, in:
$\left(\operatorname{IN}(1)_{1}, I N(1)_{2} \quad\right.$ depicted in (a) of Fig. 81 and
$\operatorname{IN}(1)_{3} \quad$ whose insurance period is an union of the above two, $\Delta d^{3}=\left(\Delta d^{1} \cup \Delta d^{2}\right)$, as shown in Fig. 82.


Hence, in the state $A^{0}=\left(d^{0}, a^{0}\right)$ the respective Beneficiary "must" choose whether his-her BEN-order will be aimed at $I N(1)^{1}$ or $I N(1)^{3}$.

### 27.3.3 Infinite number of events

Let the contractual system/process have the following structure

$$
\operatorname{SIN}(1)=\left(\operatorname{IN}(1)_{1}, I N(1)_{2}, \ldots, I N(1)_{\infty} ; C L(1)\right)
$$

where Richard-the Insurer is to cover, theoretically speaking infinite number of damages and injuries that may occur during the two years of the overall insurance period.
In Fig. 83 every $i^{\text {th }}$ intersection of the - theoretically - infinite number of yellow horizontals with green verticals represents a particular combination of time and kind for which a particular $\operatorname{IN}(1)_{i} i=1,2, \ldots, \infty$, is designed. To every such combination thus, in general is associated different coverage, whose magnitude is represented by its pink isoquants.


For dramatic effect, the white rectangle represents again the respective exclusions from Richard's task.

### 27.4 Rivalry between repeated tasks

The case of a repeated prescription provides a nice opportunity to return to the following two general topics.

We showed in Fig. 78 that - in LS-parlance - the two kinds of contracts are both formed in favor of two kinds of third persons who stand - as if - outside the contract but still are the only ones who have a right to submit the respective BEN-order.

As already explained, by using the term "right" we remind the reader that in reality any such Beneficiary can be not only nominated but also designed by conditions under which he-she will have to submit the respective BEN-order.

Here we will assume that it is rather common in the "world of insurance" that a Beneficiary has a genuine choice to decide whether and how they will submit their claims. Put differently, we will assume that a Beneficiary has the choice not to claim even if it is more than obvious that his-her claim would be justified.

Moreover, given that there are "several pieces of a given task" - such as, e.g., $\operatorname{IN}(1)_{1}, I N(1)_{2}, I N(1)_{3}$ in Fig. 82, it may cause practical problems to distinguish which of them which of the Beneficiaries decided not to claim.

## 28. SUBSTANTIVE VS. PROCEDURAL CONDITIONS

### 28.1 SP-representation

For the sake of the analysis we will expand the design of Richard-the Insurer into

$$
I N(1) \equiv[i n:\langle a ; d, t\rangle \rightarrow\langle B\rangle]
$$

where, again, $a$ represents diagnoses, $d$ is the time at which the harm/injury occurs and
$t$ is the time at which the BEN-order is submitted
Put the same in the SP-representation $I N(1) \equiv\left[i n:\left\langle\left\langle i \overrightarrow{i n}_{\text {ben }}\right\rangle,\left\langle i \overrightarrow{i n}_{\text {def }}\right\rangle,\left\langle i \vec{n}_{\text {man }}\right\rangle\right\rangle \rightarrow\left\langle\vec{T}_{m}\right\rangle\right.$ we may write:
$\left\langle\overrightarrow{i n}_{\text {ben }}\right\rangle=\langle a ; d, t\rangle$
$\left\langle\overrightarrow{i n}_{\text {def }}\right\rangle=0$
$\left\langle i \vec{n}_{\text {man }}\right\rangle=0$
and hence re-open the question how to differentiate substantive vs. procedural conditions of a task. Earlier, we suggested that it is the sub-domain $\left\langle\overrightarrow{i n}_{\text {man }}\right\rangle$ that represents the procedural "aspects" of an operational unit. At this point, intuitively speaking, we can see that also the time at which the BEN-order "must" be submitted can be ascribed the procedural character.

In this BOOK we shall not go any deeper into this terminological problem and directly turn to Fig. 84 describing how the coverage/benefit $B$ will change with the time $t$ - apart from the time $d$ and on the proviso that $a$ is held constant.


Fig. 84
For the sake of the analysis we shall assume that the required relationship between the two times will be designed as

$$
t \in\left(d+\Delta t^{+}\right)
$$

with the empirical meaning that:
if $t<d \quad$ the BEN-order is incorrect because submitted "too early", i.e. before the injury has ever occurred: cf. the state $\left(d^{0}, t^{0}\right) \in E_{2}^{1}$ in Fig. 84,
if $t>\left(d+\Delta t^{+}\right) \quad$ the BEN-order is incorrect because it is submitted "too late" - it is not submitted "without delay", i.e. within a given time limit $\Delta t^{+}$after the injury: cf. the state $\left(d^{0}, t^{0}\right) \in E_{2}^{2}$ in Fig. 84.

### 28.2 Impact on coverage

The following Fig. 85 is a mere transposition of Fig. 84 from the space $(d, t)$ into the space of Richard's states $(d, \Delta t)$, where

$$
\Delta t=(t-d)
$$

The pink downward sloping curves in Fig. 85 represent the Designer's decision to reward the speed with which a BEN-order is submitted. In words, Richard and Mary are assumed that the coverage $B$ will increase with the "promptness" with which Benjamin-the Beneficiary will claim his injury. The rationale for this bonus may be that transaction costs of the Assessor are likely to decrease with the speed.


The state $A^{0}=\left(d^{0}, \Delta t^{0}\right)$ Fig. 85 is selected so as to demonstrate the case when $\Delta t^{0}<0$, i.e. the state in which

$$
t<d
$$

because the BEN-order is submitted before the respective event ever occurred.
Similarly we could assume that Richard and Mary will want to make the coverage depend on the promptness with which Manuela-the Assessor will "process" Benjamin's BEN-order.

### 28.3 Ex ante demand

Let $I N(1) \equiv[i n:\langle a ; d, t\rangle \rightarrow\langle B\rangle]$ be somewhat modified so that, as shown Fig. 86, no limit $\Delta t^{+}$is now imposed on the above required promptness of the BEN-order and, more importantly, BEN-orders may be submitted ex ante, i.e. so that

$$
t<d \text { up to some limit } \Delta t^{\mathrm{EA}}
$$

Leaving aside details, two notes may somewhat illuminate the arrangement:

- Given that Benjamin may always choose not to submit his-her BEN-order, he may also want to let "everybody" know ex ante that he will not give up his "right to claim".
- The ex ante order submitted at $t<d$ may be also interpreted as a demand that the Assessor will repeatedly, e.g. during the insurance period $\Delta d$, attempt to validate it, for the sake that the "awaited" event occurs.



## 29. VALIDITY VS. EFFECTIVENESS OF DESIGN AND RE-DESIGN

### 29.1 Validity of design

The most trivial condition under which an operational unit may be prescribed is that the unit ever exists, that it is in a well defined stage $\operatorname{IN}(1)$ from which only the transition $I N(1) \Rightarrow I N(2)$ can be executed.
The problem concerned here is that once designed, the stage $I N(1)$ can be re-designed, e.g. for the sake of a new behavioral cycle, as explained as early as in Fig. 4 of PART I. To take hold of the problem, we will establish
time $\tau^{a c}$ at which Richard-the Insurer has been designed in the actual formula $\operatorname{IN}(1)_{a c}$
time $\tau^{r d}$ at which Richard-the Insurer has been re-designed into $I N(1)_{r d}$ due to the transition $I N(1)_{a c} \Rightarrow I N(2)_{r d}$.

In other words, we will show in what sense it will be relevant for the coverage to be paid that the actual formula $\operatorname{IN}(1)_{a c}$ "exists" only within the time interval

$$
\left\langle\tau^{a c}, \tau^{r d}\right\rangle
$$

Using the LS-parlance, the actual mapping

$$
I N(1)_{a c} \equiv\left[i n_{a c}:\left\langle a_{a c} ; d_{a c} t_{a c}\right\rangle \rightarrow\left\langle\vec{T}_{i n / a c}\right\rangle\right]
$$

can be said to be valid since $\tau^{a c}$ until the time $\tau^{r d}$ at which it is replaced with some other $I N(1)_{r d} \equiv\left[i n_{r d}:\left\langle a_{r d} ; d_{r d} t_{r d}\right\rangle \rightarrow\left\langle\vec{T}_{i n / r d}\right\rangle\right]$.

### 29.2 Effectiveness of design

### 29.2.1 Insurance period

At the time $\tau^{a c}$ Manuela-the MO confirms that the Negotiation 1 between Richard and Mary has been "successfully" completed, that their insurance contract $\operatorname{SIN}(1)$ has become valid or, put differently, has been created, formed, ... come to its existence.


Fig. 87
As shown in Fig. 87, our focus will now fall on the relationship between the time of validity $\tau^{a c}$ and the lower bound

$$
d^{b}=\mathbf{\tau}^{a c / e f}
$$

of the insurance period that will be interpreted as the time from which Richard's task becomes effective.
Summarizing then, at $\tau^{a c}$ the Richard and Mary jointly designs that $\operatorname{IN}(1)_{a c}$ will be effective since $\tau^{a c l e f}$.

### 29.2.2 Postponed effectiveness

In Fig. 87 the example is selected so as to show the "natural" arrangemen

$$
\boldsymbol{\tau}^{a c l e f}>\boldsymbol{\tau}^{a c}
$$

where $I N(1)_{a c}$ becomes valid before the start of the insured period. In LS-parlance this kind of inequality can be read so that the contracting parties decided to postpone the effectiveness of their contract. In other words $\operatorname{IN}(1)_{a c}$ that is valid from $\tau^{a c}$, becomes effective only from

$$
\boldsymbol{\tau}^{a c / e f}=\left(\boldsymbol{\tau}^{a c}+\Delta \tau^{a c}\right)
$$

where $\Delta \tau^{a c}>0$ represent the number of days by which the effectiveness is postponed. The empirical meaning of the postponement is that injuries that occur before $\tau^{a c l e f}$ will not satisfy the necessary justifiability condition.
To illustrate, the state $A^{0}=\left(d^{0}, a^{0}\right)$ in Fig. 87 is infeasible even though the respective contract is valid since $\tau^{a c}$ already and the diagnose $a^{0}$ belongs to injuries that are covered by the insurance. Hence, the state $A^{0}=\left(d^{0}, a^{0}\right)$ is as irrelevant with respect to the coverage as, e.g., again, yesterday's temperature in Rhodes, Greece.

### 29.2.3 Retro-active effectiveness

For the sake of the analysis we will now assume in Fig. 88 the opposite inequality

$$
d^{0}<\boldsymbol{\tau}^{a c}
$$

or

$$
\boldsymbol{\tau}^{a c l e f}=\left(\tau^{a c}-\Delta \tau^{a c}\right)
$$

where $\Delta \tau^{a c}>0$ represent the number of days by which the effectiveness is retroactive. Leaving aside jurisdictions that may prohibit this kind of insurance, we will illustrate in Fig. 88 on the state $A^{0}=\left(d^{0}, a^{0}\right)$ that the injury may be covered even though it had occurred "long before" the contract was formed.


### 29.2.4 Ex ante retro-active order

For dramatic effect we will now return to Fig. 84 and Fig. 85 and assume that:

- the BEN-order can be justifiably submitted ex ante, i.e. before the injury occurred, $t<d$, if within the limit $\Delta t^{E A}$,
- the injury can justifiably occur before Richard's $I N(1)_{a c}$ is designed, $d<\tau^{a c}$, if within the limit $\Delta t^{a c}$,

For completeness, the limit $\Delta t^{+}$of promptness from Fig. 84 is applied in Fig. 89 on BEN-orders submitted after the time $t^{a c}$ of the contract's formation.


### 29.3 Consensual re-design

Formally, a re-design of $I N(1)$ has been denoted as a transition $I N(1)_{a c} \Rightarrow I N(1)_{r d}$. In what follows we shall differentiate according to whether re-designs are consensual and enforced.

### 29.3.1 Consensuality of a transition

For the convenience of the reader let us recall that the stage $I N(1)$ has been established as an outcome of a "match" of two DESIGN-orders or, put differently, as an outcome of a specific election called Negotiation 1.

Let us also recall the default rule according to which if the Addressee of a DESIGN-order does not respond he-she is taken as if he-she has rejected the "proposal". In this sense the design is consensual and, by definition, cannot be "unilaterally" enforced.

Analogously, it appears natural to assume that the state that was consensually designed can be re-designed by only a bi-lateral consensus. Hence it is also natural to assume that the re-design $I N(1)_{a c} \Rightarrow I N(1)_{r d}$ is also a transition of the above consensual kind.

As a result, then, the preceding analysis of DESIGN-orders can be - mutatis mutandis - essentially applied - upon the respective pair of

## RE-DESIGN-orders

with the obvious advantage that in the case of a re-design the counter-parties are ex ante known. Put differently a RE-DESIGN-order can be justifiably submitted by only persons who are already in the roles of the actual contract parties, namely Richard and Mary.

### 29.3.2 Example

For notational convenience and concreteness:
the actual design $\operatorname{IN}(1)_{a c}$
the outcome $I N(1)_{r d}$ of the re-design
is in (a) of Fig. 90 assumed to be valid since $\tau^{a c}$ and effective since $\tau^{a c l e f}$,
is in (b) of Fig. 90 assumed to be valid since $\tau^{r d}$ and effective since $\tau^{r d / e f}$.

The content of the re-design can be summarized so that at $\tau^{r d}$ Manuela-the Manager delivered her verdict according to which - effectively from $\tau^{r d / e f}$ :

- the time exclusion $\Delta d^{e x}$ is shortened and hence Benjamin-the Beneficiary will be covered for a somewhat enlarged "insured period",
- the diagnosis exclusion $\Delta a^{e x}$ is abolished and hence Benjamin-the Beneficiary will be covered for an enlarged "range of diagnoses",
- recovery will be calculated according to a new formula.

The example has been selected so that to illustrate a case when the existing task of the Insurer has been re-designed in favor of Benjamin-the Beneficiary, e.g., under the pressure of Richard's competitor.

### 29.3.3 Rivalry between original and new design

Confusing and largely practical problems arise from the obvious fact that once the re-design is valid a clear border line must be drawn between the new design and its effectiveness.

To illustrate, let Benjamin-the Beneficiary claim an injury characterized by the combination

$$
A^{+}=\left(a^{e x}, d^{+}\right)
$$

shown in (b) of Fig. 90. Then his BEN-order will be processed on the bases of the original $I N(1)_{a c}$ even though the design $\operatorname{IN}(1)_{r d}$ is already valid. The apparent reason is that the time $d^{+}$of the injury's occurrence is

$$
\boldsymbol{\tau}^{d s l e f}<d^{+}<\boldsymbol{\tau}^{r d / l e f}
$$

and hence at $d^{+}$the re-design is valid by not effective yet.

### 29.4 Enforceable re-design

### 29.4.1 Enforceability of a transition

Intuitively speaking enforceability of a re-design $\operatorname{IN}(1)_{a c} \Rightarrow \mathrm{IN}(1)_{r d}$ is only a little more likely than the design. In the case of an enforceable RE-DESIGN-order, let us summarize that:

- an Addressee may respond with only a limited "arsenal of defenses" and
- if the Addressee remains silent he-she is taken as if he-she has fully accepted the order.


Fig. 90

### 29.4.2 Methodological notes

The interpretation of enforceable re-design in Fig. 91 is based upon the following expansion of the so far applied mapping:

$$
I N(1)_{a c} \equiv\left[i n_{a c}:\left\langle\overrightarrow{i n}, i \overrightarrow{n^{r d}}\right\rangle \rightarrow\left\langle\vec{T}_{i n \prime} \operatorname{IN}(1)_{r d}\right\rangle\right]
$$

Under two different conditions designed by $\overrightarrow{i n}$ and $\overrightarrow{i n}^{r d}$, two different agents Beneficiary ${ }^{\text {in }}$ and Beneficiary ${ }^{\text {rd }}$
in general may justifiably demand transitions towards new stages $\operatorname{IN}(2)_{a c}$ and $I N(1)_{r d}$, respectively.


As many times before, also a deeper analysis of the arrangement in Fig. 91 would deserve a separate book-size analysis:

- If Mary-the Client is both the Beneficiary ${ }^{\text {in }}$ and Beneficiary ${ }^{\text {rd }}$ her demand may be unclear as to whether she demands prescription $\operatorname{IN}(2)_{a c}$ or re-design $I N(1)_{r d}$. If conditions for both transitions are satisfied the ambiguity must be resolved by Manuela's ad hoc regulation.
- The three instance in Fig. 91 of the would be transitions are differentiated according to whether it is a behavior/action of a Designee or Designer,
- the three broken arrows will be later characterized as a pseudo-split so as to strictly separate them arrows that represent a genuine SPLIT.


## COMMENT 8.

## Multi-unit strategies; the case of a SPLIT

For the remainder of this BOOK we will abandon the case of a collective choice and return to the individual Designer named John. He will be assumed to design a SPLIT-type three-unit two-phase system/process represented by the section (c) of Fig. 8.

Our discussion will be organized as follows:
Comment 8 will corroborate the notion of a SPLIT primarily from a technical point of view with the aim to pinpoint widespread confusion between:

- a "pseudo-split" defined by two or more variant developments of a given operational unit and
- a SPLIT proper established by two or more operational units who as Followers depend on the behavior/action of their common Leader.
Comment 9 will fully focus on the empirical meaning that the pattern (c) of Fig. 8 may well represent.


## 30. PSEUDO-SPLIT

For the sake of this chapter we will return to Fig. 17 where John designed Charles and Mary as the two-unit two-phase Firm.

### 30.1 Example 1

### 30.1.1 Simplifications and notations

Let the Firm be designed as follows:
Charles is a Supplier of leather designed as $K(1) \equiv[k:\langle\vec{k}\rangle \rightarrow\langle k, d\rangle]$
Mary is a Manufacturer of shoes designed as $Q(1) \equiv[q:\langle k, d\rangle \rightarrow\langle Q, D\rangle]$, where:
$k, d \quad$ represent Charles' delivery of leather, namely its magnitude and time, respectively,
$Q$ and $D$ represent two parameters of Mary's task, namely magnitude and time of shoes, respectively.

The broken arrows in Fig. 92 then depict mutually exclusive responses of Mary to mutual different deliveries of leather. Every broken arrow in Fig. 92 thus represents a variant in which Phase 2 can be prescribed.


### 30.1.2 Variant verdicts

Put differently, the broken arrows can be interpreted as variant verdicts of Manuela-the Manager over whether and in what form Mary's task will be prescribed. Hence, in the language of Fig. 39, every broken arrow represents a particular outcome of the order routing of the respective BEN-order.
Let us stress that in Fig. 92 we assume that John has designed $Q(1)$ so that Phase 2 may be prescribed in a finite number of instances. For concreteness we assumed, that the respective BEN-order may lead to (only) four verdicts:

- three verdicts in the affirmative, i.e. three mutually exclusive prescriptions of Mary's non-empty behavior/action $Q(2)$ shown in the blue shaded area of Fig. 92,
- one verdict in the negative resulting in Mary's non-action $Q(1)$.

In sum, $Q(1)$ is designed so that the (generally) infinite number of states $(k, d)$ may lead to only four verdicts. Put formally, in Fig. 92

$$
\langle k, d\rangle^{\mathrm{i}}, \boldsymbol{i}=0,1, \ldots, 3,
$$

is the $\boldsymbol{i}$-the set of (internal) events/conditions to which is associated the same verdict of Manuela-the Manager. In other words, the states $(k, d) \in\langle k, d\rangle^{\mathrm{i}}$ are equivalent with respect to the particular verdict.

### 30.1.3 Graphical representation

The array of broken arrows in Fig. 92 will be referred to as a pseudo-split in order to differentiate it from a morphologically akin array of solid arrows called a SPLIT proper - such as that depicted as early as in Fig. 28 of Comment 3.

It is of a particular importance to emphasize that Manuela's verdicts shown in Fig. 92 are:

- exhaustive by assumption (one of them must occur),
- mutually exclusive (one and only one can occur).

Apparently, pictures like that in Fig. 92 can be "technically" drawn only on the proviso that the number of verdicts or broken arrows is not only finite but also small enough, e.g. up to 4 or 5.

### 30.1.4 Classification of verdicts

As stressed, every verdict is a multi-dimensional entity. It is then only natural to divide the set of mutually exclusive verdicts according to various criteria.

The elemental - trivial - classification is already in Fig. 92, where the four verdicts are grouped according to whether Manuela's verdict is in the affirmative $Q(2)$ or negative $Q(1)$.

Then, within the three affirmative verdicts two criteria of their classification are shown in (a) and (b) of Fig. 93. The danger of confusion is then brought up by the fact that on one branch of a given pseudosplit we establish a lower-lever pseudo-split.


### 30.2 Example 2

### 30.2.1 Macro-tasks

Major confusions are brought up by not respecting the distinction between Mary-the Designee (an Owner of the operational unit $Q(1)$ ) and particular Executors of Mary's task, e.g. Mr. Strong and Miss Weak who are the only two employees of Mary. The problem then arises when - as will be the case here - the magnitudes $Q>100$ will be prescribed so that the shoes will have to be delivered by both employees.

To illustrate the problem we will now expand the design of Mary's task by two additional parameters, namely an Executor and a place of delivery and compare the following verdicts of Manuela-the Manager:

Verdict 1 the amount is 99 pairs and the shoes must be delivered so that Mr . Strong will deliver everything to London in August.
Verdict 2 the amount is 104 pairs and the shoes must be delivered so that: - Mr. Strong will be to deliver 84 pairs to London in June and - Miss Weak will be to deliver 20 pairs to Prague in September.

Verdict 3 the amount is 65 pairs and the shoes must be delivered so that Miss Weak will deliver everything to Tokyo by the end of December.

### 30.2.2 Macro-Executor

Now we can invoke our earlier discussion of a macro-task and its partial fulfillment in (a) of Fig. 45 and assume that Mary's design is

$$
Q(1) \equiv\left[q:\langle k, d\rangle \rightarrow\left\langle Q,\left(L_{s t} \cup L_{w k}\right)\right\rangle\right]
$$

where $\left(L_{s t} \cup L_{w k}\right)$ is a macro-Executor characterized, largely intuitively, so that $L_{s t}$ and $L_{w k}$ are alpha-numeric variables each of which may obtain only two values:
$L_{s t}=\varnothing$ (nobody), and $L_{s t}=M r$. Strong
$L_{w k}=\varnothing$ (nobody), and $L_{w k}=$ Miss Weak
For example, in (a) of Fig. 94 to the set of states $\langle k, d\rangle^{1}$ corresponds one single verdict

$$
\left(Q,\left(L_{s t}, L_{w k}\right)\right)^{1}=[99,(\text { Mr. Strong, } \varnothing)]
$$

with the meaning that the magnitude $\mathrm{Q}^{1}=99$ pairs will be delivered solely by Mr. Strong.

### 30.2.3 Classification of verdicts

In section (b) of the picture a lower-lever pseudo-split is established so as to represent the classification of the three verdicts according to whether the Executor will be of a collective or individual nature.


### 30.3 Digression: analytically "friendly" arrangements

### 30.3.1 Continuum of states and prescriptions

As already stressed, analytically most comfortable designs will be such that will allow for an infinitesimal calculus. Hence, scientists will always dream about arrangements under which Mary will be designed by a mapping $\left[q:\langle\vec{q}\rangle \rightarrow\left\langle\vec{T}_{q}\right\rangle\right]$ that will have "all the good mathematical properties", namely be continuous and differentiable.

As warned, no such dream is possible in the universe or real-life phenomena where most variables are alpha-numeric vector strings. In addition to this, the obvious disadvantage of "continuity" is that there is no way how to represent graphically individual instances of the phenomenon under study.

### 30.3.2 Binary abstraction

To begin with we should remind the reader of our earlier discussion of a representative state in Comment 5, e.g. in Fig. 53. Now, using the above mapping $\left[q:\langle\vec{q}\rangle \rightarrow\left\langle\vec{T}_{q}\right\rangle\right.$ ] as an example, the binary abstraction will allow for prefabrication of two representative states

$$
\vec{q}^{b} \text { and } \vec{q}^{g}
$$

that will replace the four sets of states $\langle k, d\rangle^{i}, \boldsymbol{i}=0,1, \ldots, 3$ so that:
the bad state $\vec{q}^{b} \quad$ will represent all unfeasible states - ELSE in Fig. 92 - and the corresponding bad verdict $(Q, D)^{b}$, in the negative, i.e. the verdict $(Q, D)^{g}=Q(1)=(\varnothing, \varnothing)$,
the good state $\vec{q}^{g} \quad$ will represent all feasible states to which $(Q, D)^{g}$ can be associated as the only good verdict in the affirmative.

In Comment 10 our subject will converge towards a binary yes-or-no choice from two representative firms each of which will be represented by one single representative/referential state of its environment and hence also one single referential optimal profit.

## 31. SPLIT PROPER

### 31.1 Three-unit two-phase production

### 31.1.1 Multi-unit Phase 2

Recall that so far, namely in Fig. 94, Mr. Strong and Miss Weak have been established as two non-zero values of two endogenous variables $L_{s t}$ and $L_{w k}$ by which the THEN-component of Mary is designed. Recall that John-the Designer was allowed to involved Mr. Strong and Miss Weak into the design because they are subordinated to Mary, namely as her employees, the only two employees in particular.

Now, in Fig. 95 the two persons - Mr. Strong and Miss Weak - will be designed into entirely different roles. John-the Designer will design Phase 2 of the system/ /process so that it will consist in two operational units, namely $S T(1)$ personified by Mr. Strong and $W K(1)$ personified by Miss Weak.


Mr. Strong and Miss Weak are now two separate Designees designed as follows:
IF-components of $S T(1)$ and $W K(1)$ consist, again, in only two above defined variables $k, d$,
THEN-components of $S T(1)$ and $W K(1)$ will consist in only magnitudes $Q_{\text {st }}$ and $Q_{\mathrm{wk}}$ of shoes to be delivered, respectively.

In sum, we shall abstract from external events/conditions and hence Mr. Strong and Miss Weak will be "pure, $100 \%$ " Followers of their common Leader Charles.

### 31.2 Rivalry/competition

### 31.2.1 Overlapping domains

Whereas logic gate JOIN may establish rivalry between two or more Suppliers of Leather (cf. Fig. 25) a SPLIT may lead to a rivalry/competition between recipients of the leather.

To take hold of the topic, we have established in Fig. 96:
$\langle k, d\rangle_{\mathrm{st}} \equiv\left\langle\Delta k_{\mathrm{st}} \Delta d_{\mathrm{st}}\right\rangle \quad$ as a domain of the design of Mr. Strong,
$\langle k, d\rangle_{\mathrm{wk}} \equiv\left\langle\Delta k_{\mathrm{wk}}, \Delta d_{\mathrm{wk}}\right\rangle \quad$ as a domain of the design of Miss. Weak.

We can read from (a) of Fig. 96 that the domains are selected so as to demonstrate their overlapping

$$
\Delta d_{w l} \subset \Delta d_{s t} \text { and } \Delta k_{w k} \subset \Delta k_{s t}
$$

and hence a potential conflict between Mr. Strong and Miss Weak.


The three deliveries $A, B$ and $C$ shown in (a) of Fig. 96 should illuminate the problem of a rivalry.

| delivery | (non-empty) <br> behavior/action | interpretation |
| :--- | :--- | :--- |
| $A=(Q(+3), d(+3))^{1}$ | Mr. Strong will be <br> prescribed to deliver shoes | Miss Week remains <br> inactive |
| $B=(Q(+3), d(+3))^{2}$ | Both Mr. Strong and Miss <br> Week will be prescribed to <br> deliver shoes | Potential conflict |
| $C=(Q(-3), d(-3))$ |  | Both Mr. Strong and Miss <br> Week remains inactive |

### 31.2.2 Solutions to the conflict

By the definition of a system/process, Mr. Strong and Miss Week are designed by the same Designer, namely John. Hence, if a conflict emerges it will be mostly due to an inconsistency of John's choice.

To avoid such inconsistency:

1) Section (b) of Fig. 96 clearly demonstrates the most obvious way. The empirical meaning can be that of a "binary" arrangement when the deliveries of leather are divided into two disjoint subsets characterized as "bad" and "good". Apparently, the setting fulfilled vs. breached shown in Fig. 29 is of this kind.
2) John, as shown in Fig. 97, may add to the two designs external events/ /conditions based on whatever, e.g. weather, competition, stability of government etc.

3) As said, Beneficiary ${ }^{\text {st }}$ and Beneficiary ${ }^{\mathrm{wk}}$ may "coordinate" their BEN-orders, namely when the same person is nominated into the two roles.
4) In Comment 9 we will discuss in detail the case when the outcome of Phase 1 is strictly divided between the two OP-UNs by which Phase 2 is constituted.

### 31.2.3 Non-conflicting heterogeneity

In Fig. 98 Mr. Strong remains in the position of a Manufacturer of shoes, whereas Miss Weak will be an Advisor who is to issue her report to John if delivery of leather is "improvable", e.g. executed somewhat later in the sense explained in section (b) of Fig. 96.


### 31.3 Vertical integration and disintegration

### 31.3.1 Two-plant and two-product designs

Fig. 99, in its sections (a) and (b), seeks to show - highly intuitively - how a two-unit Phase 2 in Fig. 95 and Fig. 98, respectively, can be aggregated into one
"meta" operational unit. Focusing on the multi-product Firm depicted in (b) of Fig. 99 it can be analytically put as

$$
G(1) \equiv\left[g:\langle k, d\rangle \rightarrow\left\langle\left(Q_{\text {shoo }} Q_{\text {adv }}\right),\left(L_{\text {shor }} L_{\text {adv }}\right)\right\rangle\right]
$$

where
$Q_{\text {sho }} Q_{\mathrm{adv}} \quad$ are magnitudes of shoes and advice, respectively, $L_{\text {sho, }} L_{\text {adv }} \quad$ are Executors of the deliveries of shoes and advice, respectively.

For illustration, let in (b) of Fig. 99 the input $(k, d)$ of $G(1)$ be the one represented by the point $A$ in Fig. 96 and hence lead to the following prescription $G(2)$

$$
G(2)=\left[\left(Q_{\text {sho }}, \varnothing\right),\left(L_{\text {sho }}, \varnothing\right)\right]
$$

In words, if the input is on the level of $A$ the multi-product firm will produce only one of its two products - shoes.


### 31.4 Associated notes

### 31.4.1 Executors

Among the topics that we have fully left aside are also Executors, i.e. the agents who, in reality, could have been designed as employees who as, e.g., employees of Mr. Strong and Miss Weak can be prescribed to factually execute the respective tasks.

Moreover, we have also abstracted from the fact that Mr. Strong and Miss Weak as Designees will - as lower-level Designers - design their lower-level strategies how to fulfill their tasks including Executors of these lower-level strategies. To illustrate, a lower-level Executor will be to find and hire the Executor one level above him-her.

### 31.4.2 Beneficiaries

Given that Mr. Strong and Miss Weak from Fig. 97 are Designees as any other their designs, by definition, involve the triad of Nominees, among them

$$
\text { Beneficiary }{ }^{\text {st }} \text { and Beneficiary }{ }^{\text {wk }}
$$

In general the two Beneficiaries are fully independent in the sense that they independently decide whether and in what particular form they will demand that $S T(1)$ and $W K(1)$ be prescribed.

At the same time, we should not exclude the personal union when, e.g., the young man named Benjamin will be nominate into both of the two roles of a Beneficiary ${ }^{\text {st }}$ and Beneficiary ${ }^{\text {wk }}$.

In the obviously extremal arrangement:

- John-the Designer will nominate the "same" Benjamin into the role of a Beneficiary of all three OP-UNs, i.e. of $K(1), S T(1)$ and $W K(1)$ and
- Benjamin will decide to submit all three BEN-orders on one single document with the aim to simultaneously demand all three prescriptions of $K(1), S T(1)$ and $W K(1)$.


### 31.4.3 Further notes

1) By definition, "sub-units" of which a macro-task consists are demanded by "the same" Benjamin-the Beneficiary who does so by one single BEN-order.
2) The inverse operation to integration is disintegration due to which "subunits" become OP-UNs "of their own" (cf., e.g., (b) of Fig. 45).
3) A specific kind of a vertical integration was already mentioned in Fig. 24 in the social context of a JOIN-type three-unit two-phase production.

## COMMENT 9.

## Re-allocation of wealth between future agents

In what follows we will keep to the pattern (c) of Fig. 8. Again, John will be assumed to design a SPLIT-type three-unit two-phase system/process. In particular, under study will be the system/process in Fig. 100 that is of the same kind as that discussed in Fig. 95 in the following sense:

- it will be assumed to be also designed by an individual Designer named John and
- choices of the two Followers are designed so as to depend on what will do their common Leader.


As to the differences:

1) For notational convenience what was Phase 1 and Phase 2 in Fig. 95 will now in Fig. 100 be consistently referred to as a present Phase $\alpha$ and a future Phase $\beta$, respectively. Correspondingly, the agents operating in Phase $\alpha$ will be characterized as present, whereas those from Phase $\beta$ will be called future agents.
2) The same person named Mary is designed to belong to both Phases. In words, the present Mary will be assumed to inter-act "with herself" in the role of the future Mary.

The empirical meaning of the system/process will now be that John designs the present Mary $M_{a}(1)$ so as to transform during Phase $\alpha$ the allocation of wealth

$$
\vec{B}_{\alpha}=\left(B_{\alpha / M}, B_{\alpha / V}\right)
$$

into the re-allocation

$$
\vec{B}_{\beta}=\left(B_{\beta / M}, B_{\alpha / V}\right)
$$

to be used in Phase $\beta$. In other words, the present Mary will be designed so as to transfer some of her wealth to the two future agents.

## 32. INTRODUCTION

### 32.1 Essential identities and notation

### 32.1.1 Operational situations

Trivially, wealth of an agent can be represented in natural or monetary units. For example, if the agents are Consumers they may be characterized by magnitudes of consumer goods, e.g.

$$
\text { bread } x_{1} \text { and wine } x_{2}
$$

and their monetary value

$$
B=\vec{p} \cdot \vec{x}=\left(p_{1} \cdot x_{1}+p_{2} \cdot x_{2}\right)
$$

where $\vec{p}=\left(p_{1}, p_{2}\right)$ are the respective prices of bread and wine.
In sum:
$\vec{x}_{\alpha / M}, B_{\alpha / M}$ and $\vec{x}_{\alpha / V}, B_{\alpha / V} \quad$ will represent the present (at $T^{\alpha}$ ) situations (endowments) of Mary and Victor, respectively
$\vec{x}_{\beta / M}, B_{\beta / M}$ and $\vec{x}_{\beta / V,}, B_{\beta / V}$
will represent their respective future wealth

In more detail: $\vec{x}_{i / M}=\left(x_{1 / / / M}, x_{2 / \alpha / M}\right)$ and $\vec{x}_{i / V}=\left(x_{1 / i / V}, x_{2 / \alpha / V}\right)$

$$
i=\alpha, \beta
$$

represent bread and wine - expressed in physical units, kilograms and litters, respectively.

### 32.1.2 Allocation vs. re-allocation; transfer

An aggregation of the two present situations of Mary and Victor is in text-books often called an allocation or an endowment. We will denote it:

$$
\overrightarrow{x x}_{\alpha}=\left(\vec{x}_{\alpha / M,}, \vec{x}_{\alpha / V}\right) \text { and } \vec{B}_{\alpha}=\left(B_{\alpha / M}, B_{\alpha / V}\right)
$$

Similarly, an aggregation of two future situations of Mary and Victor will be denoted

$$
\overrightarrow{x x}_{\beta}=\left(\vec{x}_{\beta / M}, \vec{x}_{\beta / V}\right) \text { and } \vec{B}_{\beta}=\left(B_{\beta / M}, B_{\beta / V}\right)
$$

and referred to as re-allocation. As said, Mary's task will be to establish within

$$
\text { Phase } \alpha=<T^{\alpha}, T^{\beta / 1} \text { ) }
$$

a re-allocation $\overrightarrow{x x_{\beta}}=\left(\vec{x}_{\beta / M}, \vec{x}_{\beta / V}\right)$ or, to execute two transfers:

$$
\begin{aligned}
\Delta \vec{x}_{\alpha / M} & =\left(\Delta \vec{x}_{1 / \alpha / M}, \Delta \vec{x}_{2 / \alpha / M}\right)=\left(\vec{x}_{\beta / M}-\vec{x}_{\alpha / M}\right) \\
\Delta \vec{x}_{\alpha / V} & =\left(\Delta \vec{x}_{1 / \alpha / V}, \Delta \vec{x}_{2 / \alpha / V}\right)=\left(\vec{x}_{\beta / V}-\vec{x}_{\alpha / V}\right)
\end{aligned}
$$

Put in terms of monetary values the transfers are

$$
\begin{aligned}
\Delta B_{\alpha / M} & =\left(B_{\beta / M}-B_{\alpha / M}\right) \\
\Delta B_{\alpha / V} & =\left(B_{\beta / V}-B_{\alpha / V}\right)
\end{aligned}
$$

where by $B_{\beta / M}$ and $B_{\beta / V}$ is denoted the newly acquired wealth of Mary and Victor, respectively, as of the beginning $T^{\beta / 1}$ of Phase $\beta=<T^{\beta / 1}, T^{\beta / 2}$ ).

### 32.1.3 Closeness

Phase will be (here and in what follows) assumed to be closed in the sense that no wealth is lost or gained during the transfer.

Hence, in physical units we will assume $\overrightarrow{x x}_{\alpha}=\overrightarrow{x x}_{\beta}$ or

$$
\left(\vec{x}_{\beta / M}+\vec{x}_{\beta / V}\right)=\left(\vec{x}_{\alpha / M}+\vec{x}_{\alpha / V}\right)
$$

In words, the overall amounts of bread and wine will remain constant - neither of the two commodities will increase or decrease during the transfer, no bread and wine will get lost, spoiled, stolen, etc. For example, if re-allocated, it will happen on zero transaction costs.
However, in what follows the analysis will be mostly taken in monetary units where the closeness will be expressed as:

$$
\Delta B_{\alpha / M}=-\Delta B_{\alpha / V}
$$

and

$$
\left(B_{\beta / M}+B_{\beta / V}\right)=\left(B_{\alpha / M}+B_{\alpha / V}\right) \equiv B_{\alpha}^{+}
$$

In sum, the future - as of the beginning $T^{\beta / 1}$ of Phase $\beta$ - the newly acquired wealth of the two future agents $M_{\beta}(1)$ and $V_{\beta}(1)$ will be

$$
\begin{gathered}
B_{\beta / M}=\left(B_{\alpha / M}+\Delta B_{\alpha / M}\right) \\
B_{\beta / V}=\left(B_{\alpha / V}+\Delta B_{\alpha / V}\right)
\end{gathered}
$$

### 32.2 Persons and roles

### 32.2.1 Designees

As already stressed, some peculiarity of the system/process in Fig. 100 may be seen in that $M_{\alpha}(1), M_{\beta}(1)$ represent the same Mary in two roles within the same system/process.

Hence, Mary will be assumed to inter-act "with herself" in the sense that her action today will affect the wealth that she will have tomorrow.

### 32.2.2 Designer's welfare

In following chapters, John's motivation will be taken as a mixture of two (or more) kinds of motivations aggregated further into the notion of a welfare.

To illustrate, John may design the system/process so as to materialize two kinds of preferences:

- the utility "proper", e.g. the overall profit of the system/process,
- inter-personal socio cultural distance between him as a Designer and the two future Designees $M_{\beta}(1)$ and $V_{\beta}(1)$.

The latter criterion will be represented by a so-called discount rate by which will express how much more or less John likes Mary than Victor.

The relative weights of the two motivations will vary, in what follows, across social contexts within which John will select his designs. Consequently, we shall - among others - differentiate whether the present Mary is a Donor or a Collector or, put differently, whether the transfer $\Delta B_{\beta / V}$ is positive (in Victor's favor) or negative, respectively.

### 32.3 Generalized production function

Given the concept under study, $M_{\beta}(1)$ and $V_{\beta}(1)$ will be designed as follows:

- by the beginning $T^{\beta / 1}$ of Phase $\beta$ they will have to accept and use all of the transfers $\Delta B_{\alpha / M}, \Delta B_{\alpha / V,}$
- by the end $T^{\beta / 2}$ of Phase $\beta$ they will have to use their new wealth $B_{\beta / M}$ and $B_{\beta / V}$ only in line with John's ex ante regulation.

Put differently, John-the Designer will be assumed to regulate - ex ante - the behavior/action of the two operational units $M_{\beta}(1)$ and $V_{\beta}(1)$ so that they will be prohibited to reject acceptance of prospective transfers or use them for other then designed purposes.
As said, during Phase $\beta$ the future agents $M_{\beta}(1)$ and $V_{\beta}(1)$ will have a task to transform their newly - at $T^{\beta / 1}$ - acquired wealth $B_{\beta / M}$ and $B_{\beta / V}$. In order to generalize the analysis, the behavior/action of the two future agents will be in what follows characterized - regardless of the nature of their roles - by production functions

$$
Q_{\beta / M}=a_{f / M} \cdot f_{M}\left(K_{\beta / M}, L_{\beta / M}\right) \text { and } Q_{\beta / V}=a_{f / V} \cdot f_{V}\left(K_{\beta / V}, L_{\beta / V}\right)
$$

and consequently by also the respective revenue functions denoted further as

$$
\left[h_{M}:\left\langle B_{\beta / M}\right\rangle \rightarrow\left\langle T R_{\beta / M}\right\rangle\right] \text { and }\left[h_{V}:\left\langle B_{\beta / V}\right\rangle \rightarrow\left\langle T R_{\beta / V}\right\rangle\right]
$$

To illustrate the level of generalization, if $M_{\beta}(1)$ and $V_{\beta}(1)$ are designed as Consumers, the production and revenue functions will be taken as transforming non-zero inputs into zero physical outputs $Q_{\beta / M,}, Q_{\beta / V}=0$ and hence also zero monetary revenues $T R_{\beta / M}, T R_{\beta / V}=0$.

### 32.4 Two kinds of maximization " $I V$ "

By definition, the three-unit two-phase system/process in Fig. 7 is designed as

$$
\text { an optimal strategy } s t r^{J / *}=M_{\alpha}(1) ; M_{\beta}(1), V_{\beta}(1)
$$

that John obtains by solving the maximization problem $M A X_{J}$ shown in section (a) of the following Fig. 101.


In the lower section (b) of Fig. 101 we assume that John designs the three operational units in the form of "lower-case" maximization problems

$$
\max _{\alpha / M}, \max _{\beta / V} \text { and } \max J_{\beta / M}
$$

Hence, what John is assumed to design are, in fact, three "lower-case" maximization problems. Into them then the respective Manuela will substitutes the respective data so as to calculate out the tasks' prescriptions.
Our major concern in Comment 9 will be "What does John maximize?" and into what maximization problems Manuela substitutes the data.

### 32.5 Associated notes

### 32.5.1 Behavioral cycles

Invoking the graphical representation of JOIN-type production cycles, we may recall that the present Phase 1 in Fig. 11 consisted in behavior/action of 2 present Suppliers and the subsequent Phase 2 was performed by 1 future Manufacturer. Moreover, by two different colors (blue and pink) we in Fig. 11 expressed that John's strategy for CYCLE 1 could be re-designed for the sake of CYCLE 2.

Strictly analogically can now be construed SPLIT-type cycles in Fig. 102, on the proviso that the present Phase 1 consists in only 1 present agent designed so as to distribute wealth between 2 future agents.


### 32.5.2 Present vs. future behavior/action

The possible confusions over the present vs. future nature of a behavior/action can arise from the following terminology:

1) Our definition of dynamics was based on that the driving force of a system/ /process is a decision of the present Designer who designs "now" what will become the future kinetics of the system/process concerned. In brief, the Designer "today" designs behavior/action that may be observed "tomorrow".
2) Leaving aside the obvious time sequence of the cycles shown in Fig. 102, also within each of them we use the terminology of most text-books on intertemporal choice, namely that $T^{\alpha}$ is a present time, whereas at $T^{\beta / 1}$ begins the phase when the future behavior/action is to occur.
3) To make the concept still more confusing, within Phase $\beta$ we will later differentiate between behavior/action performed immediately at $T^{\beta / 1}$ and that postponed until "tomorrow" at $T^{\beta / 2}$.

### 32.5.3 Acceptance of a transfer

As to the term "rejection" or a "duty to accept" we will only remind the reader of our earlier discussion on stage (-3)-breached and the LS conceptions of:

- buyer's breach when an agent ... refuses to receive a timely tender, provided the tender is proper ....,
- buyer's rightful rejection or justifiable revocation acceptance of a delivery.

Hence, also the present Mary $M_{\alpha}(1)$ may fail to make delivery or repudiate and the future agent may fail to receive a timely tender.

### 32.5.4 Self-design

As always, anybody can become a Designer, including Mary. Hence it may be Mary herself who will impose upon herself the task to re-allocate the present distribution of wealth - between herself and Victor. Not only this, later we will show that Mary may design herself into all of the future roles. As a Robinson Crusoe indeed, she will then design herself to be both a Producer and Consumer.
As said, in the case of this kind of a personal unions it will appear to be more appropriate to interpret Mary's tasks rather as her objectives, goals, plans ...

### 32.6 Variant social contexts

In the following chapters of this Comment 9 we will organize our discussion of the system/process in Fig. 100 according to the nature of the two future agents. In other words, we shall differentiate among various kinds of the ex ante regulations that John may impose upon Mary and Victor - upon ways in which they will be allowed - in Phase $\beta=<T^{\beta / 1}, T^{\beta / 2}$ ) - to dispose of with their newly acquired wealth.
In the following 5 chapters we will consecutively discuss the following arrangements:

1) To begin with, both $M_{\beta}(1)$ and $V_{\beta}(1)$ will be Manufacturers or "ordinary Producers", e.g. Shoe-makers discussed throughout this BOOK. As elements of the system/process concerned they can be taken as two Plants of one big Firm whose overall profit is to be maximized. Hence, Mary will have to reallocate the present wealth so that the two Plants will jointly maximize the overall future profit of the system/process.
2) Next, Mary will - for the sake of Phase $\beta$ - keep her role of an "ordinary Producer", e.g. the Shoe-making Manufacturer, whereas Victor will be designed into the role of "a Producer of a special kind", namely a Bank.
3) After that, the future Mary and Victor will be designed by John so as to behave as Consumers.
4) Then, the future Mary will keep her role of a Consumer, whereas Victor will perform the role of the "extra-ordinary Producer" called a Bank.
5) Finally, Mary will keep her role of a Consumer and Victor will "return" to the role of an "ordinary" Producer, or an ordinary Shoe-making Manufacturer. We will show circumstances under which the arrangement will constitute the text-book Robinson Crusoe economy.
Apparently, the above "syllabus" can be, again, also taken as the author's (yet another) attempt to contribute to the didactics of the perfectly standard problems of microeconomics. To illustrate this kind of objectives, towards the end of this Comment 9, in Fig. 136 is demonstrated how the rather burdensome labor can lead to a diagram that any under-graduate student of economics will find in an "ordinary" text-book on inter-temporal choice. The value added to the diagram should rest in the detail in which we have corroborated assumptions and simplifications under which the classic picture can only be construed. As the reader will see, the same "didactic" note applies also to the two remaining Comments 10 and 11.

## 33. THE CASE OF TWO PLANTS

In this chapter John will be assumed to design the three-unit two phase system/ process from Fig. 100

$$
s t r^{*}=\left(M_{\alpha}(1) ; M_{\beta}(1), V_{\beta}(1)\right)
$$

so that the two future agents $M_{\beta}(1)$ and $V_{\beta}(1)$ are Manufacturers who will be to maximize John's overall profit - towards the end $T^{\beta / 2}$ of the Phase $\beta$.

### 33.1 Introduction

### 33.1.1 Essential identities and notation

Let us summarize the essentials established in the preceding Comment 8:
Phase $\alpha$ is devoted to two transfers $\Delta B_{\alpha / M}$ and $\Delta B_{\alpha / V}$ leading to - as of $T^{\beta / 1}$ - the re-allocation $\vec{B}_{\beta}=\left(B_{\beta / M}, B_{\alpha / V}\right)$ where

$$
\begin{aligned}
B_{\beta / M} & =\left(B_{\alpha / M}+\Delta B_{\alpha / M}\right) \\
B_{\beta / V} & =\left(B_{\alpha / V}+\Delta B_{\alpha / V}\right)
\end{aligned}
$$

Phase $\beta$ is established by:

- its beginning $T^{\beta / 1}$ at which $B_{\beta / M}$ and $B_{\beta / V}$ are "invested into" Mary and Victor, respectively,
- its end $T^{\beta / 2}$ at which will be available the overall profit of the system/process.

As said, we assume that the Phase $\alpha$ is fully devoted to re-allocation and is "closed" in the sense

$$
\Delta B_{\alpha / M}=-\Delta B_{\alpha / V}
$$

and

$$
\left(B_{\beta / M}+B_{\beta / V}\right)=\left(B_{\alpha / M}+B_{\alpha / V}\right) \equiv B_{\alpha}^{+}
$$

### 33.1.2 Donor vs. Collector

In Fig. 103 are summarized - given the social context under study - the following properties of the transfers:

- They are allowed to flow in both directions - to or from Victor. Hence, given that Mary is designed as both the present and future agent, she can be taken as a Donor (when $\Delta B_{\alpha / M}<0$ ) or Collector (when $\Delta B_{\alpha / M}>0$ ).
- The transfers are constrained - as to their magnitude - by what the agents factually have at $T^{\alpha}$. In plain language, Mary cannot "sacrifice" more than $B_{\alpha / M}$ as a Donor and, conversely, as a Collector cannot get "enriched" by more than $B_{\alpha / V}$.


In terms of Mary's future wealth Fig. 103 can be expressed analytically as

$$
0 \leq B_{\beta / M} \leq B_{\alpha}^{+}
$$

or, equivalently, as $0 \leq B_{\beta / V} \leq B_{\alpha}^{+}$- in terms of Victor's newly acquired wealth.

### 33.2 Two kinds of a maximization " $I V-\mathbf{a}^{\prime \prime}$

### 33.2.1 Revenue functions

John's problem becomes analytically interesting once we accept that the technologies of the two Plants are mutually different - represented by two different production functions

$$
Q_{\beta / M}=a_{f / M} \cdot f_{M}\left(K_{\beta / M}, L_{\beta / M}\right) \text { and } Q_{\beta / V}=a_{f / V} \cdot f_{V}\left(K_{\beta / V}, L_{\beta / V}\right)
$$

and hence differentiated also by their revenue functions

$$
\left[h_{M}:\left\langle B_{\beta / M}\right\rangle \rightarrow\left\langle T R_{\beta / M}\right\rangle\right] \text { and }\left[h_{V}:\left\langle B_{\beta / V}\right\rangle \rightarrow\left\langle T R_{\beta / V}\right\rangle\right]
$$

or, for simplicity

$$
T R_{\beta / M}=h_{M}\left(B_{\beta / M}\right) \text { and } T R_{\beta / V}=h_{V}\left(B_{\beta / V}\right)
$$

For completeness, let us summarize that $h_{M}\left(B_{\beta / M}\right)$ and $h_{V}\left(B_{\beta / V}\right)$ represent maximal revenues generated by the future agents on the proviso that $B_{\beta / M}$ and $B_{\beta / V}$, are given - predetermined by the two transfers

$$
\Delta B_{\alpha / M}=-\Delta B_{\alpha / V}
$$

As said, John is assumed to design the system/process with the aim to maximize the sum of the two Plants' profits - as of the end of Phase $\beta$. Given the closeness of the transfer $B_{\alpha}^{+}=\left(B_{\alpha / M}+B_{\alpha / V}\right)=\left(B_{\beta / M}+B_{\beta / V}\right)$ the maximal overall profit will occur for the reallocation $\vec{B}_{\beta}=\left(B_{\beta / M,}, B_{\beta / V}\right)$ at which the maximal revenue $\left(T R_{\beta / M}\right.$ $\left.+T R_{\beta / V}\right)$ is generated.

### 33.2.2 "Lower-case" maximizations

Invoking Fig. 101 the system/process under study can be somewhat concretized as shown in Fig. 104.
As said, the outcome of John's "upper-case" $M A X_{J}$ takes up the form of the above triad of maximization problems.

### 33.3 Present agent

### 33.3.1 Maximization problem

In the very brief, as said, John designs the present Mary with the aim to maximize the overall revenue

$$
T R_{\beta}\left(B_{\beta / M,}, B_{\beta / V}\right)=\left(T R_{\beta / M}+T R_{\beta / V}\right)=\left(h_{M}\left(B_{\beta / M}\right)+h_{V}\left(B_{\beta / V}\right)\right)
$$

on the proviso that the two future agents will maximize their individual revenues.

The looked for $\max J_{\alpha / M}$ can then be put as follows:

```
\(\max \left(h_{M}\left(B_{\beta / M}\right)+h_{V}\left(B_{\beta / V}\right)\right)\)
s.t.:
\(B_{\beta / M}, B_{\beta / V} \geq 0\)
\(\left(B_{\alpha / M}+B_{\alpha / V}\right)=\left(B_{\beta / M}+B_{\beta / V}\right)=B_{\alpha}^{+}\)
```



Fig. 104

It may be noteworthy to emphasize already here that in later chapters the problem $\max J_{\alpha / M}$ will be expanded in at least the following two ways:

- A topic of a so-called insolvency will be considered. Now we will only briefly note that the two-Plant system/process can never become "bankrupt" - on the proviso that - as we will demonstrate shortly - the revenue functions $h_{M}\left(B_{\beta / M}\right)$ and $h_{V}\left(B_{\beta / V}\right)$ are increasing both in $B_{\beta / M}$ and $B_{\beta / V}$, respectively.
- The function to be maximized in $\max J_{\alpha / M}$ will be generalized into a welfare function $W^{J / M \alpha}(\ldots)$ that will involve not only John's overall revenue but also his inter-personal preferences - the fact that he may, e.g., like Mary better than Victor.


### 33.3.2 Equality of marginal revenues

In Fig. 105, LHS and RHS stand for the "right-hand side" and "left-hand side", respectively, of the Kuhn-Tucker first-order conditions
LHS = RHS
that the solution to $\max J_{\alpha / M}$ must satisfy.

If we confine for simplicity to only interior solutions to $\max J_{\alpha / M}$ a simple application of a Kuhn-Tucker procedure shown in Fig. 105 brings up the property of the looked-for optimal wealth re-allocation

$$
\vec{B}_{\beta}^{*}=\left(B_{\beta / M}^{*}, B_{\beta / V}^{*}\right)
$$


The resultant equation represents the text-book wisdom that the system/process as a whole generates the maximal revenue (and hence also profit) if two future plants $M_{\beta}(1)$ and $V_{\beta}(1)$ operate on levels where their marginal revenues are equal.

### 33.3.3 Graphical representation

Contours of the criterion function $T R_{\beta}\left(B_{\beta / M}, B_{\beta / V}\right)$ are represented by the green curves in the following Fig. 106 on the proviso that the respective individual revenue functions $h_{M}\left(B_{\beta / M}\right), h_{V}\left(B_{\beta / V}\right)$ have the text-book properties - to be illustrated shortly in Fig. 108 for $h_{V}\left(B_{\beta / V}\right)$.
In the particular case of Fig. 106 the optimal combination ( $B_{\beta / M,}^{*}, B_{\beta / V}^{*}$ ) of investments into the future Mary and Victor exists and is unique. It is, again, determined by the tangent point of the highest iso-revenue contour with the segment representing the "budget constraint" $\left(B_{\beta / M}+B_{\beta / V}\right)=B_{\alpha}^{+}$.


The tangency condition of the optimum $\vec{B}_{\beta}^{*}=\left(B_{\beta / M}^{*}, B_{\beta / V}^{*}\right)$ is nothing else than a geometric representation of the above observation that Mary and Victor will operate on the levels where their marginal revenues are equal.

### 33.3.4 Example

For illustration, the example in Fig. 106 is selected so that

$$
B_{\beta / M}^{*}<B_{\alpha / M} \text {, or equivalently } B_{\beta / V}^{*}>B_{\alpha / V}
$$

In this particular case, the present Mary can be - in the sense of Fig. 103 characterized as a Donor vis a vis the future Victor. Contrariwise, in the opposite case $B_{\beta / M}^{*}>B_{\alpha / M}$ or $B_{\beta / V}^{*}<B_{\alpha / V}$ Mary will be a Collector of wealth from Victor.

### 33.3.5 Interior vs. corner optima

Apparently, the equality of marginal revenues holds only as long as the optimum is an interior one. For completeness a few examples of non-interior (corner) optima are sketched Fig. 107.


Fig. 107

### 33.4 Utility (production) possibility frontier

### 33.4.1 Future agents

In order to calculate out prescriptions of the two future tasks the respective Manuelas will have to solve the following problems:
For $M_{\beta}(2)$ we obtain:

```
\(\max \left[T R_{\beta / M}=h_{M}\left(B_{\beta / M}\right)\right]\)
s.t.:
    \(\max _{\beta / M}\)
\(B_{\beta / M}=\left(B_{\alpha / M}+\Delta B_{\alpha / M}^{*}\right)\)
```

where the asterisk "*" marks that the magnitude $\Delta B_{\alpha / M}^{*}$ of the transfer is provided for the sake of the future $\max J_{\beta / M}$ by Manuela as an outcome of the present maxj $j_{\alpha / M}$.

Mutatis mutandis, for $V_{\beta}(2)$ :

```
\(\max \left[T R_{\beta / V}=h_{M}\left(B_{\beta / V}\right)\right]\)
s.t.: \(\quad \max J_{\beta / M}\)
\(B_{\beta / V}=\left(B_{\alpha / V}+\Delta B_{\alpha / V}^{*}\right)\)
```

Technically speaking, Manuela simply substitutes $\left(B_{\alpha / M}+\Delta B_{\alpha / M}^{*}\right)$ and $\left(B_{\alpha / V}+\right.$ $\left.\Delta B_{\alpha / V}^{*}\right)$ into the two respective revenue functions.

### 33.4.2 Revenues

The two revenue functions $T R_{\beta / M}=h_{M}\left(B_{\beta / M}\right)$ and $T R_{\beta / V}=h_{V}\left(B_{\beta / V}\right)$ are in general mutually different but, by assumption, of the same text-book nature - increasing and concave. For illustration this property is illustrated for Victor's $T R_{\beta / V}=$ $h_{V}\left(B_{\beta / V}\right)$ in Fig. 108. The kind reader will hopefully understand that they are depicted as polygonal lines - for graphical convenience if not mere legibility.

The monetary input-output combinations selected in Fig. 108 have the following meaning:
$T R_{\beta / V}^{+} \quad$ is the revenue if Victor invests all of his own present endowment $B_{\alpha / V}$ and only this endowment,
$T R_{\beta / V}^{x} \quad$ is the revenue if Victor invests $B_{\alpha}^{+}=\left(B_{\alpha / M}+B_{\alpha / V}\right)$, i.e. all the resources of the system/process as a whole,
$T R_{\beta / V}^{x \tau} \quad$ is the optimal revenue if Victor is an independent decision-maker, i.e. if he himself - not John-the Designer - is the decision-maker and hence, among other is not constrained by the overall wealth $B_{\alpha}^{+}$of the system/process .

By $T R_{\beta / V}^{*}$ we illustrate the further discussed optimal revenue if Victor is ex ante regulated by John-the Designer.


### 33.4.3 Linear revenues

Revenue functions in Fig. 109 emphasize the following:
(1) even if Victor is stripped off some of his wealth his revenue may be still higher than costs,
(2) represent "no growth" investment - its simple maintenance/storage,
(3) represents a deterioration of the investment.

Linear revenue functions will be applied later when one of the future agents will be a provider of banking services and the variable $v$ in the function $(1+v) . B_{\beta / V}$ will represent an interest rate.


### 33.4.4 Reverse revenue function

In Fig. 106 the contours of the overall revenue $T R_{\beta}=\left(T R_{\beta / M}+T R_{\beta / V}\right)$ are nonlinear because they are depicted in the space of the future (as of the beginning of Phase $\beta$ ) individual investments $B_{\beta / M}$ and $B_{\beta / V}$.

By contrast, in Fig. 110 the contours are linear because represented in the space of the future (as of the end $T^{\beta / 2}$ of Phase $\beta$ ) individual revenues $T R_{\beta / M}$ and $T R_{\beta / V}$. Hence the contours are depicted by the red broken south-east decreasing lines

$$
T R_{\beta / V}=T R_{\beta}-T R_{\beta / M}
$$



The function $T R_{\beta / V}=\mu\left(T R_{\beta / M}\right)$ depicted in Fig. 110 by a green curve represents how Victor's maximal revenue changes with changes in Mary's maximal revenue. Its interpretation thus is that of a utility (production) possibility frontier.

Its curvature is derived with the help of the so-called

$$
\text { reversed revenue function } T R_{\beta / V}=H_{V}\left(B_{\beta / M}\right)
$$

depicted by the blue downward sloping curve in Fig. 111. Put analytically, the increasing concave function $T R_{\beta / V}=h_{V}\left(B_{\beta / V}\right)$ from Fig. 108 will be transformed as follows

$$
B_{\beta / V}=B_{\alpha / V}+\Delta B_{\beta / V}=B_{\alpha / V}-\Delta B_{\beta / M}=\left(B_{\alpha / V}+B_{\alpha / M}\right)-B_{\beta / M}
$$

As a result we obtain a decreasing concave function

$$
T R_{\beta / V}=h_{V}\left(\left(B_{\alpha / V}+B_{\alpha / M}\right)-B_{\beta / M}\right) \equiv H_{V}\left(B_{\beta / M}\right)
$$

representing how Victor's individual revenue $T R_{\beta / V}$ depends on investments into Mary's plant.


Problem $\max J_{\alpha / M}$ by which the present Mary is designed can then be re-formulated as follows:
$\max \left(T R_{\beta / M}+T R_{\beta / V}\right)$
s.t.: $T R_{\beta / V}=\mu\left(T R_{\beta / M}\right)$

### 33.4.5 Inverse problem

Inter-temporality represented by the revenue function,. e.g. that in Fig. 108, can be expressed as a question: "If today (at $T^{\beta / 1}$ ) the investment into Victor is $B_{\beta / V}$, what revenue can be obtained tomorrow at $T^{\beta / 2}$ ?"

For completeness, Fig. 112 answers the inverse question: "If Victor is prescribed to ,deliver'" $T R_{\beta / V}$ tomorrow (at $T^{\beta / 2}$ ) how much must be invested today (at $T^{\beta / 1}$ ).


### 33.5 Inter-personal preferences

### 33.5.1 Introduction

For completeness we briefly sketch in Fig. 113 various optima depending on various kinds of John's inter-personal preferences or his socio-cultural distances from the Future Mary and Victor, respectively.

The example is selected so as to show that - for reasons rather "psychological" - John prefers to invest into Mary even though it is economically "irrational".


### 33.5.2 Complex structure of a task

In Fig. 113 we show that John likes Mary's shoe-manufacturing better than Victor's. Hence Mary will be ordered to deliver more that would be efficient.

In our IT-parlance we can expand this line of argument so that, in reality, John designs:

- Mary's task to deliver a given magnitude of shoes to a given place at a given time and, mutatis mutandis,
- Victor's task to deliver a given magnitude of shoes to a given place at a given time.

Hence, apart from the obvious pure economic and inter-personal preferences, John's choice may be also affected by his affection to, e.g., a particular place of delivery.

## 34. THE CASE OF THE FIRM AND ITS BANK

Likewise in the preceding chapter, John will design the three-unit two phase system/process from Fig. 100

$$
s t r^{*}=\left(M_{\alpha}(1) ; M_{\beta}(1), V_{\beta}(1)\right)
$$

so as to maximize his overall profit. What will be different is that whereas Mary will remain an "ordinary" profit-making Manufacturer, e.g. a Shoe-maker, Victor will be assumed to make his profits in an "extra-ordinary" way - by providing banking services.

### 34.1 The nature of banking services

### 34.1.1 Collector vs. Donor

Let Mary-the Shoe-maker be Victor's Client. Put differently, the future Victor $V_{\beta}(1)$ will be a Bank in which Mary has a debit account. Hence, the transfers can now be interpreted as Mary's deposits on or withdrawals (if $\Delta B_{\alpha / M}>0$ ) from a bank account. In the very brief the two roles can be compared as follows:

- The major novelty will concern the withdrawals, or Mary's role of a Collector. It will be by nature of "banking services" that Mary $M_{\beta}(1)$ will be "obliged" to pay back to Victor her respective debts - in contrast to her role of a Collector in the preceding chapter.
- The empirical meaning of the opposite case $\Delta B_{\alpha / V}>0$ is that Mary-the Donor makes a deposit on her account. This case will return us to the "ordinary" two-Plant arrangement where John makes the present Mary invest into Victor's undertaking. Given that the undertaking has the form of a Bank, its contribution to the overall (as of the end of Phase $\beta$ ) revenue will be

$$
T R_{\beta / V}=(1+r) \cdot B_{\beta / V}=(1+r) \cdot \Delta B_{\alpha / V}
$$

where, apparently, $r$ is the (market, exogenously given) interest rate.

### 34.1.2 Simplifications

In order to make the analysis somewhat simpler we will assume that:

- Mary is the only Client of Victor and hence all that Victor "owns" - now and then - may come from only Mary, due to her deposits or withdrawals.
- $B_{\alpha / V}=0$ which states that the present (at time $T^{\alpha}$ ) wealth of Victor is "nominally" zero - that the Mary's account is empty at $T^{\alpha}$.

The present allocation (endowment) is thus assumed to be

$$
\vec{B}_{\alpha}=\left(B_{\alpha / M}, B_{\alpha / V}\right)=\left(B_{\alpha / M}, 0\right)
$$

and, trivially, the future re-allocation is

$$
\vec{B}_{\beta}=\left(B_{\beta / M}, B_{\beta / V}\right)=\left[\left(B_{\alpha / M}+\Delta B_{\alpha / M}\right), \Delta B_{\alpha / V}\right]
$$

### 34.1.3 Feasible transfers

Moreover, for simplicity, we shall assume that Victor $V_{\beta}(1)$ will have to let Mary withdraw from her account any amount of wealth. Invoking Fig. 103 now it is in Fig. 114 where we summarize the constraints and roles.


### 34.2 Digression: terminology

It may be of value to compare our terms Donor vs. Collector with the text-book concepts of an Investor vs. Dis-investor, Creditor vs. Debtor or even Lender vs. Borrower.

### 34.2.1 Investor vs. dis-Investor

Let us summarize as follows:

1) As said, the empirical meaning of $\Delta B_{\alpha / V}>0$ is that Mary makes a deposit on her account. Our more sophisticated interpretation will return to John and his choice of an optimal strategy. The case $\Delta B_{\alpha / V}>0$ will then mean that John-the Designer, within his overall strategy, has decided that - through Mary's transfer - some (if not all) of the overall wealth available at $T^{\alpha}$ will be invested into Victor-the Bank $V_{\beta}(1)$. As a result, the Bank's contribution to the overall (as of the end of Phase $\beta$ ) revenue will be

$$
T R_{\beta / V}=(1+r) \cdot B_{\beta / V}=(1+r) \cdot \Delta B_{\alpha / V}
$$

where, apparently, $r$ is the (market, exogenously given) interest rate.
2) Contrariwise, if $(-\infty) \leq \Delta B_{\alpha / V}<0$, Mary $M_{\alpha}(1)$ is a Collector who withdraws wealth from Victor-the Bank - from the account that is empty by assumption. Put in our IT-parlance, again, John-the Designer, as a part of his overall strategy, has decided that - through Mary's transfer - the investment into her shoe-making will exceed her present wealth $B_{\alpha / M}$. Given that the positive
transfer $\Delta B_{\alpha / V}>0$ has been called an investment (into the Bank), the opposite case $\Delta B_{\alpha / V}<0$ may be seen as a disinvest into Victor's Bank. Contrariwise, due to this dis-investment, the investment into shoe-making will be strengthened by the loan from Victor's Bank.

### 34.2.2 Lender/Creditor vs. Borrower/Debtor

The nature of the setting under study is such that it makes sense to call the dis-investment $\Delta B_{\alpha / V}<0$ as a bank loan. The term suggests that now, unlike in the two-Plant case, the Collector will be taken as a Borrower/Debtor who will have to, sooner or later, pay back a somehow determined debt.

Apparently, the conditions of the respective task of the Borrower/Debtor are designed by John-the Designer who may - as always - do it in an unlimited number of ways. For simplicity, let he design the rules of the "game" so that the debt must be duly paid:

- by the end of Phase $\beta$, i.e. by $\mathrm{T}^{\beta / 2}$,
- in the amount $(1+r) . B_{\alpha / V}$, where, again, $r$ is the market (exogenously given) interest rate.


### 34.3 Two kinds of a maximization " $I V-\mathrm{b}$ "

Apparently, the following Fig. 115 is a mere specification of Fig. 104, on the proviso that Victor's revenue $T R_{\beta / V}=h_{V}\left(B_{\beta / V}\right)$ obtains the above form

$$
T R_{\beta / V}=(1+r) \cdot B_{\beta / V}=(1+r) \cdot \Delta B_{\alpha / V}
$$



### 34.4 Present agent

### 34.4.1 Equality of marginal revenues

As said, John's motivation is to maximize the overall revenue

$$
T R_{\beta}=T R_{\beta}\left(B_{\beta / M}, B_{\beta / V}\right)=\left(h_{M}\left(B_{\beta / M}\right)+h_{V}\left(B_{\beta / V}\right)\right)
$$

The respective maximization problem from which the respective Manuela calculates the task of the present Mary can be put as follows.

$$
\max T R_{\beta}\left(B_{\beta / M}+B_{\beta / V}\right)
$$

s.t.:

```
\(\left(B_{\beta / M}+B_{\beta / V}\right)=B_{\alpha}^{+}\)
    \(\max J_{\alpha / M}\)
\(B_{\beta / M} \geq 0\)
\(B_{\beta / V} \leq B_{\alpha / M}\)
\(T R_{\beta}=\left(T R_{\beta / M}+T R_{\beta / V}\right) \geq 0\)
```

Invoking the procedure corroborated in Fig. 105 we will again leave aside technicalities and confine to the first order condition of a so-called interior solution to $\max J_{\alpha / M}$. Then, as explained, the respective two marginal revenues must be equal:

$$
\frac{d h_{M}\left(B_{\beta / M}^{*}\right)}{d B_{\beta / M}}=\frac{d h_{V}\left(B_{\beta / V}^{*}\right)}{d B_{\beta / V}}
$$

in the optimum. Given that the revenue function of Victor's Bank is

$$
T R_{\beta / V}=h_{V}\left(B_{\beta / V}\right)=(1+r) \cdot \Delta B_{\alpha / V}=(1+r) \cdot B_{\beta / V}
$$

the condition requires

$$
\frac{d h_{M}\left(B_{\beta / M}^{*}\right)}{d B_{\beta / M}}=(1+r)
$$

Graphically, Mary's optimum $B_{\beta / M}^{*}$ is determined in Fig. 117 by the tangent point between the revenue function and the line of a slope $(1+r)$.
The complementary optimum $B_{\beta / M}^{*}$ for Victor is obtained by the closeness condition

$$
\left(B_{\beta / M}^{*}+B_{\beta / V}^{*}\right)=B_{\alpha / M}=B_{\alpha}^{+}
$$

What must be then checked is the non-negativity condition $T R_{\beta} \geq 0$ to be discussed somewhat later under the label of a solvency.

### 34.5 Revenue functions

The bank's revenue can be represented in two equivalent forms:
in (a) of Fig. 116 by the blue increasing straight line

$$
T R_{\beta / V}=h_{V}\left(B_{\beta / V}\right)=(1+r) \cdot B_{\alpha / V}=(1+r) \cdot B_{\alpha / V}
$$

in (b) of Fig. 116 by the blue decreasing straight line representing the so-called reversed revenue function established and explained in Fig. 111.
$T R_{\beta / V}=H_{V}\left(B_{\beta / M}\right)$ shows the speed with which the revenue generated by Victor--the Bank decrease with the investment into shoe-making. Also in this case, the fact that the debt must be paid back will be considered in order to obtain the contribution of the Bank to the overall revenue of the system/process.


### 34.6 Solvency problem

### 34.6.1 Relevance of an endowment

Let - for the sake of the analysis - $B_{\beta / M}^{*}$ be non-zero. In Fig. 117 we then show its impact on the second "plant" represented by Mary's bank account in Victor's bank. Apparently, the impact crucially depends on Mary's present wealth $B_{\alpha / M}$.


It may be of value to compare the following two section of Fig. 117:
in (a) of Fig. 117 the endowment $B_{\alpha / M}^{a}$ essentially corresponds to the arrangement shown in Fig. 116. The resultant investment into shoemaking is $B_{\beta / M}^{*}>B_{\alpha / M}^{a}$ and hence $\Delta B_{\alpha / M}^{*}>0$. Put differently, Mary is a Collector/Dis-investor/Debtor/
/Borrower. On Mary's account (at the beginning $T^{\beta / 1}$ of Phase $\beta$ ) is a dis-investment $B_{\beta / V}^{*}=\left(B_{\alpha / M}^{\mathrm{a}}-B_{\beta / M}^{*}\right)<0$.
in (b) of Fig. 117 the endowment $B_{\alpha / M}^{\mathrm{b}}$ is selected so as to show the case when $B_{\beta / M}^{*}<B_{\alpha / M}^{\mathrm{b}}$ and hence a non-zero investment will be made into both "plants" - shoe-making and the bank. In other words, Mary is a Donor/Investor/Creditor/Lender.

### 34.7 Zero endowments

For the sake of future discussion we depict in Fig. 118 the case when not only $B_{\alpha / V}$ but also $B_{\alpha / M}=0$ and hence also the overall endowment is $B_{\alpha}^{+}=0$. In words, Mary starts her shoe-making activities with nothing both in a Bank and her pocket.

Mary's production can thus be launched only if a bank loan is available. A deposit $B_{\beta / V}$ on Mary's bank account will thus be never positive.


In Fig. 119 we return to the arrangement (a) of Fig. 117, where it is optimal to strengthen the investment into shoe-making by a bank loan in the amount $T R_{\beta / V}^{*}<0$.


Fig. 119

The aim of the example is to show that even though $T R_{\beta / M}^{*}<0$, the revenue $T R_{\beta / M}^{*}$ from shoe-making is big enough to pay back - by $T^{\beta / 2}$, at the latest - the debt, in the sense of a nonnegativity condition

$$
T R_{\beta}=\left(T R_{\beta / M}+T R_{\beta / V}\right) \geq 0
$$

As explained, the system/process is - as a whole - ex ante regulated so as to maximize the overall revenue. Hence, as shown in Fig. 119 the "insolvency" outcome can never occur, by design. Should "insolvency" be the outcome, the respective operational units would not be prescribed, as the above non--negativity condition $T R_{\beta / V} \geq 0$ in $\max J_{\alpha / m}$ would not be satisfied.

Mostly for interest we thus depict in Fig. 119 the level of dis-investment $T R_{\beta / M}^{s}$ in which the system/process as a whole will break even

$$
T R_{\beta}^{s}=\left(T R_{\beta / M}^{s}+T R_{\beta / V}^{s}\right)=0
$$

In sum, the combination

$$
\left(T R_{\beta / M}^{s}, T R_{\beta / V}^{s}\right)
$$

is such that the positive revenue $T R_{\beta / M}^{s}$ from shoe-making will equal the debt $T R_{\beta / V}^{s}$ by which the shoe-making was financed.

### 34.8 Utility possibility frontier

### 34.8.1 Borrowing



Invoking Fig. 116 and (a) of Fig. 117, we in Fig. 120 depict a production possibility border (or, equivalently an utility possibility frontier)

$$
T R_{\beta / V}=\mu\left(T R_{\beta / M}\right)
$$

as based on the combinations of revenues (1), (2), (3) and (4) from Fig. 121. The green iso-revenue lines are contours of the linear criterion function

$$
T R_{\beta}=\left(T R_{\beta / M}+T R_{\beta / V}\right)
$$

representing the overall revenue of the system/process as a whole.

### 34.8.2 Lending

Apparently, the graphs in Fig. 121 are fully analogous with those in Fig. 111 and the tangent point (3) represents a solution to $\max J_{\alpha / M}$ on the proviso that the arrangement is that of section (b) of Fig. 117.


Analogously with Fig. 110 we can then in Fig. 122 construed the utility possibility frontier $T R_{\beta / V}=\mu\left(T R_{\beta / M}\right)$ and the point (3) representing the respective optimum $T R_{\beta}^{*}=\left(T R_{\beta / M}^{*}+T R_{\beta / V}^{*}\right)$.


## 35. THE CASE OF TWO CONSUMERS

In this chapter John will be assumed to design the three-unit two phase system/ /process from Fig. 100

$$
s t r^{*}=\left(M_{\alpha}(1) ; M_{\beta}(1), V_{\beta}(1)\right)
$$

so that the two future agents $M_{\beta}(1)$ and $V_{\beta}(1)$ will both have to behave as Consumers. By design the system/process as a whole will be to maximize John's overall welfare.

### 35.1 Introduction

### 35.1.1 Essential identities and notation

Recall that a Consumer can be characterized by consumer goods, e.g.
bread $x_{1}$ and wine $x_{2}$
and their overall monetary value

$$
B=\vec{p} \cdot \vec{x}=\left(p_{1} \cdot x_{1}+p_{2} \cdot x_{2}\right)
$$

Hence, as explained already in Comment 8:
$\vec{x}_{\alpha / M}, B_{\alpha / M}$ and $\vec{x}_{\alpha / V}, B_{\alpha / V} \quad$ represent the present (at $T^{\alpha}$ ) situations (an endowments) of Mary and Victor, respectively,
$\vec{x}_{\beta / M}, B_{\beta / M}$ and $\vec{x}_{\beta / V}, B_{\beta / V} \quad$ represent their respective future (at $T^{\beta / 1}$ ) situations.

In more detail: $\left.\vec{x}_{i / M}=\left(x_{1 / / / M}, x_{2 / \alpha / M}\right)\right)$ and $\vec{x}_{i / V}=\left(x_{1 / / V}, x_{2 / \alpha / V}\right)$

$$
i=\alpha, \beta
$$

are endowments of bread and wine - expressed in physical units, kilograms and litters, respectively.

An aggregation of the two present situations (endowments) will be called an allocation and denoted:

$$
\overrightarrow{x x}_{\alpha}=\left(\vec{x}_{\alpha / M}, \vec{x}_{\alpha / V}\right) \text { and } \vec{B}_{\alpha}=\left(\mathrm{B}_{\alpha / M}, \mathrm{~B}_{\alpha / V}\right)
$$

In the full analogy, an aggregation of two future (at $T^{\beta / 1}$ ) situations of Mary and Victor will be denoted as

$$
\overrightarrow{x x} \vec{x}_{\beta}=\left(\vec{x}_{\beta / M}, \vec{x}_{\beta / V}\right) \text { and } \vec{B}_{\beta}=\left(\mathrm{B}_{\beta / M}, \mathrm{~B}_{\beta / V}\right)
$$

and referred to as re-allocation. By a transfer we shall thus understand

$$
\Delta \overrightarrow{x x}_{\alpha}=\left(\overrightarrow{x x}_{\beta}-\vec{x} \vec{x}_{\alpha}\right), \Delta \vec{B}_{\alpha}=\left(\vec{B}_{\beta}-\vec{B}_{\alpha}\right)
$$

or, in more detail:

$$
\begin{gathered}
\Delta \vec{x}_{\alpha / M}=\left(\Delta x_{1 / \alpha / M}, \Delta x_{2 / \alpha / M}\right)=\left(\vec{x}_{\beta / M}-\vec{x}_{\alpha / M}\right) \\
\Delta \vec{x}_{\alpha / M}=\left(\Delta x_{1 / \alpha / V}, \Delta x_{2 / \alpha / V}\right)=\left(\vec{x}_{\beta / V}-\vec{x}_{\alpha / V}\right) \\
\Delta B_{\alpha / M}=\left(B_{\beta / M}-B_{\alpha / M}\right) \\
\Delta B_{\alpha / V}=\left(B_{\beta / V}-B_{\alpha / V}\right)
\end{gathered}
$$

Again, we will assume closeness of the transfers by the following equalities

$$
\Delta B_{\alpha / M}=-\Delta B_{\alpha / V}
$$

and

$$
\left(B_{\beta / M}+B_{\beta / V}\right)=\left(B_{\alpha / M}+\mathrm{B}_{\alpha / V}\right) \equiv \mathrm{B}_{\alpha}^{+}
$$

### 35.1.2 Generalized production and revenue functions

As said, during Phase $\beta$, Mary and Victor will be obliged to use all their newly acquired wealth and that the usage must be consumption. Moreover, it has been also stated, that Phase $\beta$ will be always taken as "a kind of" a production.

The two future agents' behavior/action will be then always characterized by mutually different production functions

$$
Q_{\beta / M}=a_{f / M} \cdot f_{M}\left(K_{\beta / M}, L_{\beta / M}\right) \text { and } Q_{\beta / V}=a_{f / V} \cdot f_{V}\left(K_{\beta / V}, L_{\beta / V}\right)
$$

and consequently also revenue functions

$$
T R_{\beta / M}=h_{M}\left(B_{\beta / M}\right) \text { and } T R_{\beta / V}=h_{V}\left(B_{\beta / V}\right)
$$

As said, depending on the social context the specific nature of future agents will then be embodied in the specific form of the two functions - like in the case of Victor-the Bank whose revenue function was taken in the form $T R_{\beta / V}=(1+\mathrm{r})$. $B_{\beta / V}=(1+r) . B_{\alpha / V}$.

We have also explained that Consumers will be represented by zero revenue functions that transform non-zero inputs $B_{\beta / M}$ and $B_{\beta / V}$ into zero monetary revenues $T R_{\beta / M}, T R_{\beta / V}=0$, respectively.

### 35.2 Present agent

### 35.2.1 Generalized utility function

For the two foregoing production-type arrangements John's welfare was simplified into the overall future revenue $T R_{\beta}=\left(T R_{\beta / M}+. T R_{\beta / V}\right)$ Now, for a consumption-type system/process, the design of the present Mary will be generalized as follows:
$\max W^{I / M \alpha}\left(\overrightarrow{\Delta B}_{\alpha}\right)=W^{I / M \alpha}\left(\Delta B_{\alpha / M}, \Delta B_{\alpha / V}\right)$ $\max _{\alpha / M}$
s.t.: $\left(\Delta B_{\alpha / M}, \Delta B_{\alpha / V}\right) \in X X^{J / M \alpha}$
or, equivalently:
$\max W^{I / M \alpha}\left(\vec{B}_{\beta}\right)=W^{I / M \alpha}\left(B_{\beta / M}, B_{\beta / V}\right)$
$\max _{\alpha / M}$
s.t.: $\left(B_{\beta / M}, B_{\beta / V}\right) \in X X^{J / M \alpha}$

### 35.2.2 Welfare function - discounting

The generalization will rest in that John will now express how much more or less he likes or dislikes the two recipients of the would be transfers - Mary and Victor. Put formally, the "not-for-profit" welfare $W^{J / M \alpha}\left(B_{\beta / M}, B_{\beta / V}\right)$ will be expressed as a weighted sum of two components

$$
W^{J / M \alpha}\left(B_{\beta / M}, B_{\beta / V}\right)=\mathrm{a} \cdot U^{I}\left(B_{\beta / M}\right)+\mathrm{b} \cdot U^{I}\left(B_{\beta / V}\right)
$$

where:
$U^{J}\left(B_{\beta / M}\right)$ represents John's pleasure from Mary's consumption in the amount $B_{\beta / M}$ in Phase $\beta$,
$U^{J}\left(B_{\beta / V}\right)$ represents John's pleasure from Victor's consumption in the amount $B_{\beta / V}$ in Phase $\beta$.
We will leave to other books the "philosophical" discussion about the weights $\mathbf{a}$ and $\mathbf{b}$ and simply except their text-books representation

$$
W^{J / M \alpha}\left(B_{\beta / M}, B_{\beta / V}\right)=\left[U^{J}\left(B_{\beta / M}\right)+\frac{1}{(1+\rho)} \cdot U^{J}\left(B_{\beta / V}\right)\right]
$$

where $\rho$ stands for a so-called discount rate.

### 35.2.3 Socio cultural distance

In words, by the parameter $\rho$ are measured John's mutually different socio cultural distances from the two future agents. Hence, in $\rho$ is embodied the fact that John is likely to have his own, highly individual, mutually different sympathy or aversion towards Mary and Victor.

The parameter $\rho$ can be also called a discount rate and assumed - as in standard text-books - to lie in the interval

$$
\rho \in\langle 0, \infty)
$$

where
$\rho=\infty \quad$ means that John evaluates his social distance from Victor as infinitely large and hence Victor can be hardly "visible" by John - as if not existent, represented by a near-zero discount factor $\frac{1}{(1+\rho)}=0$,
$\rho=0 \quad$ or $\frac{1}{(1+\rho)}=1$ states that John's socio cultural distance from Mary and Victor is identical.

Intuitively, one could say that $\rho$ is positively correlated to a geographical distance. The nearer (in miles or km) will be Mary to John, the "closer to his heart" she may be.

### 35.2.4 Budget constraint

In sum, the above $\max J_{\alpha / M}$ obtains the following form:

$$
\begin{aligned}
& \max \left[W^{J}\left(\vec{B}_{\beta}\right)=\left[U^{J}\left(B_{\beta / M}\right)+\frac{1}{(1+\rho)} \cdot U^{J}\left(B_{\beta / V}\right)\right]\right] \\
& X X^{/ / M \alpha}\left\{\begin{array}{l}
\left(B_{\beta / M}+B_{\beta / V}\right)=\left(B_{\alpha / M}+B_{\alpha / V}\right) \equiv B_{\alpha}^{+} \\
B_{\beta / V} \geq B_{\alpha / V} \\
B_{\beta / M}, B_{\beta / V} \geq 0
\end{array}\right. \\
& \max J_{\alpha / M}
\end{aligned}
$$

For the sake of this analysis the set $\left(B_{\beta / M,}, B_{\beta / V}\right) \in X X^{J / M \alpha}$ will be established by the following requirements:

- closeness of the transfer $\left(B_{\beta / M}+B_{\beta / V}\right)=\left(B_{\alpha / M}+B_{\alpha / V}\right) \equiv B_{\alpha}^{+}$(nothing is lost or gained during the transfer),
- general non-negativity $B_{\beta / M}, B_{\beta / V} \geq 0$,
- social justice (fairness) $B_{\beta / V} \geq B_{\alpha / V}$.


### 35.2.5 Donor vs. Collector

The requirement of social justice (fairness) $B_{\beta / V} \geq B_{\alpha / V}$ has been applied here for only illustrative purposes. It can be nicely exemplified by John who is a father of Mary and Victor, where Mary is - according to John - disproportionately wealthier than Victor. As a result, John will design Mary so as to give up some of her present wealth in favor of her less fortunate brother Victor.

Invoking Fig. 103 we show by the broken half line in Fig. 123 that Mary will be - by design -prohibited to become a Collector. She can become only a Donor as the transfer $\Delta B_{\alpha / M}$ is allowed to be only negative ( $\Delta B_{\alpha / M}<0$ ), always in favor of Victor.


### 35.3 Analytical and graphical solution

### 35.3.1 Kuhn-Tucker in action

Invoking the procedure from Fig. 105, also here LHS and RHS will stand for the "right-hand side" and "left-hand side", respectively, of the Kuhn-Tucker first-order conditions that the solution to $\max J_{\alpha / M}$ must satisfy.

If we confine for simplicity to only interior solutions to $\max J_{\alpha / M}$ a simple application of a Kuhn-Tucker procedure shown in Fig. 124 brings the following property of the looked-for income re-allocation

$$
\vec{B}_{\beta}^{*}=\left(\mathrm{B}_{\beta / M}^{*}, \mathrm{~B}_{\beta / V}^{*}\right)
$$



Fig. 124

### 35.3.2 Equality of marginal utility

In the two-Plant arrangement we have in Fig. 105 derived that in the optimum the marginal revenues of the two Plants must be equal:

$$
\frac{d h_{M}\left(B_{\beta / M}^{*}\right)}{d B_{\beta / M}}=\frac{d h_{V}\left(B_{\beta / V}^{*}\right)}{d B_{\beta / V}}
$$

Apparently, then, the above property of the two-Consumer arrangement turns into equality

$$
\frac{d U^{J}\left(B_{\beta / M}^{*}\right)}{d B_{\beta / M}}=\frac{d U^{J}\left(B_{\beta / V}^{*}\right)}{d B_{\beta / V}}
$$

for $\rho=0$ or $\frac{1}{(1+\rho)}=1$ when the "psychological factor" is irrelevant for
John's choice. John's choice.

### 35.3.3 Graphical representation

The following Fig. 125 and Fig. 126 then illustrate the major property of the solution in Fig. 124 - on the proviso that, as assumed, the utility function $U^{J}(B)$ is concave and $\rho \geq 0$.

### 35.3.4 Digression - utility of money

In economics it is often assumed that money or profit is a reasonable criterion of choice. Various "events" can then be relatively reliably evaluated merely by the monetary wealth $B$ that can be associated to them. In this sense, wealth $B$ itself is the measure of utility and, correspondingly, the increasing concave function $U^{J}(B)$ shown in (b) of Fig. 125 is a
a utility of utility
Of our interest this note should be namely in relation to the following chapter on "generalized utility" in the subsequent Comment 10.


Fig. 125

### 35.3.5 Infeasibility and sensitivity

In Fig. 125 and Fig. 126 we also stress that the looked for re-allocation of wealth must lie on the solid pink segment of the budget line

$$
B_{\beta / V}=(-1) \cdot B_{\beta / M}+\mathrm{B}_{\alpha}^{+}
$$

i.e. the allocations between $\vec{B}^{+}$and $\vec{B}^{0}$. For other point on the budget line we obtain the following:

- the black broken segment is infeasible due to the fairness (social justice) constraint,
- the blue broken segment is impossible due to our assumption about the discount rate $\rho$.

In conclusion, the green arrow in Fig. 126 shows how the iso-welfare curves change with the increase of the discount rate. The resultant south-west shift of the resultant re-allocations is then demonstrated by red arrow.


### 35.4 Physical vs. monetary representation

### 35.4.1 Edgeworth box representation

A said, work-flow charts - such as that in Fig. 100 - are a fundamental tool for BPM. To compare, the Edgeworth box in Fig. 127 belongs to the classic tool-kit of ET.


Fig. 127

### 35.4.2 Iso-wealth lines

Due to the closeness condition, the overall wealth of Mary and Victor remains the same after the transfer

$$
\begin{aligned}
& \left(\vec{x}_{\beta / M}+\vec{x}_{\beta / V}\right)=\left(\vec{x}_{\alpha / M}+\vec{x}_{\alpha / V}\right)=x x^{+} \\
& \left(B_{\beta / M}+B_{\beta / V}\right)=\left(B_{\alpha / M}+B_{\alpha / V}\right)=B_{\alpha}^{+}
\end{aligned}
$$

Hence, to every point of the Edgeworth box in Fig. 127 is associated the same $x x^{+}$and $B_{\alpha}^{+}$. By contrast, every point of the box represents a different allocation $\overrightarrow{x x}\left(\vec{x}_{M}, \vec{x}_{V}\right)$ - a different distribution of the overall physical wealth $x x^{+}$between Mary and Victor.

Still more interesting should be that different physical allocations

$$
\overrightarrow{x x}^{1} \text { and } \overrightarrow{x x}^{2}
$$

may - under specific circumstances - represent the same monetary allocations

$$
\left(B_{M}^{1}, B_{V}^{1}\right)=\left(B_{M}^{2}, B_{V}^{2}\right)
$$

Exactly this occurs for $\overrightarrow{x x}$ lying on a straight line rectangular to the price vector $\vec{p}$ - such as the green line in Fig. 127 passing through $\overrightarrow{x x}_{\beta}^{0}$ and $\overrightarrow{x x}_{\beta}^{00}$. For obvious reasons, to both these - mutually different - re-allocations correspond the same pair of wealth $B_{\beta / M}^{0}$ and $B_{\beta / V}^{0}$ of Mary and Victor, respectively.

### 35.4.3 Efficiency, equilibrium, optimality and social justice

Firstly, we should note that the text-book topics would concern optimality, Walras equilibrium and Pareto efficiency of this or that point $\overrightarrow{x x}=\left(\vec{x}_{M}, \vec{x}_{V}\right)$ in the Edgeworth box.

Leaving these topics aside, we shall rather focus on what we already noted about John's sense of fairness or "social justice". The example in Fig. 127 was selected so that

$$
\vec{x}_{\alpha / M}>\vec{x}_{\alpha / V} \text { and hence also } B_{\alpha / M}>B_{\alpha / V}
$$

which is to correspond to our earlier assumption that John's daughter Mary is wealthier than her brother Victor.
The green-mellow shaded area then consists in only re-allocations in which Victor's wealth will increase in both commodities - bread and wine. The "social policy" of John-the Designer will then be, again, to impose on Mary an ex ante regulation such that only re-allocations from the shaded area will be feasible.

To clarify the regulation, let us note that the green line that passes through an "un-feasible" physical re-allocation $\overrightarrow{x x}_{\beta}^{0}$ is feasible in monetary terms as $B_{\alpha / V}^{0}>B_{\alpha / V}$. In sum, the transition

$$
\overrightarrow{x x}_{\alpha} \Rightarrow \overrightarrow{x x}_{\beta}^{0}
$$

would on the one hand deprive Victor of some wine but this loss would be, in monetary terms, more than compensated by a generous "donation" of bread.

### 35.4.4 Welfare function - physical representation

As stressed the concern of John-the Designer is about the so-called welfare, i.e., a utility of more than one agent, in particular the two of them personified by Mary and Victor.
Put formally, within his design of the present Mary

$$
M_{\alpha}(1) \equiv\left[\bar{m}_{\alpha}:\left\langle\vec{x}_{\alpha / M}, \vec{x}_{\alpha / M}\right\rangle \rightarrow\left\langle\vec{x}_{\beta / M}, \vec{x}_{\beta / V}\right\rangle\right]
$$

John designs the following maximization problem

$$
\begin{aligned}
& \max W^{J}\left(\overrightarrow{x x}_{\beta}\right)=W^{J}\left(\vec{x}_{\beta / M}, \vec{x}_{\beta / V}\right) \\
& \text { s.t.: }\left(\vec{x}_{\beta / M}, \vec{x}_{\beta / V}\right) \in X X
\end{aligned}
$$

where:
$X X \quad$ is the shaded area in Fig. 127,
$W^{J}\left(\vec{x}_{\beta / M}, \vec{x}_{\beta / V}\right) \quad$ will be taken in its text-book form of a weighted sum

$$
W^{J}\left(\vec{x}_{\beta / M}, \vec{x}_{\beta / V}\right)=\left[A \cdot U^{J}\left(\vec{x}_{\beta / M}\right)+B \cdot U^{J}\left(\vec{x}_{\beta / V}\right)\right]
$$

where the weights $A$ and $B$ are often normalized as

$$
W^{J}\left(\vec{x}_{\beta / M}, \vec{x}_{\beta / V}\right)=\left[U^{I}\left(\vec{x}_{\beta / M}\right)+\frac{1}{(1+\rho)} \cdot U^{J}\left(\vec{x}_{\beta / V}\right)\right]
$$

where $U^{J}\left(\vec{x}_{\beta / M}\right)$ and . $U^{J}\left(\vec{x}_{\beta / V}\right)$ are - again - assumed in their text-book pseudocardinal increasing and concave form.

### 35.4.5 Optimal iso-wealth line

Let us simply assume that the red-colored allocation $\overrightarrow{x x}_{\beta}$ in Fig. 127 represents Mary's task how to re-allocate the system/process. We will mark this re-allocation by " ++ " as $\overrightarrow{x x}_{\beta}^{++}$and denote Mary's task by

$$
\overrightarrow{x x}_{\beta} \Rightarrow \vec{x} \vec{x}_{\beta}^{++}
$$

However, we know, that all re-allocations from the red iso-wealth line passing through $\overrightarrow{x x}_{\beta}^{++}$are equivalent with respect to the corresponding income distribution

$$
\left(B_{\alpha / M}^{++}, B_{\alpha / V}^{++}\right)
$$

Given that both Mary and Victor are consumers, any point of the red iso-wealth line is to any of them as good as $\overrightarrow{x x}_{\beta}^{++}$.
Hence, what is looked-for is, in fact, not a particular $\overrightarrow{x x}_{\beta}^{++}$but a straight line passing through $\overrightarrow{x x}_{\beta}^{++}$rectangular to the price vector $\vec{p}$.
Fig. 128 is thus our graphical representation of the search for an resultant physical allocation and its equivalent translation into the search for an optimal iso-wealth line.


### 35.4.6 Digression: two-Plant case

We will leave it to the reader to explain the analogy between the above Fig. 128 and Fig. 129. We will only suggest that the following picture represents the same kind of the relationship between natural and monetary representations as Fig. 128, but for the case of the production-type system/process from Fig. 104.


### 35.5 Associated notes

### 35.5.1 Post-transfer developments

Recall that within the social context concerned, John is assumed to have no ambition to interfere into post-transfer decisions of Mary and Victor - as long as they keep to their roles of Consumers.

Let us return to Fig. 127 where the re-allocation (behavior/action)

$$
\overrightarrow{x x}_{\alpha} \Rightarrow \overrightarrow{x x}_{\beta}
$$

represents in physical units a prescription of a task of the present Mary. Hence, if fulfilled the newly established physical wealth of the two future agents will be

$$
\vec{x}_{\beta / M,}, \vec{x}_{\beta / V}
$$

and the question under study here is how each of them will dispose of with the newly acquired endowment of bread and wine.

To illustrate, we shall confine to Victor in the following section.

### 35.5.2 Variant choices

Let us emphasize again that in the case under study the only interest of Johnthe Designer was the "social goal" of awarding Victor by some extra wealth. As a small digression we may thus note that there may exist other Designers with the power to affect Victor's post-transfer behavior/action. Still more importantly, one of such Designers may be Victor himself. As any other Designer indeed, Victor may design his own post-transfer IF-THEN rule with the following, variant and highly speculative, empirical consequences:

1) IF the transfer of bread and wine is completed by the end of June, he will retain his role of a Consumer and, consequently, proceed as explained in Fig. 72 of Comment 6 and now shown again in (a) of Fig. 130.
2) Alternatively, IF the transfer is completed in August or even later, Victor will take on the role of a Seller and sell all his bread $x_{1 / \beta / V}$ and wine $x_{2 / \beta / V}$ and "invest" thus collected income $B_{\beta / V}$ into a consumption of rice $x_{R C}$ and brandy $x_{B D}$ - as shown in (b) of Fig. 130.
3) IF the transfer - regardless of the time of completion - is executed personally by Ms. White (Mary's assistant) - Victor will pay little attention to bread and wine and purchase a bunch of flowers, instead.


### 35.5.3 Designer's motivation

John-the Designer - as a head of a family - was characterized by such terms as fairness, social justice and solidarity.

The obvious alternative to the above politically correct social objectives we could have as well assumed that John's genuine - more realistic (?) - motivation was based - at least partly - on economic efficiency. In particular, by his regulatory move he may as well attempt to minimize the transaction costs brought up by a social revolt of the less fortunate sibling.

## 36. THE CASE OF A CONSUMER AND A BANK

Likewise in the preceding chapter, John will design the three-unit two phase system/process from Fig. 100

$$
s t r^{*}=\left(M_{\alpha}(1) ; M_{\beta}(1), \mathrm{V}_{\beta}(1)\right)
$$

so as to maximize his overall future welfare. What will be different is that whereas Mary will remain a Consumer, Victor will be assumed to contribute to the welfare through profit-making banking services.

### 36.1 The nature of banking services

### 36.1.1 Collector vs. Donor

The system/process will be analyzed in the strict analogy with the chapter on the Firm and its Bank. To begin with, Mary will be again assumed to have a debit account in Victor's Bank and the transfers will obtain the form of bank deposits $\left(\Delta B_{\alpha / M}<0\right)$ or withdrawals ( $\Delta B_{\alpha / M}>0$ ). Also here, we will assume, for simplicity again, that at $\alpha$ the account will be empty

$$
B_{\alpha / V}=0
$$

Invoking Fig. 114, we can summarize Mary's roles of a Donor vs. Collector in an identical picture in Fig. 131.


### 36.1.2 Reverse overall revenue

The fact that Mary is a Consumer will be characterized, again, by the above established zero production and revenue functions $Q_{\beta / M}=a_{f / M} \cdot f_{M}\left(K_{\beta / M}, L_{\beta / M}\right)=0$ and $T R_{\beta / M}=h_{M}\left(B_{\beta / M}\right)=0$.

As to the revenue function $T R_{\beta / V}=h_{V}\left(B_{\beta / V}\right)$ of the other future agent Victor, the fact that he is a Bank will be represented so that $T R_{\beta / V}=(1+r) . B_{\beta / V}$ or, given that $B_{\alpha / V}=0$

$$
T R_{\beta / V}=(1+r) \cdot \Delta B_{\alpha / V}
$$

where, apparently, $r$ is a (market, exogenously given) interest rate.
In sum, the overall revenue of the system/process is $T R_{\beta}=\left(T R_{\beta / M}+T R_{\beta / V}\right)=$ $T R_{\beta / V}=(1+r) . \Delta B_{\alpha / V}$. Equivalently, in terms of a reverse revenue function we obtain further applied "budget constraint"

$$
T R_{\beta / V}=H_{V}\left(B_{\beta / M}\right)=-(1+r) \cdot\left(B_{\beta / M}-B_{\alpha / M}\right)
$$

### 36.2 Present agent

### 36.2.1 Welfare function

The mixed nature (consumption vs. banking) of the system/process under study will be reflected so that

$$
W^{J / M \alpha}\left(B_{\beta / M}, T R_{\beta / V}\right)=\mathrm{a} \cdot U^{J}\left(B_{\beta / M}\right)+\mathrm{b} \cdot U^{J}\left(T R_{\beta / V}\right)
$$

where, in contrast to the two-Consumer arrangement from preceding chapter:
$U^{J}\left(B_{\beta / M}\right) \quad$ will now represent John's pleasure generated immediately at $T^{\beta / 1}$ (when Phase $\beta$ begins) - by $B_{\beta / M}$ that Mary will spend on her consumption,
$U^{I}\left(T R_{\beta / V}\right) \quad$ will now represent John's postponed pleasure generated later - at $T^{\beta / 2}\left(\right.$ by the end of Phase $\beta$ ) - by the overall revenue $T R_{\beta}=T R_{\beta / V}$ of the system/process.

In plain terms, John compares two mutually different things that he both likes Mary's consumption and Victor's economic efficiency. This mixture leads to the following design of the present Mary

$$
\begin{aligned}
& \max W^{J M \alpha}\left(B_{\beta / M}, T R_{\beta / V}\right)=\left[U^{J}\left(B_{\beta / M}\right)+\frac{1}{(1+\rho)} \cdot U^{J}\left(T R_{\beta / V}\right)\right] \\
& \text { s.t.: } \\
& T R_{\beta / V}=H_{V}\left(B_{\beta / M}\right)=-(1+r) \cdot\left(B_{\beta / M}-B_{\alpha / M}\right) \\
& B_{\beta / M} \geq 0
\end{aligned}
$$

### 36.2.2 Intra-vs. inter-temporal choice

Invoking the welfare function from the preceding chapter, what is new here is that in addition to John's inter-personal preference (e.g. aversion to Victor), the parameter $\rho$ will now also embody John's inter-temporal preference, e.g. his highly probable aversion to postponements of pleasure.

The intuition behind the two kinds of motivation may be the following:

- whereas, as already noted, the inter-personal $\rho$ is likely to be positively correlated to a geographical distance between John and Victor,
- the inter-temporal $\rho$ may depend on the length of Phase $\beta$ or the time distance between the moment of the immediate pleasure $U^{J}\left(B_{\beta / M}\right)$ and the time of the postponed pleasure $U^{J}\left(T R_{\beta / V}\right)$.

Given that the latter case is a subject of a so-called inter-temporal analysis, the inter-personal preferences could be also characterized as an analysis of an intra--temporal nature.

### 36.3 Analytical and graphical solution

### 36.3.1 Kuhn-Tucker in action

If we confine - for simplicity again - to only interior solutions to $\max J_{\alpha / M}$ the application of the procedure shown in Fig. 105 and Fig. 124 will now obtain the following form:


Fig. 132

Assuming that the utility of money $U^{J}(B)$ is given, the resultant classic equation suggests how the optimum $\left(B_{\beta / M}^{*}, T R_{\beta / V}^{*}\right)$ depends on the only two exogenous parameters $-r$ and $\rho-$ of $\max J_{\alpha / M}$. Apparently, as the respective text-books will
stress, the two parameters have an opposite impact upon the outcome. It is briefly sketched in Fig. 135 that can be well compared to the case shown in Fig. 126.

### 36.3.2 Utility possibility frontier

Relying on the outcomes of our foregoing preparatory labor, e.g. in Fig. 116, we will dare take as self-explanatory the graphical representations in Fig. 133, where:
section (a) represents the revenue functions of the two future agents where Mary's $T R_{\beta / M}=h_{M}\left(B_{\beta / M}\right)$ is trivially everywhere zero as shown by the pink horizontal line,
section (b) represents the reversed revenue function $T R_{\beta / V}=H_{V}\left(B_{\beta / M}\right)$ which now - given the specific circumstances of the Consumer--Bank case - can be also taken as a utility possibility frontier (production possibility boarder).


The green curves in (b) of Fig. 133 represent contours of the above established welfare function $W^{J / M \alpha}\left(B_{\beta / M}, T R_{\beta / V}\right)$.

### 36.3.3 Zero endowments

For completeness, as in Fig. 118, the following Fig. 134 is to cast some light on the case of a completely "broke Mary" who has nothing not only on her account ( $B_{\alpha / V}=0$ ) but also "in her pocket" $\left(B_{\alpha / M}=0\right)$.


### 36.3.4 Lender/Creditor vs. Borrower/Debtor

One of our terminological notes compared the terms Donor vs. Collector with the text-book concepts of an Investor vs. Dis-investor, Creditor vs. Debtor or even Lender vs. Borrower.

In Fig. 135 we briefly sketch how relative changes in $r$ and-or $\rho$ may turn a Lender into Borrower and vice versa.


### 36.4 Additional external income

### 36.4.1 Consumer's debt

For the Producer-Bank arrangement we stressed that the system/process can never become "insolvent" - due to the non-negativity condition $T R_{\beta} \geq 0$ contained in $\max J_{\alpha / M}$. Hence, in Fig. 119 the dis-investment $B_{\beta / M}^{s}$ represented where the system/process as a whole breaks even

$$
T R_{\beta}^{s}=\left(T R_{\beta / M}^{s}+T R_{\beta / V}^{s}\right)=0
$$

Contrariwise, now, when Consumer-Bank arrangement is under study, the optimal re-allocation $\left(B_{\beta / M}^{*}, T R_{\beta / V}^{*}\right)$ in (b) of Fig. 133 illustrates the case when

$$
T R_{\beta / V}^{*}=(1+r) \cdot\left(B_{\alpha / M}^{+}-B_{\beta / M}^{*}\right)<0
$$

and hence also the resultant overall revenue/wealth of the system/process is negative

$$
T R_{\beta}^{*}=\left(T R_{\beta / M}^{*}+T R_{\beta / V}^{*}\right)=\left(0+T R_{\beta / V}^{*}\right)<0
$$

The reason is that the resultant indebtedness/insolvency is not prohibited by $\max J_{\alpha / M}$.

### 36.4.2 Present agent

With the aim to remedy the above "deficiency" $\max _{\alpha / M}$ will be expanded by:

1) a non-negativity constraint $T R_{\beta} \geq 0$,
2) an additional wealth $B_{\beta M}^{\mathrm{e} x}$ that will be "infused" into the system/process from some external resource by the end $T^{\beta / 2}$ of Phase $\beta$.

Consequently, John may dare prohibit insolvency of the system/process by this design of the present Mary as follows:

$$
\begin{array}{ll}
\max W^{I M \alpha}\left(B_{\beta / M}, T R_{\beta / V}\right)=\left[U^{J}\left(B_{\beta / M}\right)+\frac{1}{(1+\rho)} \cdot U^{J}\left(T R_{\beta / V}\right)\right] \\
\text { s.t.: } & \max J_{\alpha / M} \\
T R_{\beta}=\left(T R_{\beta / V}+B_{\beta / M}^{\mathrm{ex}}\right)=-(1+r) \cdot\left(B_{\beta / M}-B_{\alpha / M}\right)+B_{\beta / M}^{\mathrm{ex}} \\
B_{\beta / M}, T R_{\beta} \geq 0 &
\end{array}
$$

In text-books the "flow of money" such as $B_{\alpha / M}$ and $B_{\beta M}^{\mathrm{ex}}$ is often called an inter-temporal distribution (allocation) of income
and the relationship between the present and future income is usually presented as the following inter-temporal budget constraint.


The expansion of the analysis is summarized in Fig. 136. As promised, the picture seeks to demonstrate perhaps the major methodological objective of this BOOK. It hopefully shows how our rather burdensome and largely non-standard labor can lead to a diagram that any under-graduate student of economics must learn.


### 36.4.3 Edgeworth box representation

The following Fig. 137 should be self-explanatory as it is "only" an "inter--temporal" version of the above "intra-temporal" Fig. 128.


## 37. THE CASE OF A ROBINSON CRUSOE ECONOMY

Finally, John will be assumed to maximize his welfare by designing the three-unit two phase system/process from Fig. 100

$$
s t r^{*}=\left(M_{\alpha}(1) ; M_{\beta}(1), V_{\beta}(1)\right)
$$

so that Victor will return to the role of an "ordinary" Producer, e.g. a Shoe--making Manufacturer, whereas Mary will remain in her role of a Consumer.

### 37.1 The case of a Consumer and Manufacturer

### 37.1.1 Endowments

For concreteness and simplicity, let the endowments - at $T^{\alpha}$ - be:

$$
\vec{B}_{\alpha}=\left(B_{\alpha / M}, B_{\alpha / V}\right)=\left(0, B_{\alpha / V}\right)
$$

where the zero endowment $B_{\alpha / M}$ means that - at $T^{\alpha}$ - Mary enters the system/ /process with "empty pockets" in the sense of Fig. 134.

As to Victor, the fact that he is now a non-bank agent implies that at $T^{\alpha}$ his wealth must be $B_{\alpha / V}>0$, should he be able to support Mary's consumption as early as at $T^{\beta / 1}$. In other words, as a non-bank agent Victor can no longer issue bank loans - not to mention the infinitely large loans allowed by Fig. 114.

Similarly, the non-banking Victor will be a "Donor proper" in the sense that Mary-the Collector will not turn into a Borrowed/Debtor with an obligation to pay back her debt.

### 37.1.2 Donor vs. Collector

Given the above distribution of roles and their endowments, the transfers will be constrained as follows:

$$
0 \leq \Delta B_{\alpha / M} \leq B_{\alpha / V}
$$

Invoking Fig. 123 we show in Fig. 138 by the broken half line that - due to $B_{\alpha / M}=0$ - it is "physically impossible" for Mary to become a Donor. She can become only a Collector, albeit constrained from above by Victor's endowment $B_{\alpha / V}$.


### 37.1.3 Behavioral cycles

Fig. 139 provides us with an opportunity to refresh our understanding of behavioral cycles and, moreover, offer to the reader the following interpretation of the above established range of feasible transfers:
if $\Delta B_{\alpha / M}=0 \quad$ then $\left(B_{\beta / M}, B_{\beta / V}\right)=\left(0, B_{\alpha / V}\right)$ and Mary will die from hunger by the end of CYCLE 1,
if $\Delta B_{\alpha / M}=B_{\alpha / V} \quad$ then $\left(B_{\beta / M}, B_{\beta / V}\right)=\left(B_{\alpha / V}, 0\right)$ and Mary will die during CYCLE 2 as Victor will be unable to generate new revenue.


### 37.1.4 Present agent - a graphical representation

After all the preparatory work in foregoing chapters on $\max _{\alpha / M}$ we dare hope that Fig. 140 is self-explanatory.

The example has been, for dramatic effect, selected so as to show that:

- even though $B_{\beta / M}^{*}<B_{\beta / V}^{*}$ and hence a larger part of Victor's endowment $B_{\alpha / V}$ is not at $T^{\beta 1}$ invested into shoe-making,
- at the end $T^{\beta 2}$ of the production Phase $\beta$, Victor's newly accumulated wealth $T R_{\beta / V}^{*}$ exceeds his endowment $B_{\alpha / V}$.



### 37.2 Persons and roles

### 37.2.1 Desert island

Repeatedly or even continuously we emphasize that any person can become a Designer and as such design or nominate anybody into any role. Hence, it may be Mary herself - not John - who will design the three-unit two phase system/process from Fig. 100. Not only that, she may design herself into all the roles associated to the three operational units.

To illustrate the social context, Mary can be seen as someone dragged to a desert island after a shipwreck - exactly like the classic Defoe's Robinson Crusoe. Once on the desert island, Mary simply cannot avoid designing some strategy how to survive - how to allocate her time between "work" and "leisure", as many text-books characterize the choice. Put differently, Mary-the Robinson will simply have to become a Consumer and Manufacturer and for that matter impose some ex ante regulation on her performance of the two roles.

### 37.2.2 Nominees

The social context of a desert island brings to fore the obvious fact that except Mary there is also no one else who can become a Beneficiary, Defendant and Manager for the sake of the three operational units.

For example, it must be also Mary who will - as a Beneficiary - every morning decide whether and when she will "blow the whistle" so as to launch the respective behavioral cycle.

Needless to emphasize that every time, before the whistle, Mary may - as a Designer - consider whether and how her existing survival strategy will be re-designed.

### 37.3 Four-unit two-phase system/process

The foregoing note on persons and roles may no more than sketch the trivial fact that the agove analysis, as any other analysis indeed, can be expanded in an "infinite" number of ways.

To illustrate, the SPLIT-type four-unit two-phase system/process in Fig. 141 is a kind of aggregation of the "ordinary" and "extra-ordinary" kinds of production.

For simplicity the overall endowment of the system (at $\mathrm{T}^{\alpha}$ ) is taken to be zero

$$
B_{\alpha / M}=0, B_{\alpha / Z}=0 \text { and } B_{\alpha / V}=0
$$

Summarizing, then:

- non-bank operations are at $T^{\beta / 1}$ financed by a an overall bank loan $\Delta B_{\alpha / Z}=-\left(\Delta B_{\beta / M}+\Delta B_{\alpha / V}\right)$,
- "at the end of the day" - at $T^{\beta / 1}$ - it is Victor-the Shoe-maker who is responsible for the economic growth or decline of the system/process.


Fig. 141

## COMMENT 10.

## Designer's choices; the case of an uncertain profit

Problems concerned in this BOOK should be firstly classified according to whether they concern dynamics or kinetics of a system/process - whether their subject is the Designer's choice or the consequent behavior/actions of the Designees.

So far, a Designer's choice - in itself - was dealt with in Comment 1, where, however, the analysis mainly focused on only the "mechanics and logistics" of the respective decision making. The reason was that the Designer was, for the sake of the analysis, taken to be of a collective nature. As a result, the core of the problem lied in the election held on the floor of the respective organization - on the aggregation of its members' "votes". In fact, then, essentially no mention was made about the "upper-case" maximization through which a Designer, by definition, selects his optimal strategy.
This debt is to be paid now within the following Comments 10 and 11.
In contrast to Comment 1, the role of a Designer will be now of an individual nature, personified again by John. As before, he will solve his "upper case" $M A X_{J}$ with the aim to design "other agents' tasks" so as to support fulfillment of his own "higher-level" task $J(1)$. Also here John will believe that "money matters" and hence his strategy should always rest in a procurement of a big enough dose of financial resources. In other words, John will be assumed to select his strategy from among variant profit-making systems/processes.
Rather independently to the above introduction we will note that our interest in the "upper case" $M A X_{J}$ will be also taken as an opportunity to further corroborate what has been already said about the general inter-temporality of social choice and behavior, namely the concepts of:

- a discount rate or relative socio cultural distances between the present Designer and the future Designees,
- incompleteness of information or uncertainty about future consequences of the Designer's present choices.

In sum, the subject of this Comment 10 may be condensed into a simple question how certain or uncertain will John see profits of the variant Firms included in his $M A X_{J}$.

## 38. ASSUMPTIONS, SIMPLIFICATIONS AND NOTATIONS

### 38.1 A choice from a set of profit-making Firms

The social arrangement under study will be summarize as follows:

1) Under study will be a Designer of an individual (non-collective) nature, namely John-the Designer.
2) John will select his optimum from a discrete set of $N$ mutually exclusive strategies denoted as $\left\{F_{1}, F_{2}, \ldots, F_{i}, \ldots, F_{N}\right\}$.. Every $\boldsymbol{i}$-th variant strategy $F_{i}$ :

- will be assumed to be a one-unit one-phase system/process - in the sense of the pattern (a) of Fig. 8,
- will have the empirical meaning of a profit-making Firm.

3) Environments of the variant strategies/Firms $F_{i}, i=1,2, \ldots, N$

- will be mutually different (strategies/Firms will be heterogeneous),
- will consist in whatever exogenous variables, including "non-economic" ones.

Put formally, John-the Designer will select his optimal strategy/Firm $s t r^{*} \equiv F^{*}$ through solving:

```
max UU
```



```
\(M A X ~_{J}\)
```

where:
$F(0) \quad$ is John's sphere of interest consisting in $N$ firms, such as, e.g., Victor's grocery, Mary's shoe-making enterprise,Nina's bank-robbery gang, ...
$U^{J}(F) \quad$ is a utility function representing John's preferences.
As said, the graphical and analytical representation of thus established $M A X_{J}$ will be the major subject of this Comment 10.

### 38.1.1 Heterogeneous strategies

Confining to only one-unit strategies makes it relatively easy, in principle, to represent the environment of every $i$-th variant strategy $F_{i}$. In what follows we will characterize the environment by vectors $\vec{e}_{i}$ and $\vec{E}_{i}$, where:
$\vec{e}_{i}=\left(e_{i / 1}, e_{i / 2}, \ldots, e_{i / n / i)}\right) \quad$ is a set of exogenous variables by which the environment of every $i^{\text {th }}$ firm is constituted. The variables represent, e.g., prices, technological efficiency, states of weather, level of criminality, .... The mutual heterogeneity of the $N$ variant firms is determined by their mutually different environments $\vec{e}_{r} \neq \vec{e}_{s}, r, s=1$, $2, \ldots, N$.
$\vec{E}_{i}=\left(\vec{e}_{i}^{1} \vec{e}_{i j}^{2} \ldots, \vec{e}_{\nu}^{j} \ldots, \vec{e}_{i}^{m / i)} \quad\right.$ is a finite set of $(m / i)$ mutually exclusive conceivable states of the $i^{\text {th }}$ firm - the states of its environment.

### 38.1.2 Discrete variants and their discrete states

The scientific status of ET to a large degree depends on its capacity to apply differential calculus on nicely conceptualized maximization problems. In other words, economics could have produced its admirable text-books namely due to the artistry with which it managed to convert its subject into the infinitesimal representation.

Contrariwise, in this Comment 10, not only the variants $F_{1}, F_{2}, \ldots, F_{i j}, \ldots, F_{N}$ but also their environments do not constitute compact sets upon which the analyst could conceptualize well behaved continuous and differentiable functions.

Let it be stressed that this apparent complication has been admitted here for mostly didactic reasons. For example, the discrete probability distribution of the states in $\vec{E}_{i}$ will later make it, hopefully, somewhat easier to understand some deeper roots of the subject under study.

### 38.2 Two kinds of maximizations " $V$ "

### 38.2.1 Formula for the optimal firm

With the aim to keep in touch with standard text-books, as many times before, we will assume that every $i$-th one-unit variant strategy $F_{i}$ will take the form of a single "lower-case" maximization problem.

Having assumed that John relies on only financial resources, the looked-for maximization problem can be taken in its most elemental profit-maximizing formula
$\max \psi\left(K_{i}, L_{i}, Q_{i}\right)$
s.t.: $Q_{i} \leq a_{f i} \cdot \mathrm{f}^{0 / i}\left(K_{i}, L_{i}\right)$
$\operatorname{maxj}_{F / i}$
where, let us recall - for the convenience of the reader:
$\begin{array}{ll}\psi\left(K_{i}, L_{i}, Q_{i}\right) & \text { is profit } \psi=\left(p_{Q / i} \cdot Q_{i}-p_{K / i} \cdot K_{i}-p_{L / i} \cdot L_{i}\right), \\ Q_{i}=a_{f / i} \cdot \mathrm{f}^{0 / i}\left(K_{i}, L_{i}\right) & \text { is a production function. }\end{array}$
Apparently, also the optimal strategy

$$
F^{*} \in F(0)
$$

will obtain the form of the above formula. We shall denote it

$$
\max J_{F^{*}}
$$

The asterisk represents - as always - the fact that by this particular "lower-case" maximization is represented a solution to the respective "upper-case" $M A X_{J}$.

### 38.2.2 Two-step "procedure"

As stressed many times before, the differentiation between the "upper and lower--case" maximizations $M A X_{J}$ and $\max J_{F^{*}}$ is to be taken as our key contribution to the general analysis of choice and behavior/action.

However, on the rather intuitive level of argument, the relationship between the two kinds of maximizations can be interpreted so that:

- firstly, profit functions are construed for each of the variant firms on the basis of their their respective $\max J_{F / i}$
- secondly, the firms with the most "favorable" profit function is selected as the optimum strategy $F^{*}$.

The concept of "favorability" will be further developed in detail on the basis of what we shall call a "utility of profit", or, in a still more sophisticated way, an "expected utility of profit".

### 38.3 Associated notes

### 38.3.1 Existing (actual) vs. new (offered) strategy

Invoking our concept of the production cycles from Fig. 4 we may also recapitulate how this BOOK sees Designer's behavior/action as opposed to a behavior/action of a Designee. Recall, then, that at the beginning of every cycle John-the Designer has an opportunity to choose whether he will keep to his existing strategy - his actual Firm $F_{\alpha c}$ - or re-design it.

John's choice has then been taken as:

- Designer's action ("non-empty" behavior) if $F^{*} \neq F_{\alpha c}$
- Designer's non-action ("empty" behavior) if $F^{*}=F_{\alpha c}$.

For illustration, we may return to Fig. 3 and the associated $M A X_{J / a}$ where John selects from $\operatorname{str}^{J}(0)=\left\{M(1)_{1}, M(1)_{2}, V(1)\right\}$. Let then:
$F_{a c}=M(1)_{1} \quad$ be Mary-the Grocer and
$F^{*}=V(1) \quad$ be Victor-the Shoe-maker.
The empirical meaning of this (non-empty) action $F^{*} \neq F_{\alpha c}$ can be that John terminates his contract with Mary and signs a new one with Victor.

### 38.3.2 Change of notation

A text-book firm has been above characterized by a profit function $p f 1$ or a mapping

$$
\vartheta:\left\langle p_{\mathrm{K}}, p_{\mathrm{L}} p_{\mathrm{Q}^{\prime}}, a_{f}\right\rangle \rightarrow\left\langle\psi^{*}\right\rangle
$$

representing how an optimal profit $\psi^{*}$ of the Firm changes with changing environment - established by prices and efficiency only, in this particular case.

Unfortunately and with a deep apology to the reader this notation will now have to be changed. Every $i^{\text {th }}$ variant firm $F_{i}$ will be thus - in what follows characterized by a profit function denoted, somewhat confusingly, as

$$
B_{i}^{*}=\psi_{i}\left(\overrightarrow{e_{i}}\right)
$$

that shows what will be the optimal profit $B_{i}^{*}$ of the Firm in the state $\vec{e}_{i}$ of the Firm's environment.

## 39. GENERALIZED UTILITY

### 39.1 Utility of money

The topic concerned here was briefly mentioned in the association with the increasing concave function $U^{J}(B)$ shown in (b) of Fig. 125. We noted that, in fact the function represent
utility of utility
given that a monetary wealth $B$ can be - in itself - often taken as a reasonable measure of utility, namely when the utility of the Firm is under study.

Also here, as suggested, one of the major points of this Comment 10 will be that mutually different profit functions of variant Firms can be compared according to their "utility of profit", or, still more interestingly, "expected utility of profit". To begin with, for the sake of this stage purely formally, we will establish a function

$$
u\left(B_{i}^{*}\right) \equiv u\left(\psi_{i}\left(\overrightarrow{e_{i}}\right)\right) \equiv u_{i}^{*}
$$

that will represent the already mentioned fact that not the profit itself but its somehow generalized utility will represent preferences of John-the Designer. Apparently, then, substantial amount of what follows will be devoted to the bases upon which this generalization can be reasonably construed.
We will see that the function $u\left(B_{i}^{*}\right)$ will embody individual "psychological portrait" of the respective Designer, very much in the sense that the so-called discount rate $\rho$ reflected the particular "psychology" of the Designer in our earlier chapters.

### 39.2 Utility of the Firm

### 39.2.1 Text-book notation

With the aim to approach the classic (somewhat less accurate and in fact redundant) text-book notation, every $i^{{ }^{\text {th }}}$ firm $F_{i}$ will be put as

$$
F_{i}=\left(\vec{e}_{i}^{1} \vec{e}_{i j}^{2}, \ldots, \vec{e}_{i}^{\mathrm{j}}, \ldots, \vec{e}_{i}^{(m / i)} ; u_{i}^{* / 1}, u_{i}^{* / 2}, \ldots, u_{i}^{* / j}, \ldots, u_{i}^{* /(m / i)}\right), i=1,2, \ldots, N
$$

where, let us summarize

$$
u_{i}^{* / j}=u\left(\psi_{i}\left(\vec{e}_{i}^{j}\right)\right)
$$

is the optimal utility of the $\boldsymbol{i}$-th Firm $F_{i}$ in its environment's state $\vec{e}_{i}$.
For short we will sometime represent the Firm as $F_{i}=\left(\vec{E}_{i} ; \vec{u}_{i}\right)$. Hence, John will be selecting his optimal strategy

$$
F^{*}=\left(\vec{E}^{*} ; \vec{u}^{*}\right)
$$

from the set $F(0)=\left\{F_{1}, F_{2}, \ldots, F_{i}, \ldots, F_{N}\right\}$ where

$$
F(0)=\left\{\left(\vec{E}_{1} ; \vec{u}_{1}^{*}\right),\left(\vec{E}_{2} ; \vec{u}_{2}^{*}\right), \ldots,\left(\vec{E}_{i} ; \vec{u}_{i}^{*}\right), \ldots,\left(\vec{E}_{N} ; \vec{u}_{N}^{*}\right)\right\}
$$

### 39.2.2 Utility vector as an overall measure of utility

Then, it will be our major thesis that John's evaluation of every $i^{\text {th }}$ firm $F_{i}$ will be expressed by the vector

$$
\vec{u}_{i}^{*}=\left(u_{i}^{* / 1}, u_{i}^{* / 2}, \ldots, u_{i}^{* / j}, \ldots, u_{i}^{* /(m / i)}\right)
$$

Hence, the vector as a whole - and only this vector - expresses how much John likes or dislikes the particular variant of his choice. Purely formally, John's preferential ordering of the $N$ firms should then be - by definition - put as

$$
\left(F_{r} \gtrsim F_{s}\right) \Leftrightarrow\left(\vec{u}_{r}^{*} \gtrsim \vec{u}_{s}^{*}\right), r, s=1,2, \ldots, N
$$

In words, John will be better off with the firm $F_{r}$ than $F_{s}$ if and only if he will "like" the vector $\vec{u}_{r}^{*}$ of optimal utilities better than the vector $\vec{u}_{s}^{*}$.

Apparently, the methodological mystery is how to compare vectors, namely the above two vectors $\vec{u}_{r}^{*}$ and $\vec{u}_{s}^{*}$. As always, the solution rests in finding an appropriate substitute for the analytically unfriendly relationship between vectors with an equivalent relationship between values of a given representative scalar.

Hence what we look for is a representative scalar $u^{R}$ such that

$$
\left(\vec{u}_{r}^{*} \gtrsim \vec{u}_{s}^{*}\right) \Leftrightarrow\left(u_{r}^{R E P} \geq u_{s}^{R E P}\right), r, s=1,2, \ldots, N
$$

## 40. UNCERTAINTY

As said, the empirical meaning of the term uncertainty is that John-the Designer is not certain which of the $m / i$ states ( $\vec{e}_{i}^{1} \vec{e}_{i}^{2}, \ldots, \vec{e}_{i}^{\mathrm{j}}, \ldots, \vec{e}_{i}^{(m / i)}$ ) of the $i^{\text {th }}$ firm $F_{i}$ will factually occur.

### 40.1 Representative level of utility

### 40.1.1 Representative state

As said, our analytical problem is to replace the vector $\vec{u}_{i}^{*}=\left\{u_{i}^{* / 1}, u_{i}^{* / 2}, \ldots, u_{i}^{*}, \ldots\right.$, $\left.u_{i}^{* /(m / i)}\right\}$ with a scalar $u_{i}^{R E P}$. What we look for in fact is

$$
\text { a representative state } \vec{e}_{i}^{R E P}
$$

of the $m / i$ states $\vec{E}_{i}=\left(\vec{e}_{i}^{1} \vec{e}_{i}^{2}, \ldots, \vec{e}_{i j}^{j} \ldots, \vec{e}_{i}^{(m / i)}\right)$ of the environment of the $i^{\text {th }}$ firm $F_{i}$. If we find such state the representative scalar by which John will evaluate the $i^{\text {th }}$ Firm will be simply

$$
u_{i}^{R E P}=u\left(\psi_{i}\left(\vec{e}_{i}^{R E P}\right)\right)
$$

Apparently, $\vec{e}_{i}^{R E P}$ may be a state that - in itself - need not be feasible, need not be an element of the set of states $\vec{E}_{i}$.

### 40.1.2 Example

In order to illustrate the concept and with respect to our highly methodological objectives we will for a moment deviated from the classic text-book line of the topic and return to Fig. 53 depicting Manuela's efforts to express at present five future efficiencies $a_{f}^{1}, a_{f}^{2}, \ldots, a_{f}^{2}$ of Mary's firm. Also Manuela - this way or another - had to establish some kind of

$$
\text { a representative efficiency } a_{f}^{R E P}
$$

in order to substitute its value into the respective IF-THEN rule so as to calculate the prescription of Mary's task.

Already there we noted that any such $a_{f}^{R E P}$ will have to be - at the end of the day - confronted with the factual efficiency $a_{f}^{f_{c}}$.

### 40.1.3 Preferential ordering of variant firms

The reason for recalling Manuela's procedure here is that John-the Designer can in principle follow her example and consider how to represent the ( $\mathrm{m} / \mathrm{i}$ ) variant states $\left(\vec{e}_{i j}^{1} \vec{e}_{i j}^{2} \ldots, \vec{e}_{i j}^{j} \ldots, \vec{e}_{i}^{(m / i)}\right)$ by one single representative $\vec{e}_{i}^{R E P}$ in order to substitute it into the function $u\left(\psi_{i}\left(\vec{e}_{i}^{j}\right)\right)$ so as to obtain $u_{i}^{R E P}=u\left(\psi_{i}\left(\vec{e}_{i}^{R E P}\right)\right)$ as the looked-for scalar criterion for the evaluation of the $i^{\text {th }}$ firm $F_{i}$.

However analogous may appear the above two kinds of a representative states, $a_{f}^{R E P}$ and $\vec{e}_{i}^{R E P}$, their substance is of different nature:

- Manuela seeks to establish $a_{f}^{R E P}$ in order to prescribe Mary's particular task,
- John is in search for his $\vec{e}_{i}^{R E P}$ in order to evaluate the $i^{\text {th }}$ variant of his choice.

What John is after is the scalar criterion to be applied within the obvious simple rule

$$
\left(F_{r} \gtrsim F_{s}\right) \Leftrightarrow\left(u_{r}^{R E P} \geq u_{s}^{R E P}\right), r, s=1,2, \ldots, N
$$

However, also here, the representative state $\vec{e}_{i}^{\text {REP }}$ will be - at the end of the day - confronted with what actually occurred in reality.

### 40.2 Problems trivialized

Before we seriously start our search for John's scalar criterion, we shall illustrate the problem by simplifying it as shown in this section.

### 40.2.1 Profit as Designer's utility

Our first methodological question has aimed at a subjective utility of money/ /profit. By far easiest way how to answer this kind of a problem is by assuming that every $i^{\text {th }}$ firm $F_{i}=\left(\vec{e}_{i j}^{1} \vec{e}_{i j}^{2}, . . \vec{e}_{i}^{j}, \ldots, \vec{e}_{i}^{(m / i)} ; u_{i}^{* / 1}, u_{i}^{* / 2}, \ldots, u_{i}^{* / j}, \ldots, u_{i}^{* /(m / i)}\right)$ can be put as only

$$
F_{i}=\left(\vec{e}_{i}^{1}, \vec{e}_{i}^{2} \ldots, \vec{e}_{j}^{j} \ldots, \vec{e}_{i}^{(m / i)} ; B_{i}^{* / 1}, B_{i}^{* / 2}, \ldots, B_{i}^{* / j}, \ldots, B_{i}^{* /(m / i)}\right) \equiv\left(\vec{E}_{i}, \vec{B}_{i}^{*}\right)
$$

Due to this simplification we would in fact return to the social context where

$$
\left[u_{i}^{* / j}=u\left(\psi_{i}\left(\vec{e}_{i}^{j}\right)\right)\right]=\left[\psi_{i}\left(\vec{e}_{i}^{j}\right)=B_{i}^{* / j}\right]
$$

with the meaning that profit itself can be seen as a reliable embodiment of John's preferences.

### 40.2.2 Probability distribution

The second methodological question has asked how to replace the vector of optimal utilities - profits $\vec{B}_{i}^{*}=\left(\vec{B}_{i}^{* / 1}, B_{i}^{* / 2}, \ldots, B_{i}^{* j}, \ldots, B_{i}^{* /(m / i)}\right)$ in this particular case - by a representative scalar $B_{i}^{R E P}$ that will be equivalent in the sense $B_{i}^{R E P} \sim \vec{B}_{i}^{*}$.

The easiest way how to establish such $B_{i}^{\text {REP }}$ is based upon an assumption that John - apart from $\vec{E}_{i}$ and $\vec{B}_{i}^{*}-$ also knows, for every $i^{\text {th }}$ firm $F_{i}$, the probability distribution

$$
\vec{\pi}_{i}=\left(\pi_{i}^{1} \pi_{i,}^{2}, \ldots, \pi_{i}^{j}, \ldots, \pi_{i}^{(m / i)}\right)
$$

of the firm's states and, hence, also its optimal profits. If so, text-books often re-write the above $F_{i}=\left(\vec{e}_{i}^{1}, \vec{e}_{i}^{2}, \ldots, \vec{e}_{i}^{j}, . . \vec{e}_{i}^{(m / i)} ; B_{i}^{* / 1}, B_{i}^{* / 2}, \ldots, B_{i}^{* / j}, \ldots, B_{i}^{* /(m / i)}\right)$ into the following probability distribution of optimal profits

$$
\left.F_{i}=\left(\pi_{i}^{1}, \pi_{i j}^{2} \ldots, \pi_{i}^{j}, \ldots, \pi_{i}^{(m / i)}\right) ; B_{i}^{* / 1}, B_{i}^{* / 2}, \ldots, B_{i}^{* / j}, \ldots, B_{i}^{* /(m / i)}\right) \equiv\left(\vec{\pi}, \vec{B}_{i}^{*}\right)
$$

### 40.2.3 Mean (expected) value of profit

If John knows all the data contained in $\left(\vec{\pi}, \vec{B}_{i}^{*}\right)$, the trivialized way of obtaining the looked-for scalar criterion $B_{i}^{\text {REP }}$ is

$$
B_{i}^{R E P}=\sum_{j=1}^{(m / \mathrm{i})} \pi_{i}^{j} \cdot B_{i}^{* / j} \equiv \tilde{B}_{i}^{*}
$$

where $\widetilde{B}_{i}^{*}$ is a usual "mathematical" notation for an "ordinary" mean (expected) value of the respective variable - mean optimal profit in this particular case.

To conclude, two notes may be of value:

- Given that $\vec{E}_{i}=\left(\vec{e}_{i}^{1}, \vec{e}_{i j}^{2} \ldots, \vec{e}_{j}^{j} \ldots, \vec{e}_{i}^{(m / i)}\right)$ is, by assumption, an exhaustive list of mutually exclusive states, it is by definition, that one and only one of the states will certainly occur and hence $\sum_{j=1}^{(m / i)} \pi_{i}^{j}=1$.
- Purely formally, $\tilde{B}_{i}^{*}$ has also the meaning of a particular weighted average of utilities. As already suggested, apart from probabilities or along with them many other weights can be applied by John-the Designer, in principle.


### 40.2.4 Example

To illustrate, the concept of a mean (expected) value of profit, in Fig. 142 is a graphical representation of a particular probability distribution of $(\mathrm{m} / \mathrm{i})=6$ exhaustive and mutually exclusive states and optimal profits.


Two things may be note-worthy:

- In Fig. 142 we can see that $B_{i}^{* / 1}, B_{i}^{* / 2}<0$. This is to illustrate that optimal profits may be negative, which will become of some deeper interest in later chapters.
- The representative profit $B_{i}^{R E P}$ is not an element of the set $B_{i}^{* / 1}, B_{i}^{* / 2}, \ldots B_{i}^{* / 6}$, by which the $i^{\text {th }}$ firm $F_{i}$ is defined. The same thus applies to the representative state $\vec{e}_{i}^{R E P}$ formally obtainable as an outcome of the inverse operation $\vec{e}_{i}^{R E P}=\psi_{i}^{(-1)}\left(B_{i}^{R E P}\right)$.


### 40.3 Uncertainty vs. risk

As said, if the expected profit is the weighed average, John will calculate $\widetilde{B}_{i}$ for every strategy and determine

$$
\widetilde{B}^{*}=\max \left\{\widetilde{B}_{1}^{*}, \widetilde{B}_{2}^{*}, \ldots, \widetilde{B}_{i}^{*} \widetilde{B}_{N}^{*}\right\}
$$

to find the optimum strategy.
Given the discrete and finite distribution of states, the maximum always exists. If it is not unique, then either John is not a rational agent, or the criterion of his choice cannot be based solely on $\widetilde{B}_{i}$.

In the latter case the ambiguity may be resolved by adding to $\widetilde{B}_{i}$ a variance

$$
\left(\sigma_{i}\right)^{2}=\sum_{\mathrm{j}=1}^{(m / i)} \pi_{i}^{j} \cdot\left(B_{i}^{* / j}-\tilde{B}_{i}^{*}\right)^{2}
$$

Its square root $\sigma_{i}$ - the so-called standard deviation - is then sometimes interpreted as a so-called risk. John's preferences can then be represented by an utility function

$$
U=U(\widetilde{B}, \sigma)
$$

which is increasing in $\tilde{B}$ and decreasing in the "risk" $\sigma$. These properties of the utility function are graphically illustrated by the green indifference curves. The shaded area called bullet is depicted in Fig. 143 only with the aim to remind the reader of its application in the realm of financial economics.


## 41. MEAN/EXPECTED UTILITY OF PROFIT

As said, substantial amount of this Comment will focus on a construction of the above established "psychological portrait" $u\left(\psi_{i}\left(\vec{e}_{i}^{j}\right)\right) \equiv u\left(B_{i}^{* j}\right)$.

### 41.1 Referential interval of profits

In Fig. 144 we can see a set $F(0)$ consisting of $N=3$ variant strategies, i.e. mutually exclusive strategies $F_{1}, F_{2}$ and $F_{3}$. By $B_{L}$ and $B_{H}$ at the bottom of the picture are denoted the $L$-lowest and $H$-highest profits attainable within the set of strategies

$$
B_{L}=(-140) \text { usd and } B_{H}=(+200) \text { usd }
$$

All profits $B_{i}^{j}$ attainable within $F(0)$ thus necessarily fall into the interval

$$
\left\langle B_{L}, B_{H}\right\rangle
$$

that we shall further call a referential interval of profits.

Two notes may be of value here:

- Later we will see that it is of no accident that some profits $B_{i}^{j}$ from $\left\langle B_{L}, B_{H}\right\rangle$, e.g. $B_{3}^{2}$ shown in Fig. 144 - are negative, represent losses of the respective strategies/firms.
- The interval $\left\langle B_{L}, B_{H}\right\rangle$ is, as will be shown shortly, a major tool for homogenization of the strategies' assumed heterogeneity.



### 41.2 Fifth Axiom of Rationality

### 41.2.1 Axioms of rationality

In this BOOK the concept of a rational decision maker has been established upon four axioms. It may then be of value to briefly recapitulate that John-the Designer will be rational if able:

1) to provide a well articulated specification of $J(1)$ by which he is designed and hence also of the variant forms of $J(2)$ in which his task may be prescribed,
2) to specify Firms that are relevant with respect to the support of fulfillments $J(2) \Rightarrow J(+3)$ and determine which of them are feasible,
3) to impose a preferential ordering upon the given list $F(0)=\left\{F_{1}, F_{2}, \ldots, F_{i}, \ldots\right.$, $\left.F_{N}\right\}$ of all $N$ feasible Firms, i.e. establish their ranking $\left(F_{r} \succsim F_{s}\right)$,
4) to select from $F(0)$ one and only one (unique) optimal firm $F^{*}$.

Now - in the line of a text-book wisdom - we have come to the point where the concept of uncertainty enforces a fundamental expansion of the list of rationality axioms. We will require that John - to be rational - must be able to associate to every profit $B_{i}^{j} \in\left\langle B_{L}, B_{H}\right\rangle$, one and only one (subjective) probability $u\left(B_{i}^{j}\right)$ so that

$$
B_{i}^{j} \sim\left[\left(1-u\left(B_{i}^{j}\right)\right), u\left(B_{i}^{j}\right) ; B_{L} B_{H}\right]
$$

The intuition behind this indifference is the following:
Let John's actual profit be guaranteed (for certain) on the level $B_{i}^{j}$ and let him be offered a lottery ticket with the two outcomes, a bad one $B_{L}$ and a good one $B_{H^{\prime}}$ where

$$
B_{L} \text { and } B_{H}
$$

are the lower and upper limits of the above established referential interval of profits.
Then, to be regarded as rational, John must be able to determine the lowest probability $u\left(B_{i}^{j}\right)$ at which he would regard the certainty of the profit $B_{i}^{j}$ as good as the lottery ticket. Put differently, John, if rational, will be able to determine the probability $u\left(B_{i}^{j}\right)$ of the winning $B_{H}$ such that at any probability higher than $u\left(B_{i}^{j}\right)$ he will accept the offer and exchange the guaranteed profit $u\left(B_{i}^{j}\right)$ for the lottery ticket.
If we return to the re-allocation of wealth thoroughly discussed throughout Comment 9 John - to be rational - has to be able to state at what probability of winning $B_{H}$ he will decide to withdraw the respective amount of his savings from a bank in order to buy the ticket and thus sacrifice the presumably guaranteed interest-rate-based profit.

### 41.2.2 Subjective probability of winning

The above equivalency, or indifference will be for notational convenience further put as

$$
B_{i}^{j} \sim S T R_{i}^{j}
$$

where

$$
\operatorname{STR}_{i}^{j} \equiv\left[\left(1-u\left(B_{i}^{j}\right)\right), u\left(B_{i}^{j}\right) ; B_{L}, B_{H}\right]
$$

is sometimes called an equivalent (standardized) strategy ${ }^{2}$.
Apparently, $S T R_{i}^{j}$ has a general structure of a strategy, however hypothetical the strategy is, in the sense that, in general

$$
S_{T R}^{j} \notin F(0)
$$

with the meaning that in reality no-one offers John this particular strategy how to make profit and that the hypothetical exchange of $B_{i}^{j}$ for $S T R_{i}^{j}$ is thus only a laboratory experiment organized purely with the aim to disclose John's psychological characteristics, namely a particular value for $u\left(B_{i}^{j}\right)$.

Accepting the Fifth Axiom as a meaningful characteristic of a rational agent, we will further express it in the form of an invertible, one-to-one function/mapping

$$
u:\left\langle B_{L} B_{H}\right\rangle \rightarrow\langle 0,1\rangle
$$

from the domain, let us recall, determined by the referential interval of profits and the range consisting in all conceivable probabilities from zero to one.

To illustrate, a graphical representation of one such invertible function/mapping is in Fig. 145.


[^1]
### 41.2.3 Subjective-objective probability

It is noteworthy that in addition to the "standard" "objective" $\pi\left(B_{i}^{j}\right) \equiv \pi_{i}^{j}$ a subjective probability $u\left(B_{i}^{j}\right) \equiv u_{i}^{j}$ has been established by the Fifth Axiom of John's rationality.

We will see shortly that the subjective probability can be then interpreted as a specific kind of an utility function and that that a product

$$
\pi\left(B_{i}^{j}\right) \cdot u\left(B_{i}^{j}\right) \equiv \pi_{i}^{j} \cdot u_{i}^{j}
$$

of the two kinds of probabilities is the base upon which the notion of an "expected utility".

However, somewhat confusing may be the "objectiveness" of $\pi\left(B_{i}^{j}\right) \equiv \pi_{i}^{j}$. Needless to stress that John in fact may only expect the probability with which this or that profit will be generated - depending on this or that state's occurrence.

## 41.3 "Psychology" of uncertainty

### 41.3.1 Properties of subjective probability

To the extent that whatever kind of profit is taken as John's "good", it is more than apparent that, as shown in Fig. 145, the function $u(B)$ must be strictly increasing: the higher is $B_{i}^{j}$, the higher probability $u\left(B_{i}^{j}\right)$ will be demanded by John, should he exchange the profit $B_{i}^{j}$ for the strategy $S T R_{i}^{j}$.

Less obvious is the curvature of $u(B)$. For dramatic effect, function $u(B)$ in Fig. 145 is depicted so that it is convex up to the point $B^{i n f}$ and concave after this inflection point. The standard, intuitively well acceptable text-book interpretation of the curvature is the following:

- a convex part of $u(B)$ represents, in marginal terms, John's "risk-loving" attitude to uncertainty,
- a concave part of $u(B)$ represents, in marginal terms, John's "risk-aversion".

In sum, the curvature represents John's psychological, purely subjective characteristic that we may call an uncertainty preference, in the obvious parallel to the already discussed concepts of time and personal preferences.

### 41.3.2 Economics vs. "psychology"

In what follows, the particular curvature/elasticity of the function $u(B)$ is often referred to as John's psychological portrait.

In general, John may exhibit whatever attitude to risk and, if he is of the kind shown in Fig. 145, he may even exhibit both as if contradicting characteristics -
depending on what part of the interval $\left\langle B_{L}, B_{H}\right\rangle$ is at stake. However, mainly for graphical convenience, we will further mostly assume that John is risk-averse and his $u(B)$, correspondingly, concave.

Summarizing the "psychological" aspects of our - as if purely economic discussions, we should return to Fig. 126 demonstrating the sensitivity of an agent to parameters $r$ and $\rho$, where:
interest rate $r$ can now be seen as a variable whose actual value is - so to speak - objectively determined by the market,
discount rate $\rho$ is a fully subjective (individual) characteristic of the decisionmaker concerned - his socio-cultural distance from other agents - in space or time.

As an individual psychological characteristic the rate $\rho$ may be assumed as stable over a relatively long period of time. Contrariwise, the rate $r$ may change even on daily basis thus bringing to fore the issue of uncertainty that raises the currently discussed aversion or love to risk.

### 41.4 Homogenization of heterogeneity

### 41.4.1 Fifth Axiom in action

The procedure shown in Fig. 146 rests in replacing a heterogeneous $i^{i t h}$ strategy $s t r_{i}$ by an homogenous ("comparable") $S T R_{i}$ that is - in the eyes of John equivalent to $s t r_{i}$ in the sense of the indifference "as good as"

$$
\operatorname{STR}_{i} \sim F_{i}
$$

The clue is the equivalence (indifference) derived from the Fifth Axiom of rationality

$$
B_{i}^{j} \sim S T R_{i}^{j}
$$



Fig. 146

Strictly formally, $\tilde{u}_{i}$ is a mean (expected) value of the variable $u(B)$, i.e. of the subjective probability. Given the context the subjective probability is in textbooks traditionally called a "utility" and hence, $\tilde{u}_{i}$ is the expected utility which is to be maximized by John. The reason is that, in the eyes of John - characterized by his specific $u(B)$ - the bigger is $\tilde{u}_{i}$ the better will be the $i^{\text {th }}$ strategy

$$
\left[F_{r} \gtrsim F_{s}\right] \Leftrightarrow\left(\tilde{u}_{r} \geq \tilde{u}_{s}\right), r, s=1,2, \ldots, N
$$

Hence, if we find formula for $\tilde{u}_{i}$ we will obtain the first criterion of John's evaluation of his strategies.

### 41.4.2 Simultaneous occurrence of events

We can easily prove that the looked-for formula is

$$
\tilde{u}_{i}=\sum_{\mathrm{j}=1}^{(m / i)} \pi_{i}^{j} \cdot u\left(B_{i}^{j}\right)
$$

To begin with we will interpret every $i^{\text {th }}$ element of the sum, i.e. the $j^{\text {th }}$ product.


### 41.4.3 Resultant criterion

However, in order to preserve the concept of profit maximization, our aim is to derive the criterion in the form of a somehow weighted average $\bar{B}_{i}$ of profits attainable within the $i^{\text {th }}$ strategy

$$
\left[F_{r} \gtrsim F_{s}\right] \Leftrightarrow\left(\bar{B}_{r} \geq \bar{B}_{s}\right), r, s=1,2, \ldots, N
$$

Such average is easy to find given that

$$
F_{i} \sim\left[\left(1-\tilde{u}_{i}\right), \tilde{u}_{i} ; B_{L}, B_{H}\right] \equiv S \widetilde{T} R_{i}
$$

Apparently, $S \widetilde{T} R_{i}$ has the structure of a standardized strategy and hence must be a standardized strategy as any other. Therefore, there must - by the Fifth Axiom of rationality - exist exactly one profit, let us denote it, for the moment as $\hat{B}_{i}$, such that

$$
\hat{B}_{i} \in\left\langle B_{L}, B_{H}\right\rangle \text { and at the same time } \hat{B}_{i} \sim S \widetilde{T} R_{i}
$$

Then, trivially, because $S \widetilde{T} R_{i} \sim F_{i}$, the profit $\hat{B}_{i}$ is the looked-for referential profit $\bar{B}_{i j}$ for which applies the required equivalence/indifference $F_{i} \sim \bar{B}_{i}$.

The graphical representation of the criterion will be discussed in the following chapter.

### 41.5 Multi-dimensionality reduced

Let us recall and summarize that:
$N \quad$ is the number of variant strategies/firms $F_{1}, F_{2}, \ldots, F_{i}, \ldots, F_{N}$,
$(n / i) \quad$ is the number of exogenous variables, i.e. the dimensions of the environment of the $i^{\text {th }}$ firm $F_{i}$,
$(m / i) \quad$ is the number of states of the environment of the $i^{\text {th }}$ firm $F_{i}$.
It is certainly one of the imminent tasks of any science to provide students with a meaningful reduction of the "space under study". It is in fact the genuine art of science to get over (abstract from) the real-world multidimensionality in the way that will not lose contact with the substance of the phenomena under study.

If the science succeeds in this challenge, then may come to fore the sophistications of contemporary IT-technology that will reasonably embrace the true, unavoidable complexity of social phenomena.

In the following chapters, firstly the number of states will be reduced to $(\mathrm{m} / \mathrm{i})=2$ and then we shall proceed towards the Designer's choice from only two variant strategies $-N=2$.

## 42. BINARY CHOICE " $I$ "

### 42.1 Introduction

As said, complexity of the real world is the major constraint to realistic application of science - be it theoretical physics or micro-economics. It is thus the art of abstraction that may give ways to notions and concepts through which some of the complexity can be overcome.

Social analysts and practitioners thus simply have to experiment with conversions of the realworld multidimensionality into as few dimensions as possible, where, moreover every such dimension will be allowed to obtain as few values as possible - preferably only two, such as "true or false", "yes or no", "good or bad", "black or white", "right or left", ...

In this sense, the problem under study will be now trivialized as follows:

- to begin with, the environment of John's whatever strategy will be allowed to obtain only two states of which one will be seen as bad (due to the lower profit provided) and the other good,
- later, the set of John's variant strategies will be assume to consist in only two variants of which one will be his actual strategy/Firm and the other will be interpreted as the only strategy/Firm that is offered to John as his prospective behavior/action.


### 42.2 Two-state strategy

### 42.2.1 Bad vs. good states

By restricting the analysis to only two states of the environment we in fact state that a real-world Designer is ready to divide the generally continuous set of states into two sub-sets each of which will be then meaningfully replaced with one representative state - good and bad, respectively.
In general, there are two extreme ways how to simplify the problem of numerous - $(m / i)$ - states of the firm's environment. Firstly, we may assume a continuum of such states and apply the infinitesimal calculus based on "well behaved" mathematical functions. The opposite extreme, adopted in what follows, assumes that it makes sense to aggregate the many states $\vec{e}_{i}^{1}, \vec{e}_{i j}^{2} \ldots, \vec{e}_{i j}^{j} \ldots$, $\vec{e}_{i}{ }^{(m / i)}$ into only two representatives differentiated as

$$
\text { bad and good state }-\vec{e}_{i}^{\mathrm{b}}, \vec{e}_{i}^{\mathrm{g}} \text {, respectively }
$$

Apparently, if the environment of $F_{i}$ may obtain only two states $\vec{e}_{i}^{\mathrm{b}}, \vec{e}_{i}^{\mathrm{g}}$, their respective probabilities $\pi_{i}^{\mathrm{b}} \equiv \pi\left(\vec{e}_{i}^{\mathrm{b}}\right)$ and $\pi_{i}^{\mathrm{b}} \equiv \pi\left(\vec{e}_{i}^{\mathrm{g}}\right)$ can be put as

$$
\pi_{i}^{\mathrm{b}}=\left(1-\pi_{i}^{g}\right) \text { and } \pi_{i}^{\mathrm{g}}
$$

on the proviso, let us recall, that the two aggregate states $\vec{e}_{i}^{\mathrm{b}}, \vec{e}_{i}^{\mathrm{b}}$ are exhaustive and mutually exclusive.
A two-state strategy $F_{i}=\left[\pi_{i}^{\mathrm{b}}, \pi_{i}^{g} ; B_{i}^{\mathrm{b}}, B_{i}^{\mathrm{g}}\right]$ is then in text-books often put as

$$
F_{i}=\left[\left(1-\pi_{i}^{\mathrm{g}}\right), \pi_{i}^{\mathrm{g}}, B_{i}^{\mathrm{b}}, B_{i}^{\mathrm{g}}\right] \text { or only as } F_{i}=\left[\pi_{i}^{\mathrm{g}}, B_{i}^{\mathrm{b}}, B_{i}^{\mathrm{g}}\right]
$$

where, to complete this introductory section, $B_{i}^{\mathbf{b}}=\psi_{i}\left(\vec{e}_{i}^{\mathrm{b}}\right)$ and $B_{i}^{\mathrm{g}}=\psi_{i}\left(\vec{e}_{i}^{\mathrm{b}}\right)$.

### 42.2.2 Example - two-state lottery

Let us return to the $N=3$ and hence John's choice from the three variant strategies $F_{1}, F_{2}$ and $F_{3}$ in Fig. 144 to which has been ascribed the referential interval of profits

$$
\left\langle B_{L}, B_{H}\right\rangle=\langle(-140),(+200)\rangle
$$

As can be read from Fig. 144 it is the second strategy/firm $F_{2}$ whose environment may obtain only two states. To be as concrete as possible, let the technology and the respective revenue function of this two-state $F_{2}$ be exemplified by the following lottery ticket:

- its price - representing John's investment/costs $T C_{2}$ - is certain, given as 220 USD,
- its revenue $T R_{2}$ is uncertain, may be bad or good, for concreteness: $T R_{2}^{b}=130 U S D$ and 400 USD, respectively.
In sum, $F_{2}$ as a two-state strategy $\left[\left(1-\pi_{2}^{g}\right), \pi_{2}^{g}, B_{2}^{b}, B_{2}^{g}\right]$ takes the form:

$$
F_{2}=\left[\left(1-\pi_{2}^{g}\right), \pi_{2}^{g} ;(-90),(+180)\right]
$$

given that the two - bad and good - profits are:
$B_{2}^{\mathrm{b}}=(130-220)=(-90)$ USD,
$B_{2}^{\mathrm{g}}=(400-220)=(+180)$ USD.

### 42.2.3 Expected (mean value of) profit

If John is risk neutral, he will, to evaluate each strategy/firm by the "objective" criterion $\ddot{B}_{i}=\tilde{B}_{i}$

$$
\tilde{B}_{i}=\sum_{j=1}^{(m / i)} \pi_{i}^{j} \cdot B_{i}^{j}
$$

In the specific case of the two-state $F_{2}$ the criterion will become a two-component algebraic sum

$$
\tilde{B}_{2}=\left[\left(1-\pi_{2}^{\mathrm{g}}\right) \cdot B_{2}^{\mathrm{b}}+\pi_{2}^{\mathrm{g}} \cdot B_{2}^{\mathrm{g}}\right]=\left[\left(B_{2}^{\mathrm{g}}-B_{2}^{\mathrm{b}}\right) \cdot \pi_{2}^{\mathrm{g}}+B_{2}^{\mathrm{b}}\right] \equiv \tilde{b}\left(\pi_{2}^{\mathrm{g}}\right)
$$

If the two profits (bad and good) are given, the value of the criterion $\widetilde{B}_{i}$ depends only on the their probability distribution or simply on $\pi_{2}^{g}$; as demonstrates in Fig. 147 the linear function

$$
\widetilde{B}_{2}=\left[270 \cdot \pi_{2}^{g}+(-90)\right]
$$



Two examples are depicted in Fig. 147 may be illustrative:

- if $\pi_{2}^{g}=\frac{2}{3}$, the value of the looked-for criterion will be $\widetilde{B}_{2}=(+90)$,
- if $\pi g_{2}=\frac{1}{3}$ the expected (mean value of) profit will be $\widetilde{B}_{2}=(0)$.


### 42.2.4 Subjective probability

Given the two (bad and good) profits, the the above example can be completed by adding data about:

- a concrete value of the objective probability $\pi_{2}^{g}$ of winning (+180) USD,
- a concrete form of $u(B)$.

To begin with the latter, we will assume, in the spirit of most text-books, that John is riskaverse and hence the function $u(B)$ in Fig. 148 is concave.


### 42.2.5 Expected utility

Apparently, the following Fig. 149 is a mere two-state adaptation of the general analysis corroborated in Fig. 146.


Fig. 149

To obtain $\tilde{u}_{2}$, we - again - adapt the general formula

$$
\tilde{u}_{i}=\sum_{j=1}^{(m / i)} \pi_{i}^{j} \cdot u\left(B_{i}^{j}\right)
$$

In the specific case of the two-state $F_{2}$ the criterion will become a two-component algebraic sum

$$
\left.\tilde{u}_{2}=\left[\left(1-\pi_{2}^{\mathrm{g}}\right) \cdot u\left(B_{2}^{\mathrm{b}}\right)+\pi_{2}^{\mathrm{g}} \cdot u^{\left(B_{2}^{\mathrm{g}}\right)}\right)\right]=\left[\left(u_{2}^{\mathrm{g}}-u_{2}^{\mathrm{b}}\right) \cdot \pi_{2}^{\mathrm{g}}+u_{2}^{\mathrm{b}}\right] \equiv \tilde{u}_{2}\left(\pi_{2}^{\mathrm{g}}\right)
$$

or - again - a linear function of the "objective" probability $\pi_{2}^{g}$ - like in the case of $\tilde{b}_{2}\left(\pi_{2}^{g}\right)$ from Fig. 147. The following green line $\tilde{u}_{2}\left(\pi_{2}^{g}\right)$ in Fig. 150 thus fully corresponds to the line $\tilde{b}_{2}\left(\pi_{2}^{g}\right)$ in Fig. 147.


Fig. 150

### 42.2.6 Referential profit

In the following Fig. 151 we assume that John is as risk averse as in Fig. 150 and that the good probability is $\pi_{2}^{g}=\frac{1}{4}$.

As a result, the value of the looked-for criterion $\tilde{u}_{2}$ can thus be simply transferred from Fig. 150 to Fig. 151. Given the value $\tilde{u}_{2}$, we can establish the standard (equivalent) strategy

$$
\operatorname{STR}_{2}=\left(\left(1-\tilde{u}_{2}\right), \tilde{u}_{2}, B_{L}, B_{H}\right)
$$

that is as good as $F_{2}$.
Given $S T R_{2}$ is a standard strategy as any other, the "inverse" of the "invertible" Fifth Axiom will state that there must exist one and only one profit $\hat{B}_{2}$ such that $\widehat{B}_{2} \sim S \widetilde{T} R_{i}$. Because, as said, $S \widetilde{T} R_{2} \sim F_{2}$, the profit $\hat{B}_{2}$ must be our looked-for criterion, i.e., what we called a referential profit

$$
\bar{B}_{2}=\hat{B}_{2}
$$



Fig. 151

As already said, the dotted green line is the inverse to $\widetilde{b}\left(\pi_{2}^{g}\right)$ from Fig. 147. Hence, $\widetilde{B}_{2}$ is the expected (mean value of) the pair $B_{2}^{\mathrm{b}}$ and $B_{2}^{\mathrm{g}}$

$$
\widetilde{B}_{2}=\left[\left(1-\pi_{2}^{g}\right) \cdot B_{2}^{b}+\pi_{2}^{g} \cdot B_{2}^{g}\right]
$$

given that the objective probability $\pi_{2}^{g}=\frac{1}{4}$. As can be read from Fig. 147 the relationships between the two kinds criteria ("objective" $\widetilde{B}_{2}$ and "subjective" $\bar{B}_{2}$ ) is

$$
\tilde{B}_{2}>\bar{B}_{2}
$$

clearly due to the concavity of $u(B)$. This inequality will be interpreted later, e.g. with respect to the so-called fairness of lotteries.

### 42.2.7 The third criterion

Purely formally, $\tilde{\bar{B}}_{2}$ in Fig. 152 is the hypothetical expected (mean value of) the pair $B_{L}$ and $B_{H}$

$$
\tilde{\bar{B}}_{2}=\left[\left(1-\tilde{u}_{2}\right) \cdot B_{L}+\tilde{u}_{2} \cdot B_{H}\right]
$$

on the proviso that the objective probability of $B_{H}$ is $\tilde{u}_{2}$.


It can be relatively easily shown this hypothetical mean value $\tilde{\bar{B}}_{2}$ is a criterion as good as $\tilde{u}_{2}$ and $\bar{B}_{2}$. Put differently, the three criteria are equivalent with respect to the looked-for optimal strategy.
To conclude, let us pin-point the fact that the example has been chosen so that the referential profit be negative. Hence, the risk aversion of John is such that he would exchange his guaranteed loss $\bar{B}_{2}$ for a lottery offering $B_{L}$ and $B_{H}$ only if the probability of $B_{H}$ were no smaller than $\tilde{u}_{2}$ and - consequently - the mean value of $B_{L}$ and $B_{H}$ be no smaller that the highly positive $\tilde{\bar{B}}_{2}$.

### 42.3 Two-strategy (yes-or-no) choice

### 42.3.1 Notation

Having simplified the problem in the preceding chapter by assuming $(m / i)=2$, we will now proceed further to $N=2$. In words, John - in order to resolve $M A X_{J}$ - will succeed in aggregating his $N$ real-world heterogeneous strategies/firms $F_{1}, F_{2}, \ldots, F_{i}, \ldots, F_{N}$ into only two - denoted further as $F_{a c}$ and $F_{o f}$ where:
$F_{a c}$ is John's actual strategy, i.e. the strategy that John has applied so far, namely for the sake of the preceding production cycle,
$F_{o f} \quad$ is the (only) strategy that is offered to John as a feasible option how to change his actual "technology of resolving problems" - how to change/ /redesign his responses to the "developments of the outside world".
$M A X_{J}$ thus obtains the following form
$\max U^{J}\left(F^{J}\right)$
s.t.: $F^{J} \in\left[F^{J}(0)=\left\{F_{o f}, F_{a c}\right\}\right]$
$M A X ~_{J}$

Put differently, as already stressed, John's choice is of the binary (yes-or-no) nature when:
if $F^{*}=F_{\text {of }} \quad$ John decides yes - in the affirmative for a change/redesign of his strategy/firm,
if $F^{*}=F_{a c} \quad$ John decides no so as to continue with his existing (actual) strategy/firm.

To illustrate we will further use the example when $J(1)$ represents John's task to pay alimony and his two variant strategies $F_{o f}$ or $F_{a c}$ how to fulfill the task can will take the form of Mary's shoe-making firm ("M-shoes") and $F_{a c}$ is Mary's grocery ("M-grocery").

### 42.3.2 Two-state strategies

As expected and already signaled, we will now assume that not only $N=2$ but also both variant strategies will be of a "binary" nature:

$$
\begin{aligned}
& (m / a c)=2 \\
& (m / o f)=2
\end{aligned}
$$

In short, the environment of each of the two variant strategies/firms $F_{a c}$ and $F_{o f}$ can obtain only two states In sum, for John's:
offered strategy $F_{o f}$ the only two states of Mary-the Shoe-maker are:

- bad state $\vec{e}_{o f}^{\mathrm{b}}$,
- good state $\vec{e}_{\text {of }}^{g}$.
actual strategy $F_{\text {ac/J }}$ the only two states of Mary-the Grocer are:
- bad state $\vec{e}_{a c}^{\mathrm{b}}$,
- good state $\vec{e}_{a c}^{g}$.


### 42.3.3 Representative states

As said many times already, to evaluate each of his two strategies/firms, Johnthe Designer should know which of the two conceivable states will occur.

In text-books' models all that he is assumed to known are the respective probability distributions of the states:
of $F_{o f} \pi\left(\vec{e}_{o f}^{b}\right), \pi\left(\vec{e}_{o f}^{g}\right)$ or, for short, $\pi_{o f}^{b} \pi_{o f}^{g}$,
of $F_{a c} \pi\left(\vec{e}_{a c}^{b}\right), \pi\left(\vec{e}_{a c}^{g}\right)$ or, for short, $\pi_{a c}^{b} \pi_{a c}^{g}$.
where, as explained

$$
\pi_{a c}^{b}=\left(1-\pi_{a c}^{g}\right) \text { and } \pi_{o f}^{b}=\left(1-\pi_{o f}^{g}\right)
$$

In sum, John is taken as choosing between:

$$
\begin{gathered}
F_{a c}=\left[\left(1-\pi_{a c}^{g}\right), \pi_{a c}^{g} ; B_{a c}^{b} B_{a c}^{g}\right] \\
F_{o f}=\left[\left(1-\pi_{o f}^{g}\right), \pi_{o f}^{g} ; B_{o f,}^{b}, B_{o f}^{g}\right]
\end{gathered}
$$

In Fig. 153, for the sake of concreteness again, we assume that

$$
B_{a c}^{b}<B_{a c}^{b} \text { and } B_{o f}^{g}>B_{a c}^{g}
$$

and hence, by definition of the referential interval of profits, its lower and upper bounds are

$$
B_{L}=B_{a c}^{b} \text { and } B_{H}=B_{o f}^{g}
$$



### 42.3.4 Action vs. non-action

By definition John will decide to act if

$$
\left(F_{o f} \gtrsim F_{a c}\right) \Leftrightarrow\left(\tilde{u}_{o f} \geq \tilde{u}_{a c}\right)
$$

The rule is in detail corroborated in Fig. 154.


Fig. 154

### 42.4 Associated notes

### 42.4.1 Pseudo-design of a Designer

As explained, John resolves $M A X_{J}$ with the aim to select a strategy that would best support fulfillment of his own task $J(1)$ designed by his own Designer, say Lena.

Having adopted the concept of an expected utility, $M A X_{J}$ can be re-written as the IF-THEN rule in Fig. 154 or as the rule

IF: A) John's psychological portrait is given as $u(B)$,
B) John's "objective" probabilities are determined by $\pi_{o f}^{g}$ and $\pi_{a c r}^{g}$
C) John's variant profits are $\left(B_{1}^{b}, B_{1}^{g}\right),\left(B_{2}^{b}, B_{2}^{g}\right)$.

THEN: John's optimal strategy will be $\operatorname{str}{ }_{J}^{*} \equiv F^{*}$, i.e.:

$$
F_{o f} \text { or } F_{a c}
$$

The rules can be read so that John in fact has a task to select as his optimal strategy a pre-determined - as if designed - firm $F^{*}$ which contradicts our thesis that his task is only to pay alimony and for that matter he is "free to choose" whatever strategy.
The confusion is a consequence of our - not John's - choice to impose upon him the text-book concept of an expected utility. This pseudo-design of his task is of the same kind as the rule

| IF: | $\max 1$, |
| :--- | :--- |
| THEN: | $\left(K^{*}, L^{*}, Q^{*}\right)=\boldsymbol{d s}\left(p_{K}, p_{L} p_{Q^{\prime}} a_{f}\right) \equiv \boldsymbol{d s}(\vec{p})$. |
| IT $2 \boldsymbol{a}$ a |  |

discussed in the very initial sections of PART I. Put differently the external assumption about an agent's interests and preference pre-determine his-her behavior/action.

Summarizing, then, our - not John's - adoption of the concept of expected utility is of the same kind as the text-book assumption that the profit maximization Firm is in fact pseudo-designed by a Designer called an Invisible Hand of the Market.

Understandably, this rather philosophical discussion, unfortunately, will not find room in this BOOK.

### 42.4.2 Facts finder

The above interpretation of John-the Designer as a pseudo-Designee leads to the question who and namely how will find the respective data and substitute them into the formulas in Fig. 154.
Apparently, already the "objective" probabilities are not easy to find. However, the true mystery is associated to the curvatures (elasticity) of the subjective probabilities $u(B)$.
We should then warn again that our agents will be often taken as risk averse and that this particular psychological profile is assumed for only a graphical convenience. Their genuine attitude to risk will serve in this BOOK as yet another illustration of the real-world limits to our understanding of social choice and behavior.

## 43. ONE STRATEGY CERTAIN

### 43.1 Introduction

### 43.1.1 Notation

The two strategies under John's choice will now be established as follows:

1) One of them will be $\left.\left[\left(1-\pi^{+}\right), \pi^{+} ; \infty, B^{+}\right)\right]$, which unorthodox notation will be understood as a strategy/firm which will provide:

- the good profit $B^{+}$with the probability $\pi^{+}=1$,
- the bad profit ," $\infty^{\prime \prime}$, where the sign for infinity can be read as anything due to its probability $\left(1-\pi^{+}\right)=0$.

2) The other strategy/firm will remain its uncertain nature. Given the circumstances, we can omit the unnecessary sub-scripts and put simply as

$$
\left[\left(1-\pi^{\mathrm{g}}\right), \pi^{\mathrm{g}} ; B^{b}, B^{g}\right]
$$

where, by definition of the referential interval of profits, its lower and upper bounds are

$$
B_{L}=B^{b} \text { and } B_{H}=B^{g}
$$

### 43.1.2 Associated notes

1) As any outcome of any strategy, also $B^{+}$may be positive or negative. To illustrate, the case $B^{+}=0$ represents non-profit strategy such as "doing nothing", e.g. "watching TV only".
2) In what follows we shall differentiate according to which of the two variant strategies is actual or offered.

### 43.2 Action towards uncertainty

### 43.2.1 Introduction

Let as shown in Fig. 155:
$F_{a c}=\left(0,1 ; \infty, B_{a c}^{+}\right)$
$F_{o f}=\left[\left(1-\pi^{\mathrm{H}}\right), \pi^{\mathrm{H}} ; B_{L / o f}, B_{H / o f}\right]$
To illustrate, let John's choice be between:
$F_{a c} \quad$ that will, for simplicity, guarantee $B_{a c}^{+}=0$ and
$F_{o f} \quad$ that will offer to John the above discussed lottery ticket $\left[\left(1-\pi^{\mathrm{H}}\right), \pi^{\mathrm{H}}\right.$ (-90), (+180)].

In plain terms, John can either "watch TV" with the fully guaranteed zero profit or pay 220 usd for the lottery ticket and thus buy two uncertain, mutually exclusive revenues 130 usd and 400 usd.


As already noted and now shown in Fig. 155 the lower and upper bounds of the referential interval of profits are

$$
B_{L / o f}=B_{o f}^{b} \text { and } B_{H / o f}=B_{o f}^{g}
$$

and hence represent the unique case when the standard (equivalent) strategy $S T R$ is - as to its profits - a real-world strategy.

### 43.2.2 The criterion of action

A simple application of the procedure in Fig. 154 will bring up the following values of the respective criterions $\tilde{u}_{a c}$ and $\tilde{u}_{o f}$ :



The condition

$$
\left(F_{o f} \gtrsim F_{a c}\right) \Leftrightarrow\left(\tilde{u}_{o f} \geq \tilde{u}_{a c}\right)
$$

of John's behavioral change thus takes the form

$$
\pi^{H} \geq u\left(B_{a c}^{+}\right)
$$

In words, should John "move from the TV screen" the objective probability of the offered uncertain winning must be bigger that the subjective probability of his certainty of a zero profit.

### 43.2.3 Variety of arrangements

The above rule can be also put as follows:

- if John's actual - fully guaranteed - profit is ex ante given as $B_{a c}^{+}$he will change his behavior, at any level of $\pi^{H}$ for which, $\pi^{H} \geq \pi^{H /+}$, where $\pi^{H /+}=$ $u\left(B_{a c}^{+}\right)$,
- if the probability $\pi^{H}$ of the lottery is ex ante given as $\pi^{H}=\pi^{H /+}$, John will change his behavior, only if his guaranteed profit $B_{a c}^{+}=\bar{B}$ where $\bar{B}$ is the so-called referential profit.

From many other interesting questions, we could assume that what is ex ante given are the values of $\pi^{H}$ and $B$ and the owner of the lottery will ask at what profits $B_{L}$ and $B_{H}$ would John buy the ticket. Hence, if the uncertain revenues of the lottery are ex ante given, the looked for variable is the price of the ticket.

### 43.2.4 Fairness

In the social context under study here we may generalize the text-book convention about fairness by the equality

$$
\tilde{B}_{o f}=B_{a c}^{+}
$$

where by

$$
\tilde{B}_{o f}=\left[\left(1-\pi^{H}\right) \cdot B_{L / o f}+\pi^{H} \cdot B_{H / o f}\right]
$$

is denoted the "objective" expected (mean) value of the two profits offered by the above discussed lottery.
In Fig. 155 the fair probability of the offered lottery is marked as $\pi^{\mathrm{H} / \text { fair }}$. As explained, the risk-averse John will buy the respective ticket only if the probability is at least $\pi^{H /+}$ which is substantively bigger than $\pi^{H / f a i r}$. All that, even though the existing $B^{+}$assumed in Fig. 155 is visibly negative.

If $B^{+}=0$, the above condition requires that

$$
\left[\left(1-\pi^{H}\right) \cdot B_{L / o f}+\pi^{H} \cdot B_{H / o f}\right]=0
$$

and hence

$$
\pi^{H / f a i r}=-\frac{B_{L / o f}}{\left(B_{H / o f}-B_{L / o f}\right)}
$$

The intuition behind the term fairness then rests in that $B_{a c}^{+}=0$ represents John doing nothing and keeping his money in his pocket.

### 43.2.5 Digression: pseudo-normativity of economics

Recall, that the concept of social justice (fairness) was already applied in association with the asymmetric constraint imposed by John on transfers of wealth in favor of a poorer future agent (cf. Fig. 123).
Unlike "fairness" used as a characteristic of a fully "subjective" choice of a concrete decision-maker, the above notion of a "fair lottery" may confuse students if taken - incorrectly indeed - as an "objective" or even normative imperative of a specific kind of business.

### 43.3 Action towards certainty

The arrangement when the risk-averse John is deciding between:
$F_{a c}=\left[\left(1-\pi^{\mathrm{H}}\right), \pi^{\mathrm{H}} ; B_{L / a c}, B_{H / a c}\right]$
$F_{o f}=\left(0,1 ; \infty, B_{o f}^{+}\right)$
will trivially lead to the condition

$$
\pi^{H} \leq u\left(B_{a c}^{+}\right)
$$

which is, obviously, the exact opposite to the condition derived in the preceding analysis for the transfer from certainty.

### 43.3.1 Example 1: Renting of an uncertain firm

Let John's choice be between the following two two-state strategies $F_{a c}$ and $F_{o f}$ represented graphically in Fig. 156 and analytically as:
$F_{a c}=\left[\left(1-\pi^{\mathrm{H}}\right), \pi^{\mathrm{H}} ; B_{L / a c}, B_{H / a c}\right] \quad$ and $B_{L / a c}, B_{H / a c}$ are two uncertain profits to be generated by his firm in the coming year and
$F_{o f}=\left(0,1 ; \infty, B_{o f}^{+}\right) \quad$ is a magnitude of a rent to be paid by a prospective, perfectly reliable tenant.

All questions we may ask in this arrangement are fully analogous to our previous discussion:

- If the rent is ex ante given (e.g. by law) what probability $\pi^{H}$ will make the profits $B_{L / a c} B_{H / a c}$ acceptable so as John will not change his strategy?
- If the probability $\pi^{H}$ is ex ante given as $\pi^{H}=\pi^{H /+}$, what rents will be acceptable for him to change the existing strategy?

In Fig. 156 is demonstrated that if $\pi^{H /+}$ is ex ante given then the rent $\tilde{B}$ will be fair in the above explained sense.


### 43.3.2 Example 2: Insurance of an uncertain firm

If John decides to fully insure his firm - at the level $P R$ of the respective premium - his profit will be always

$$
B^{+}=\left[B_{H / a c}-\text { premium }\right]
$$

regardless of which of the profits $B_{L}$ and $B_{H}$ will factually occur.
The proof of the equality is as trivial as follows:
if $B_{\text {L/ac }} \quad$ occurs as the firm's bad outcome, John's profit will be:
$\left[B_{L / a c}+\right.$ full compensation - premium $]=\left[B_{L / a c}+\left(B_{H / a c}-B_{L / a c}\right)-P R\right]=$ [ $B_{H / a c}$ - premium $]$,
if $B_{H / a c} \quad$ occurs as the firm's good outcome John's profit will be:
[ $B_{H / a c}$ - premium].
We show in Fig. 157 that if the probability of a good outcome is ex ante given as $\pi^{H /+}$ then the fair magnitude of premium is

$$
\left(B_{H / a c}-\tilde{B}\right)
$$



Fig. 157

## COMMENT 11.

## Designer's choices; the case of a game

Apart from other already mentioned criteria the topics concerned so far can be also differentiated according to the nature of social inter-actions under study, namely those:
among co-Designers who as voters seek to collectively design their collective strategy; e.g. who, as contracting parties in a negotiation jointly decide whether and in what form a contractual system/process will be designed,
among Designees
among Nominees within a given system/process or between Designees and some external agents, namely between a Beneficiary and Defendant who as voters seek to collectively determine whether and in what form a Defendant's task will be prescribed.

Hence, what remains to complete this taxonomy is an inter-action between Designers of two or more separate systems/processes. Apart from this admittedly limited objective we will also briefly sketch how this social setting relates to the text-book concept of a mixed-strategy dealt with within a so-called game theory.

## 44. ASSUMPTIONS, SIMPLIFICATIONS AND NOTATIONS

### 44.1 A choice from a set of profit-making Firms

The social arrangement under study will be summarized as follows:

1) The number of inter-acting Designers will be reduced to only two, namely

John-the Designer and Nina-the Designer
2) The production cycles of John and Nina will be "synchronized", namely so that the two Designers will make their choices simultaneously - at the same time.
3) Both Designers under study will be of the kind established in the first chapters of the preceding Comment 10. Hence, John will select his optimal strategy $\operatorname{str}_{j}^{*} \equiv F_{j}^{*}$ from a set of one-unit profit-making Firms, i.e. by solving:

## $\max U^{J}\left(F^{J}\right)$

s.t.: $F_{J} \in\left\{F_{J}(0)=F_{1 / J}, F_{2 / J}, \ldots, F_{n / J}\right\} \quad M A X_{J}$
and, mutatis mutandis, Nina will select her optimal strategy $\operatorname{str}_{N}^{*} \equiv F_{N}^{*}$ by solving:

## $\max U^{N}\left(F_{N}\right)$

MAX $_{N}$
s.t.: $F_{N} \in\left\{F_{N}(0)=F_{1 / N}, F_{2 / N}, \ldots, F_{n / N}\right\}$
4) Choices of John and Nina will be both of a binary nature in the sense of:

- Fig. 153 for a two-strategy choice,
- Fig. 148 for a two-state choice.


### 44.2 Binary choice "II"

### 44.2.1 Two-strategy (yes-or-no) choice

A Designer's behavior/action has been established with respect to his choice whether he-she will keep to the existing strategy or change/re-design it.

Hence, the two-strategy arrangement can be summarized as a choice from a pair of strategies $F_{a c}$ and $F_{o f}$ where:

- $F_{a c}$ is the actual strategy or the strategy applied in the preceding production cycle,
- $F_{o f}$ is the offered strategy, i.e. the strategy that may replace $F_{a c}$ for the sake of a coming production cycle.

In sum, John and Nina will be assumed to select their optimal strategies $F_{J}^{*}$ and $F_{N}^{*}$ by solving the following problems:


### 44.2.2 Two-state and homogeneous strategies

With the aim to approach the setting of a game, we will further assume that both $M A X_{J}$ and $M A X_{N}$ consist in strategies $F_{o f}$ and $F_{a c}$ that are homogeneous and, in particular, whose common environment is constituted by choices of the respective counter-Designer.

Then, the second characteristic of a binary choice is that the environment may obtain only two states - usually classified as bad and good.

In the aggregate we thus assume that the variant profits generated by $F_{o f}$ and $F_{a c}$ are affected by the yes-or-no choice of the respective counter-Player and only this choice.

For illustration, if Nina selects $F_{\text {of/N }}$ her actual profit will depend on only whether John will select $F_{o f / J}$ or $F_{a c / J}$.

### 44.2.3 Certainty vs. uncertainty

For the sake of the following analysis we will assume that every Designer:

- is certain about the variant strategies from which the other Designer selects his strategy; put equivalently, every Designer is certain about the only two states that may affect the outcomes of his-her only two variant strategies,
- is uncertain about the other Designer's choice - about whether the other Designer will re-design his-her strategy or keep to it.

Due to the above uncertainty, John and Nina will not be in the position of a Leader and Follower. They will be mostly referred to as Players, counter--Players in particular.

### 44.2.4 Expected utility

As before, namely in Fig. 154 we will impose upon both Designers the text-book concept of an expected utility based upon:

- a subjective probability $u(B)$ by which a psychological profile of a Player is represented,
- "objective" probabilities $\pi_{o f}^{g}$ and $\pi_{a c,}^{g}$ by which a Player represent his-her "expectations" about the counter-Player choice.

We should therefore also remind the reader about our note on the pseudo-design of both Designers.

### 44.3 Example

The following concretization of the problem may somewhat ease the burden of complexity.

### 44.3.1 Persons and roles

Unfortunately, for the reader, we will now have to somewhat expand the standard text-book analysis. To make the added complexity relatively manageable, to John-the and Nina-the Designer will be added only two other persons in their roles of Designees:

- Mary will personify, for simplicity, both John's variant strategies $F_{a c / J}$ and $F_{\text {ofjJ }}$ and similarly,
- Victor, mutatis mutandis, will personify, for simplicity, both Nina's variant strategies $F_{a c / \mathrm{N}}$ and $F_{o f / N}$.

Mary and Victor can be seen as Owners of the respective profit-making Firms that may support fulfillment of John's task $J(1)$ and Nina's task $N(1)$, respectively.

### 44.3.2 Actual vs. offered strategies

During the preceding production cycle, we will assume that:
John's actual strategy $F_{a c / J}$ had the form of Mary-the Grocer ("M-grocery", for short). Put differently, John financed his budgetary needs through Mary's grocery shop.

Nina's actual strategy $F_{a c / N}$ had the form of Victor-the Grocer ("V-grocery", for short). Put differently, Nina financed her budgetary needs through Victor's grocery shop.

At the beginning of the coming production cycle concerned:

- John considers whether he should "convert" Mary-the Grocer $F_{a c / J}$ into Mary-the Shoe-maker $F_{\text {of } J}$ ("M-shoes", for short) and
- Nina, analogously, thinks about "converting" Victor-the Grocer $F_{a c / N}$ into Victor-the Shoe-maker $F_{\text {off } N}$ ("V-shoes").


### 44.3.3 Competitiveness/rivalry

For the sake of the analysis we shall take it as intuitively well acceptable that if the Players choose to enter the same industry the respective price $p_{Q}$ of the output will decrease.

Apparently, then, with the decreasing $p_{Q^{\prime}}$, also the Players' respective profits will decrease.

### 44.3.4 Other persons and roles

Invoking our concept of an operational unit, every single strategy under study

$$
F_{o f j J}, F_{a c / J}, F_{o f / N}, F_{a c / N}
$$

includes, by definition, not only the name of a Designee, but also the names of a Beneficiary, Defendant and Manager - leaving aside the names of respective Executors.

Apparently, the example will have to involve one more dose of drastic simplifications namely that John and Nina will perform also the roles of the respective Nominees.

## 45. PAY-OFF MATRIX

Based on the above assumptions, John's and Nina's choices can be graphically summarized in Fig. 159 and Fig. 160. Technically speaking, the two pictures only replicate the analysis presented in Fig. 153.

The difference between Fig. 159 and Fig. 160 is almost unnoticeable as it rests in only colors - brown or blue - of the four curved arrows, where:
in Fig. 159 the brown curved arrows indicate the profits on the proviso that the respective Designer decides not to act - to keep to hisher actual strategy,
in Fig. 160 the blue curved arrows represent the opposite case - the profits when the Designer decides to act - to re-design his-her actual strategy.

In the center of each picture are depicted - in the format of a so-called pay-off matrix - four pairs of profits that may be generated depending on John's and Nina's mutually independent choices. Above and below the matrix are depicted John's and Nina's „psychological portraits" expressed by $u_{J}(B)$ and $u_{N}(B)$. Both functions are assumed to be concave mainly for only a graphical convenience. In reality, neither of the two Players need be risk averse as the concavity (elasticity) suggests. In general, any Player can be also risk loving, or as explained in Fig. 145 of a mixed "orientation".
The nature of the bad and good outcomes represents the assumed competitiveness/ /rivalry between the two Designers. Put differently, the game under study is of a non-cooperative nature.



## 46. A SET OF OPTIMAL STRATEGIES

### 46.1 Analytical symmetry of counter-Players

As already seen, we can proceed as follows:

1) We can confine to only one of the two Players as their choices are - analytically speaking - fully symmetric. Hence, for concreteness, we shall focus on John and his two variant strategies $F_{o f f J}$ and $F_{a c / J \text {. }}$. Nina's choice from her variant strategies $F_{o f / N}$ and $F_{a c / N}$ will be then supplemented mutatis mutandis.
2) As shown in Fig. 154, it will be also enough to focus on the condition under which John will decide to act (to re-design his actual strategy) and then only add the complementary reverse condition for the opposite outcome - his non-action.

Then, based on the text-book concept of an expected utility the condition under which John will decide to "actively" re-design his actual strategy $F_{a c / J}$ can be put as


To begin with, the respective "objective" probabilities will be dealt with.

### 46.2 Probability distribution

### 46.2.1 Bad vs. good state

The common environment of John's variant strategies $F_{a c / J}$ and $F_{o f / J}$ is constituted by Nina's choices and may obtain only two states. Put in more detail:

1) John's actual $F_{\text {ac/J }}$ (M-grocery) has the environment, where:

- its bad state $\vec{e}_{a c / J}^{\mathrm{b}}=F_{a c / N}$ is brought up by Nina's choice not to act and keep to her actual strategy,
- its good state $\vec{e}_{a c / J}^{\mathrm{g}}=F_{\text {of/N }}$ is brought up by Nina's choice to act and re-design her actual strategy,

2) John's offered $F_{\text {ofJ }}$ (M-shoes) has the environment, where:

- its bad state $\vec{e}_{\text {of/J }}^{b}=F_{\text {of/N }}$ is brought up by Nina's choice to act and re-design her actual strategy,
- good state $\vec{e}_{o f / J}{ }^{g}=F_{a c / N}$ is brought up by Nina's choice not to act and keep to her actual strategy.

Summarizing, then:
Nina's choice $F_{a c / \mathrm{N}}=\vec{e}_{a c / J}^{\mathrm{b}}=\vec{e}_{o f / J}{ }_{\mathrm{g}}$,
Nina's choice $F_{o f f N}=\vec{e}_{a c / J}^{\mathrm{b}}=\vec{e}_{o f f J}^{\mathrm{b}}$.

### 46.2.2 "Objectiveness" of probabilities

A text-book concept of uncertainty rests in assuming the following level of information:
$\pi_{o f}^{\mathrm{J} / \mathrm{N}}$
$\pi_{a c}^{J / N}=\left(1-\pi_{o f}^{J / N}\right)$
is John's idea about the probability with which Nina will decide to act and re-design her actual $F_{a c / N}$ into $F_{\text {offN }}$ (V-shoes),
is John's idea about the probability with which Nina will decide not to act (refrain from acting) and keep to $F_{a c / N}$ (V-grocer).

Summarizing, then:

$$
\begin{aligned}
& \pi_{o f}^{J / N}=\left[\pi\left(\vec{e}_{o f / J}^{\mathrm{b}}\right) \equiv \pi_{o f / J}^{\mathrm{b}}\right]=\left[\pi\left(\vec{e}_{a c / J}^{\mathrm{g}}\right) \equiv \pi_{a c / J}^{\mathrm{g}}\right] \\
& \pi_{a c}^{J / N}=\left[\pi\left(\vec{e}_{a c / J}^{\mathrm{b}}\right) \equiv \pi_{a c / J}^{\mathrm{b}}\right]=\left[\pi\left(\vec{e}_{o f / J}^{\mathrm{g}}\right) \equiv \pi_{o f / J}^{\mathrm{g}}\right]
\end{aligned}
$$

As already noted, somewhat confusing may be the textbook terminology according to which probabilities $\pi_{o f}^{J / N}$ and $\pi_{a c}^{J / N}$ are traditionally characterized as "objective". Needless to stress that in reality each Player may only expect the probability with which a counter-Player will make this or that "move".
Unfortunately, the term subjective probability is most often used for only the function $u\left(B_{i}^{j}\right)$ established above by the Fifth Axiom of rationality.

### 46.2.3 The counter-Player

The above analysis about John's strategies can now be mutatis mutandis applied upon Nina and her $F_{a c / N}$ and $F_{o f / N}$, as follows:
$\pi_{o f}^{N / J} \quad$ is Nina's idea about the probability with which John will re-design $F_{a c / J}$ into $F_{o f / J}$, i.e. (M-shoes),
$\pi_{a c}^{N / J}=\left(1-\pi_{o f}^{N / J}\right)$
is Nina's idea about the probability with which John will keep to $F_{a c / J}$, i.e. (M-grocer).

In sum:

$$
\begin{aligned}
& \pi_{o f}^{N / J}=\left[\pi\left(\vec{e}_{o f / N}^{\mathrm{b}}\right) \equiv \pi_{o f / N}^{\mathrm{b}}\right]=\left[\pi\left(\vec{e}_{a c / N}^{\mathrm{g}}\right) \equiv \pi_{a c / N}^{\mathrm{g}}\right] \\
& \pi_{a c}^{N / J}=\left[\pi\left(\vec{e}_{a c / N}^{\mathrm{b}}\right) \equiv \pi_{a c / N}^{\mathrm{b}}\right]=\left[\pi\left(\vec{e}_{o f / N}^{\mathrm{g}}\right) \equiv \pi_{o f / N}^{\mathrm{g}}\right]
\end{aligned}
$$

### 46.3 Summary

### 46.3.1 IF-THEN representation

Having clarified the "objective" probabilities of the uncertain states and keeping to the text-book concept of expected utility, the maximization problems $M A X_{J}$ and $M A \boldsymbol{X}_{N}$ from Fig. 158 can be re-written as the following pair of IF-THEN rules:


Fig. 161

### 46.3.2 The simplest analytical question

For simplicity we could assume that all the above parameters A), B) and C) are fixed except for John's psychological portrait $u_{J}(B)$. The analyst could then be asked to identify the portraits that would be equivalent in the sense that to each of them will correspond the same pair of profits from the pay-off matrix.

For dramatic effect we could also ask on what level must be held the fixed parameters so as to obtain a game where only one pair of the matrix can be its outcome regardless of John's psychological portrait $u_{j}(B)$. In other words we will ask, under what circumstances a Player's strategy will be called dominant in most text-books as its choice will be "independent on what the Player thinks that the other Player may do".

### 46.3.3 Action vs. non-action

In terms of a behavior/action we can summarize our observations as follows:
John-the Designer:
action: $\quad\left[\left(1-\pi_{a c}^{J / N}\right) \cdot u_{J}\left(B_{o f / J}^{b}\right)+\pi_{a c}^{J / N}\right]>\left(\pi_{o f}^{J / N}\right) \cdot u\left(B_{a c / J}^{g}\right)$
non action: $\left[\left(1-\pi_{a c}^{J / N}\right) \cdot u_{J}\left(B_{o f / J}^{b}\right)+\pi_{a c}^{J / N}\right]<\left(\pi_{o f}^{J / N}\right) \cdot u\left(B_{a c / J}^{g}\right)$
Nina-the Designer:
action $\quad\left[\left(1-\pi_{a c}^{N / J}\right) \cdot u_{N}\left(B_{o f / N}^{b}\right)+\pi_{a c}^{N / J}\right]>\left(\pi_{o f}^{N / J}\right) \cdot u\left(B_{a c / N}^{g}\right)$
non action

$$
\left[\left(1-\pi_{a c}^{N / J}\right) \cdot u_{N}\left(B_{o f / N}^{b}\right)+\pi_{a c}^{N / J}\right]<\left(\pi_{o f}^{N / J}\right) \cdot u\left(B_{a c / N}^{g}\right)
$$

Alternatively, the rules can be expressed as follows:

| JOHN |  |  | NINA |  |
| :---: | :---: | :---: | :---: | :---: |
| IF: | $\begin{aligned} & {\left[\left(1-\pi_{a c}^{J / N}\right) \cdot u_{l}\left(B_{o f / J}^{b}\right)+\right.} \\ & \left.\pi_{a c}^{J / N}\right]>\left(\pi_{o f}^{J / N}\right) \cdot u\left(B_{a c / / J}^{g}\right) \end{aligned}$ |  | IF: | $\begin{aligned} & {\left[\left(1-\pi_{a c}^{N / J}\right) \cdot u_{N}\left(B_{f o / N}^{b}\right)+\right.} \\ & \left.\pi_{a c}^{N / I}\right]>\left(\pi_{o f}^{N / f}\right) \cdot u\left(B_{a c / N}^{g}\right) \end{aligned}$ |
| THEN: | John will re-design his strategy into "M-shoes", |  | THEN: | Nina will re-design her strategy into "V-shoes", |
| ELSE: | John will keep to his actual strategy "M-grocery", |  | ELSE: | Nina will keep to her actual strategy "V-grocery", |

### 46.4 Example

### 46.4.1 Distribution of labor and profit

Based on Fig. 5, in the following Fig. 162 John and Nina are assumed to support fulfillments of their respective tasks $J(1)$ and $N(1)$ by strategies "M-grocery" and "V-shoes", respectively.


It is noteworthy that the example in Fig. 162 has been chosen so that incidentally:

- both counter-Players enjoy the good variants of their profits, as they avoided entries into the same industry,
- Nina's profit is larger than John's, $B_{a c}^{g / J}<B_{o f}^{g / N}$, as can be read from the pay off matrix in Fig. 159 and Fig. 160.


### 46.4.2 Distribution of wealth

Using the terminology of Comment 9, we may also summarize that - by the end of the production cycle concerned - the game under study will lead to the following transfer of wealth:

John's endowment

Nina's endowment
will increase-decrease by profit $\vec{B}^{*} / J=B_{a c}^{g / J}$ generated by $F_{j}^{*}=F_{a c / j}$ will increase-decrease by profit $\vec{B}^{*} / N=B_{o f}^{g / N}$ generated by $F_{N}^{*}=F_{o f / N}$.

## 47. EXTERNAL INTERVENTIONIST

### 47.1 External preferential ordering

Let it be stressed again that so far the social setting concerned has not involved anybody who would be interested in the variant distributions of profits, e.g. their efficiency let alone fairness. Hence, contrary to the above Comment 9 and its concept of a re-allocation, the currently discussed distribution $\overrightarrow{B B}^{+}$ is "nobody's choice". By using superscript "+" instead of "*" we want to demonstrate that the respective distribution of labor and profit is a "technical" aggregation of two individual and fully independent choices.

Now it will be only natural to expand the analysis by exactly the agent who, for whatever reason, "will care" about the distribution. In particular, let

## Ruxandra-the Interventionist

be the agent who will establish her preferential ordering

$$
\left[\overrightarrow{B B}^{i}=\left(B_{J}^{i}, B_{N}^{i}\right)\right] \gtrsim\left[\overrightarrow{B B}^{j}=\left(B_{J}^{j}, B_{N}^{j}\right)\right], i, j=1, \ldots, 4
$$

upon the pay-off matrix under study, where, for notational convenience:

$$
\begin{aligned}
& \left(B_{a c}^{b / J}, B_{a c}^{b / N}\right) \equiv\left(B_{J}^{1}, B_{N}^{1}\right)=\overrightarrow{B B}^{1} \\
& \left(B_{a c}^{g / J}, B_{o f}^{g / N}\right) \equiv\left(B_{J}^{2}, B_{N}^{2}\right)=\overrightarrow{B B}^{2} \\
& \left(B_{o f}^{g / J}, B_{a c}^{g / N}\right) \equiv\left(B_{J}^{3}, B_{N}^{3}\right)=\overrightarrow{B B}^{3} \\
& \left(B_{o f}^{b / J}, B_{o f}^{b / N}\right) \equiv\left(B_{J}^{4}, B_{N}^{4}\right)=\overrightarrow{B B}^{4}
\end{aligned}
$$

Purely formally, again, Ruxandra's interests and preferences will be represented by a maximization problem

```
max}[\mp@subsup{W}{}{R}(\vec{BB})=\mp@subsup{W}{}{R}(\mp@subsup{B}{|}{},\mp@subsup{B}{N}{})
s.t.: }\vec{BB}\in{\mp@subsup{\vec{BB}}{}{1},\mp@subsup{\vec{BB}}{}{2},\vec{B\mp@subsup{B}{}{3}},\vec{B\mp@subsup{B}{}{4}}
MAX 
```

The solution to $M A X_{R}$ will be denoted

$$
\overrightarrow{B B^{*} / R}=\left(B_{J}^{* / R}, B_{N}^{* / R}\right)
$$

where, as before, the superscript "*/R" will demonstrate that the vector of profits represents Ruxandra's (fully subjective) conception about how the wealth should be distributed between John and Mary.

### 47.2 Interventionist's discontent; examples

It should then be of no surprise that into our prominent analytical interest will fall the prospective inequality

$$
\left[\overrightarrow{B B}^{* / R}=\left(B_{f}^{* / R}, B_{N}^{* / R}\right)\right] \neq\left[\overrightarrow{B B}^{+}=\left(B^{* / J}, B^{* / N}\right)\right]
$$

representing Ruxandra's "discontent" with the distribution constituted by John and Nina.

The following examples should only illustrate the infinite variety of this kind of a "negative" outcome:

1) A text-book rationale for an "objective disappointment" with $\overrightarrow{B B}^{+}$may be that the distribution $\left(\vec{B}^{*} / J, \vec{B}^{*} / N\right)$ is not Pareto-efficient in the sense that there exists among $\left\{\overrightarrow{B B}^{1}, \overrightarrow{B B}^{2}, \overrightarrow{B B}^{3}, \overrightarrow{B B}^{4}\right\}$ at least one other pair of profits that will make at least one of the two Players better off without making the other Player worse off.
2) Ruxandra's discontent may reflect her inter-personal preference, e.g. the fact that she simply likes Nina better than John in the sense of a welfare function

$$
W^{R}(\overrightarrow{B B})=\left[U^{R}\left(B_{J}\right)+\frac{1}{(1+\rho)} \cdot U^{R}\left(B_{N}\right)\right]
$$

where, as explained in Comment 9 , the variable $\rho$ is the inter-personal discount rate reflecting Ruxandra's relative socio cultural distance from John and Nina.
3) Let ${\overrightarrow{B B^{+}}}^{+}={\overrightarrow{B B^{2}}}^{2}$ and $\overrightarrow{B B}^{*} R=\overrightarrow{B B}^{1}$ because Ruxandra plans to enter herself the industry of shoe-making and hence would prefer the distribution of labor when both John and Nina remain Grocers.
4) Let $\overrightarrow{B B}^{+}=\overrightarrow{B B}^{2}$ and $\overrightarrow{B B}^{\star / R}=\overrightarrow{B B}^{3}$, and hence

- Ruxandra is satisfied with the fact that John and Nina are not competitors,
- Ruxandra's dissatisfaction is two fold:
as to the distribution of wealth Ruxandra believes that the distribution should such that $B_{o f}^{g / J}>B_{a c}^{g / N}$, i.e. that John's profit should be larger than Nina's (John not Nina is to pay alimony),
as to the distribution of labor Ruxandra claims that Mary not Victor should be a Shoe-maker (Mary should avoid grocery because the hygienic conditions in this industry are more dangerous to women).


### 47.3 Interventionist's task

As stressed already on various occasions, terminology such as "discontent", "disappointment" or "dissatisfaction" should be applied with an extreme caution in treatises declaring scientific ambitions.

Our defense towards the danger of value loaded normativity will be again provided by the value-free and hence rather cynical concept of an operational unit. In the very brief Ruxandra will be taken as a Designee designed by her respective Designer in the standard form

$$
R(1) \equiv\left[r:\langle\vec{r}\rangle \rightarrow\left\langle\vec{T}_{r}\right\rangle\right]
$$

where:
the IF-component $\langle\vec{r}\rangle \quad$ will be assumed to include outcomes of John's and Nina's choices $B^{* / J}$ and $B^{*} / N$, respectively, will represent Ruxandra's task to respond to the inequality in a particular way.


Fig. 163
By calling Ruxandra an Interventionist we suggest that from the many realistically imaginable kinds of Ruxandra's responses to $\overrightarrow{B B}^{+}$, of our particular interest will be her attempts to modify the distribution by affecting the parameters A), B) and-or C) depicted in Fig. 161.

To illustrate, Ruxandra may fully leave aside all parameters except one, namely John's psychological portrait $u_{J}(B)$. For dramatic effect she may attempt to make John a more courageous man, convert him from a risk-averse to a risk-loving Player.

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[^0]:    1 Roman numerals "I, II, III, ..." attached to the title of a section we suggest that the topic will be repeatedly corroborated throughout this BOOK.

[^1]:    2 GR.

