On an error in applying feedback theory to climate

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Abstract

Temperature feedbacks have hitherto been thought to contribute to equilibrium sensitivity up to ten times the direct warming or reference sensitivity that induced them, because it had been erroneously assumed that the Earth's emission temperature would induce no feedbacks, though any subsequent direct warming, such as the response to the forcing caused by the presence of the naturally-arising, non-condensing greenhouse gases, would induce them. Accordingly, the large feedback-driven warming induced by emission temperature has been mistakenly counted as part of the feedback induced by the comparatively small direct warming from the non-condensing greenhouse gases, leading to substantial overstatements of the feedback fraction and thus of all climate sensitivities. The official estimate of equilibrium sensitivity to doubled CO_2 ("Charney sensitivity") had been $\mathbf{3.3}$ $[2.0, 4.5]$ K, implying a feedback fraction on $\mathbf{0.67}$ $[0.45, 0.75]$. However, an empirical comparison of several published official estimates of industrial-era net anthropogenic forcings with observed warming coheres with two theoretical methods in finding the feedback fraction to be only $\mathbf{0.12}$ [0.08, 0.16], implying Charney sensitivity of order $\mathbf{1.25}$ K.

Keywords

Feedback theory, climate modeling, climate sensitivity, global warming, temperature feedbacks

1. Purpose and scope

With the five *Assessment Reports* [1-5] of the Intergovernmental Panel on Climate Change (IPCC) as context, the present work corrects a long-standing error in applying feedback theory to the derivation of Charney sensitivity ΔT, defined as the total change in annual global mean surface temperature T_S at re-equilibration after a radiative forcing ΔQ_0 equivalent to doubling atmospheric CO_2 concentration when all subdecadal-scale feedbacks have acted [see e.g. 5, p. 1452].

The present results, which principally concern sensitivity-altering temperature feedbacks affecting the balance of radiative flux densities between incoming solar irradiance and outgoing long-wave radiation in the diagnostic context of a zero-dimensional sensitivity model [6], apply to all studies of Charney sensitivity in that broad category, including the discussion of feedback in [1-5; e.g., 5, §9.7], and also to effective-sensitivity estimates or integrations to equilibrium of coupled atmosphere-ocean general-circulation models, insofar as the simple zero-dimensional model faithfully reproduces their projected Charney-sensitivity interval.

The zero-dimensional model handles temperature feedbacks in the climate system via the Bode formulism codified in [7, ch. 3], which arose from the study of signal feedbacks in telephone amplifier circuits and is widely cited [e.g., by 6, eq. (21); 8-15; 16, eq. (3)] as being no less fundamental to climate than to dynamical systems generally.

It has hitherto been assumed [e.g. in 1-5] that, owing to positive or amplifying feedbacks, Charney sensitivity ΔT might exceed reference sensitivity ΔT_S up to fourfold (some authorities, e.g. [17-22], say tenfold). It is on such predictions that mitigation policy has been founded. Here, however, it will be shown that, though individual feedbacks may be substantial, after correction of a substantial error their sum will be small and ΔT will not greatly exceed ΔT_S.

A note on notation: mid-range estimates are in **bold**, and feedback-driven contributions to temperature changes are indicated by subscripts in parentheses: e.g. $\Delta T_{(0)}, \Delta T_{(b)}$.

2. The current zero-dimensional equilibrium-sensitivity model

Mainstream climatology's current approach to temperature feedback will now be described. It will be illustratively assumed that all global warming before 1850, when pre-industrial temperature T_N (= 287.5 K) prevailed, was natural; that there was no warming from 1750-1850; and that all warming ΔT_A (≈ 0.9 K) since 1850 was anthropogenic. The equation (1) for the zero-dimensional model that diagnoses equilibrium sensitivities ΔT (in Kelvin) from model ensembles (see [6, 13]), is

$$\Delta T = \lambda_0 \Delta Q_0 / (1 - \lambda_0 \lambda) = \Delta T_S / (1 - f) = \lambda_0 (\Delta Q_0 + \lambda \Delta T), \tag{1}$$

where the Planck reference-sensitivity parameter λ_0, the radiative forcing ΔQ_0, the feedback sum λ, the feedback factor f and the reference sensitivity ΔT_S are as described below.

Today's net incoming radiative flux density Q_0 at the top of the atmosphere is given by (2).

$$Q_0 = S_0(1 - \alpha)/4 = 1364.625(1 - 0.293)/4 = 241.2 \text{ W m}^{-2}. \tag{2}$$

where S_0 is total solar irradiance [23], α is the Earth's Bond albedo today [24] and the divisor 4 allows for the ratio of the area of the disk presented by the Earth to solar radiation to the area of the rotating spherical surface of the planet. Since the albedo feedback embodies any anthropogenic change in albedo (e.g. from diminished snow or cloud extent), for simplicity Q_0 will be taken as invariant from 1750 onward.

A radiative forcing ΔQ_0, denominated in W m^{-2}, is an externally-driven perturbation in the net down-minus-up emission-altitude radiative flux density Q_0. The estimated value of ΔQ_0 in response to a proportionate change in CO_2 concentration is derived via (3),

$$\Delta Q_0 = k \ln(C/C_0), \tag{3}$$

where C/C_0 is the proportionate change and k is a constant, valued at 6.3 until [25], after an intercomparison between three models, reduced k by 15% to 5.35. Based on the fifth-generation models (CMIP5) of the Climate Model Intercomparison Project, [26] gives the current CO_2 forcing ΔQ_0 in response to doubled CO_2 (the standard metric) as $3.5\ \mathrm{W\,m^{-2}}$, implying that $k = 5.05$, as in (4).

$$\Delta Q_0 = 5.05 \ln(2) = 3.5\ \mathrm{W\,m^{-2}}. \tag{4}$$

It is currently thought that the emission temperature T_0 that would prevail at the top of the atmosphere today in the absence of non-condensing greenhouse gases and before accounting for feedbacks would be as derived in (5) from the fundamental equation of radiative transfer, where σ ($= 5.6704 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$) is the Stefan-Boltzmann constant [27]. Emissivity ε at the emission altitude is, as usual, assumed to be unity.

$$T_0 = \left(\frac{Q_0}{\epsilon\sigma}\right)^{0.25} = 255.4\ \mathrm{K}. \tag{5}$$

However, [28-29], modeling the climate in the absence of the non-condensing greenhouse gases, conclude that two-thirds of the ocean would be ice-covered, so that the Earth's albedo α would be 0.418, whereupon, substituting this value in (2), Q_0 would be $198.6\ \mathrm{W\,m^{-2}}$ and, from (5), T_0 would be $243.3\ \mathrm{K}$. In [30] a snowball-Earth albedo 0.6 is assumed, implying $Q_0 = 136.5\ \mathrm{W\,m^{-2}}$ and $T_0 = 221.5$ K. All of these values will be modeled here.

Before any greenhouse-gas forcings or any feedbacks acted, the mean altitude at which the emission temperature obtained was the Earth's surface itself. As forcings and feedbacks act, this emission altitude rises. In today's climate, it is near the tropopause. Since the linear lapse-rate of tropospheric temperature with altitude is near-invariant, changes in temperature at one altitude will tend to be near-identical to those at another. Today's lapse-rate of $6.5\ \mathrm{K\,km^{-1}}$ is here assumed to be constant during the industrial era.

It is evident from [28] that, if T_0 were $\sim$255 K, some feedbacks would be present even in the absence of any non-condensing greenhouse gases: for it is there stated that the equator would be ice-free and that up to 10% of today's concentration of water vapor would subsist in the atmosphere, since climate-sensitivity studies treat any change in the atmospheric burden of water vapor as a feedback rather than as a direct forcing.

However, in [28-29], and in climatology generally, it has hitherto been erroneously assumed that T_0 would induce no feedback $\Delta T_{(0)}$ at all, and yet that even the smallest direct warming ΔT_B driven by greenhouse-gas forcings would induce feedbacks $\Delta T_{(b)}$. Since feedbacks are thus treated as though they were induced by ΔT_B alone rather than by $T_0 + \Delta T_B$, the feedback fraction f has been greatly overstated. The extent of the overstatement of warming arising from this substantial error will be established by an empirical method and by two distinct theoretical methods. The results of all three methods will be found to cohere.

Pre-industrial temperature T_N in 1850, the difference between today's surface temperature T_S (= 288.4 K) and the anthropogenic warming ΔT_A (= 0.9 K) since 1850, is derived in (6) using the least-squares trend on the monthly global mean surface temperature anomalies in [31].

$$T_N = T_S - \Delta T_A = 288.4 - 0.9 = 287.6 \text{ K}. \tag{6}$$

To a first approximation, today's Planck reference sensitivity parameter λ_0, the ratio of ΔT_S to ΔQ_0, is the first derivative of the fundamental equation of radiative transfer, as (7) shows.

$$\lambda_0 \approx \Delta T_0 / \Delta Q_0 = T_0/(4Q_0) = \frac{255.4}{4(241.2)} = 0.3 \text{ K W}^{-1}\text{ m}^2 \approx 3.2^{-1} \text{ K W}^{-1}\text{ m}^2. \tag{7}$$

Though λ_0 varies with albedo and was greater in pre-industrial times than at present, over the period of anthropogenic influence λ_0 is assumed to be constant.

A temperature feedback λ_i, denominated in Watts per square meter per Kelvin of the temperature that induces it, is a modification of surface temperature induced by the fact of that temperature. Though there are some interactions between feedbacks, for simplicity they will be assumed to be additive, so that (8) gives the feedback sum λ.

$$\lambda = \sum_{i=1}^{n} \lambda_i. \tag{8}$$

Since many feedbacks λ_i, summing to λ, subsist in the climate, one may rewrite (1) as (9).

$$\Delta T = \lambda_0(\Delta Q_0 + \lambda_1 \Delta T + \lambda_2 \Delta T + \cdots + \lambda_n \Delta T). \tag{9}$$

The unitless feedback factor f, equivalent to the feedback factor $\mu\beta$ in [7, ch. 3] assuming that the feedback fraction β is the fraction of the output signal returned to the input node and that the simple-gain factor $\mu \coloneqq 1$, is given in (10), while (11) gives the system-gain factor G and (12) shows that feedbacks account for the difference between reference sensitivity ΔT_S and equilibrium sensitivity ΔT. Where feedbacks are net-zero, $G \coloneqq 1$, whereupon $\Delta T \coloneqq \Delta T_S$. Where feedbacks are net-positive, $\Delta T > \Delta T_S$. It had hitherto been thought that $\Delta T \gg \Delta T_S$.

$$f = \lambda_0 \lambda. \tag{10}$$

$$G = 1/(1 - f). \tag{11}$$

$$\Delta T = G\ \Delta T_S. \tag{12}$$

Table 1 lists the chief climate-relevant temperature feedbacks in the CMIP5 models. In the final row, Charney sensitivities ΔT are derived from the feedback factors f using (1).

Table 1. Feedbacks estimated from 30 CMIP5 models [based on 5, p. 818, table 9.5]

Temperature feedback	Lower bound	**Mid-range**	Upper bound	Timescale
Water vapor	$+1.3$ W m^{-2} K^{-1}	**$+1.6$ W m^{-2} K^{-1}**	$+1.9$ W m^{-2} K^{-1}	Hours
Lapse rate	-1.0 W m^{-2} K^{-1}	**-0.6 W m^{-2} K^{-1}**	-0.2 W m^{-2} K^{-1}	Hours
Cloud	-0.4 W m^{-2} K^{-1}	**$+0.3$ W m^{-2} K^{-1}**	$+1.1$ W m^{-2} K^{-1}	Days
Surface albedo	$+0.2$ W m^{-2} K^{-1}	**$+0.3$ W m^{-2} K^{-1}**	$+0.4$ W m^{-2} K^{-1}	Years
Feedback sum λ	$+0.1$ W m^{-2} K^{-1}	**$+1.6$ W m^{-2} K^{-1}**	$+3.2$ W m^{-2} K^{-1}	Years
Feedback factor $f = \lambda_0\lambda$	$+0.0$	**$+0.5$**	$+1.0$	
System gain factor G	1.0	**2.0**	(Undefined)	
Charney sensitivity	1.1 K	**2.2 K**	∞	

Where $f = 1$, derived from the sum of the upper bounds of the individual feedbacks, equilibrium sensitivity is momentarily undefined, while $f > 1$ implies global cooling.

Surface temperature T_S today Δ 288.4 K [33]. Reference sensitivity ΔT_S is the direct change in T_S in response to a forcing ΔQ_0 before taking account of temperature feedbacks λ_0, λ_1, … λ_n, summing to λ. Given that the standard metric is the temperature response to doubled CO_2, reference sensitivity before taking account of feedbacks is estimated in (13), where the CO_2 forcing $\Delta Q_0 = 3.5$ W m^{-2} is as derived in [26] from the CMIP5 ensemble.

$$\Delta T_S = \Delta Q_0 \lambda_0 = 3.5/3.2 = 1.1 \text{ K}. \tag{13}$$

Equilibrium sensitivity ΔT is the final warming after all feedbacks of sub-decadal duration have acted. Charney sensitivity is equilibrium sensitivity to doubled CO_2 concentration with all other forcings fixed. Its interval is given in [1] as $\mathbf{2.5}$ [1.5, 4.5] K; in [5] as [1.5, 4.5 K]; in [34] as $\mathbf{3.0}$ [1.5, 4.5] K; and in [35] as $\mathbf{3.3}$ [2.0, 4.7] K officially diagnosed from 11 CMIP5 models.

Since the mid-range estimate ΔT_{mid} of Charney sensitivity is $\mathbf{3.3}$ **K**, finding ΔT_S in (13) and rearranging (1) as (14) gives the implicit CMIP5 mid-range feedback fraction f_{mid}.

$$f_{\text{mid}} = 1 - \Delta T_S/\Delta T = 1 - 1.1/3.3 = \mathbf{0.67}. \tag{14}$$

The interval of CMIP5 feedback factors implicit in the published Charney-sensitivity interval ΔT on $\mathbf{3.3}$ [2.0, 4.7] K is not $\mathbf{0.5}$ [0, 1], as shown in Table 1, but **0.67** [0.46, 0.76].

3. Calibration of (1)

Using published official inputs calibrates (1), insofar as it reproduces the official interval of climate sensitivities from recent generations of general-circulation models. In [5, fig. 9.43], [35] is cited as having diagnosed ΔQ_0, λ_0, λ from simulated abrupt 4-fold increases in CO_2 concentration in 11 CMIP5 models via the linear-regression method given in [36]. In [35] it is said that ~85% of the uncertainty in equilibrium sensitivity ΔT arises from uncertainty in the feedback sum λ, and hence in the feedback factor f $(= \lambda_0 \lambda)$.

In [35] the 11 models' mid-range estimate λ_{mid} of the feedback sum was $\mathbf{1.57}$ W m^{-2} K^{-1}, implying $f_{\text{mid}} = \lambda_0 \lambda_{\text{mid}} = \mathbf{0.49}$; the 2 σ bounds of f were f_{mid} $\pm$ 40%, i.e. $\mathbf{0.49} \pm 0.20$; and the implicit CO_2 forcing ΔQ_0, in which fast feedbacks were included, was 4.5 W m^{-2} compared with the 3.5 W m^{-2} in [26]. Reference sensitivity ΔT_S, there taken as 1.41 K, was ~20% above the CMIP5 mid-range estimate 1.09 K in (13). Using these values, (1) proves well calibrated, yielding Charney sensitivity ΔT on $\mathbf{3.3}$ [2.0, 4.5] K, near-exactly coextensive with the several published official intervals in Table 2.

Table 2. Calibration of (1) against Charney sensitivities in CMIP3/5 models and in [4-5]

-40%	$\boldsymbol{f_{\mathrm{mid}}}$	$+40\%$	Source	ΔT_{min}	$\boldsymbol{\Delta T_{\mathrm{mid}}}$	ΔT_{max}
0.29	$\mathbf{0.49}$	0.68	**Diagnosed from (1) for $\boldsymbol{f_{\mathrm{mid}} \pm 2\,\sigma}$**	$\mathbf{2.0\ K}$	$\mathbf{2.8\ K}$	$\mathbf{4.5\ K}$
Four published Charney sensitivity intervals ΔT_x			CMIP5 models [5, p. 818, table 9.5]	1.9 K	3.2 K	4.5 K
			CMIP5 models: [36, table 1]	1.9 K	$\mathbf{3.3\ K}$	4.4 K
			CMIP5 models [26]	2.1 K	3.4 K	4.7 K
			CMIP3 [4, p. 798]; CMIP5 mean	$\mathbf{2.0\ K}$	$\mathbf{3.3\ K}$	$\mathbf{4.5\ K}$

From this successful calibration it follows that, though (1) assumes feedbacks are linear but some feedbacks are actually nonlinear, (1) nevertheless reproduces the interval of Charney sensitivities predicted by the CMIP5 models, which do account for nonlinearities.

Though models generally treat ΔQ_0, λ_0, λ as emergent properties, diagnoses of their feedbacks via the zero-dimensional formulism derived from [7, ch. 3] closely reflect (1), confirming empirically that that equation is relevant to the study of feedbacks' contribution to global temperature. Indeed, [4, p. 631 fn.], with notation adjusted to conform to usages herein shown in square brackets, describes (1):

> "Under … simplifying assumptions the amplification of the global warming from a feedback [sum] λ (in W m^{-2} K^{-1}) with no other feedbacks operating is $[G =]\ 1/(1 - [\lambda_0\lambda])$, where $[\lambda_0]$ is the 'uniform temperature' radiative cooling response (of value approximately 3.2 W m^{-2} K^{-1} [32]. If n independent feedbacks operate, $[\lambda]$ is replaced by $[\lambda_1 + \lambda_2 + \cdots + \lambda_n]$."

Insofar as calibration shows (1) to be effective as a black-box diagnostic providing reliable estimates of the Charney-sensitivity intervals that general-circulation models would currently predict, it is also capable of diagnosing the Charney sensitivities that models programmed to reflect climatology's current understanding of feedback theory would be expected to predict over any period for which anthropogenic forcings have been published. Predictions markedly at odds with the outputs from (1) are likely to be inconsistent with the Charney sensitivities predicted by the CMIP3/5 model ensembles.

4. Empirical diagnosis of $\boldsymbol{f_{\text{mid}}}$ from observed anthropogenic warming

IPCC, acknowledging in [5] that models exaggerate, substituted its "expert assessment" for their predictions. In [1], medium-term warming of $\mathbf{0.3} \pm 0.1$ K decade^{-1} had been predicted; but, following observed warming of only 0.14 K decade^{-1} since 1990, in [5] IPCC all but halved its medium-term prediction to $\mathbf{0.17} \pm 0.1$ K decade^{-1}: but it did not commensurately cut its Charney-sensitivity interval $\mathbf{3} \pm 1.5$ K, which is as it was four decades ago in [34, p. 4].

Likewise, though [5, p. 15] says, "The long-term climate model simulations show a trend in global-mean surface temperature from 1951-2012 that agrees with the observed trend (very high confidence)", applying $f_{\text{mid}} = \mathbf{0.67}$ from (14) to the 1.7 W m^{-2} period forcing in [5, fig. SPM.6], (1) gives a period warming of 1.6 K, more than twice the 0.68 K least-squares linear-regression trend on the observed monthly anomalies in [31], confirming that f_{mid} is too high.

Though reference sensitivity ΔT_S to doubled CO_2 before accounting for feedbacks is currently thought to be $\mathbf{1.1} \pm 0.1$ K [based on 26], compared with $\mathbf{1.2}$ **K** in [5, p 676, §8.3.2.1; 37], and $\mathbf{0.9}$ **K** in [38], the CMIP3 and CMIP5 ensembles predict Charney sensitivity ΔT on $\mathbf{3.3}\,[2.0, 4.5]$ K (Table 3), implying a $\mathbf{2.2}\,[1.0, 3.4]$ K feedback-driven contribution to warming. Owing to feedbacks, then, the system gain factor $G \coloneqq \Delta T/\Delta T_S \approx \mathbf{3}\,[2, 4]$. Yet for 810,000 years surface temperature T_S ($= 288.4$ K today) has varied by only $\pm\,\mathbf{3.3}$ **K** from the period mean [39], suggesting either that large feedbacks amplified small forcings in geological time or that, as will be demonstrated here, feedbacks' net influence on temperature is small.

If for any industrial-era period the net anthropogenic forcing ΔQ_0 and warming ΔT_A are known, the feedback fraction f applicable to that period may be derived using (1). In table 3, the mean value $f_{\text{mid}} = \mathbf{0.12}$ (against the CMIP5 estimate $f_{\text{mid}} = \mathbf{0.67}$) is found empirically from authoritative, published values of anthropogenic forcing ΔQ_0 and warming ΔT_A for ten industrial-era periods, and is used in (1) to derive the Charney sensitivity $\Delta T_{\text{mid}} = \mathbf{1.25}$ **K** (against the CMIP5 estimate $\mathbf{3.3}$ **K**) that the models may be expected to have predicted.

The rationale for this approach is that the feedback fraction f_{mid} applicable to a CO_2 doubling compared with today is likely to be close to the value that has obtained throughout the industrial era, so that the application of any significantly lesser value of f_{mid} to past anthropogenic warming will imply a commensurately reduced value for Charney sensitivity ΔT.

In Table 3, Col. A gives the source, col. B the end date of the period, col. C the net anthropogenic forcing, col. D $(= \text{C}/3.2)$ the reference sensitivity for the given period, col. E $[= \text{D}/(1 - \mathbf{0.67})]$ the expected equilibrium sensitivity, col. F the observed least-squares warming trend on the monthly surface temperature anomalies from [31], col. G $(= \text{E}/\text{F})$ the ratio of expected to observed warming, col. H $(= 1 - \text{D}/\text{F})$ the feedback fraction f_{mid} implicit in the observed warming, col. I the equilibrium sensitivity where $f_{\text{mid}} = 0.12$, and col. J the revised ratio of expected to observed warming for $f_{\text{mid}} = \mathbf{0.12}$.

Table 3. Ratios of expected to observed warming ΔT_A in the industrial era

	A	B	C	D	E	F	G	H	I	J
#	Source	End date	ΔQ_0 W m^{-2}	$\Delta T_N = \lambda_0 \Delta Q_0$	Expected ΔT_A	Obsrvd. ΔT_A	Ratio E / F	$1 -$ D / F $= f_{\text{mid}}$	New ΔT_A	Ratio I / F
1	[40]	2012	2.95	0.92 K	2.77 K	0.76 K	3.6	–0.21	1.05 K	1.4
2	[41]	2016	3.10	0.97 K	2.91 K	0.84 K	3.5	–0.15	1.10 K	1.3
3	[5, SPM.6]	1980	1.25	0.39 K	1.17 K	0.38 K	3.1	–0.03	0.44 K	1.2
4	[42]	2001	1.90	0.59 K	1.78 K	0.62 K	3.0	+0.01	0.67 K	1.1
5	[5, SPM.6]	2011	2.29	0.72 K	2.15 K	0.76 K	2.8	+0.06	0.81 K	1.1
6	[43]	2006	1.93	0.60 K	1.81 K	0.68 K	2.7	+0.11	0.69 K	1.0
7	[4]	2005	1.60	0.50 K	1.50 K	0.67 K	2.2	+0.25	0.57 K	1.9
8	[5, SPM.6]	1950	0.57	0.18 K	0.53 K	0.26 K	2.1	+0.31	0.20 K	1.8
9	[44]	2010	1.40	0.44 K	1.31 K	0.74 K	1.8	+0.41	0.50 K	0.7
10	[45]	2000	1.00	0.31 K	0.94 K	0.58 K	1.6	+0.46	0.36 K	0.6
Expected / observed warming based on $f_{\text{mid}} = \mathbf{0.67}$:							**2.6**	$f_{\text{mid}} = \mathbf{0.12}$:		**1.0**

Col. G of Table 3 shows that the mean ratio of expected to observed warming over the ten periods is not the expected $\mathbf{1.0}$ but $\mathbf{2.6}$. Committed but unrealized warming does not explain this substantial divergence between expectation and observation. Anthropogenic forcings act near-immediately and all feedbacks relevant to Charney sensitivity operate on timescales of years at most [5, p. 128, fig. 1.2]. Though (1) takes no account of the delay in warming owing to the vast heat capacity of the ocean, the timescale of ocean overturning is millennial and little ocean heat will be returned to the surface within a policy-relevant time-horizon.

The above comparison reveals, contrary to published suggestions that the predictions in [1] were accurate [see e.g. 46], that models run hot. Many explanations have been offered. For instance, [47] suggests "global warming holes" (regions warming more slowly than average), and [48] suggests that filtering out short-term cooling influences like La Niña brings IPCC's original predictions in [1] into conformity with observation. Here a simpler explanation will be offered: that the models embody a significant and long-standing methodological error that has led to substantial overestimates of global warming throughout the 120 years since Arrhenius [49, table VII] first concluded that what is now known as Charney sensitivity was ~5.5 K.

5. Current feedback methodology

In Fig. 1, a standard feedback loop (a) is compared with climatology's variant (b). Experiments at a government laboratory and separately on a circuit designed and built by one of the authors (Appendixes A, B) have confirmed in detail the feedback theory discussed here.

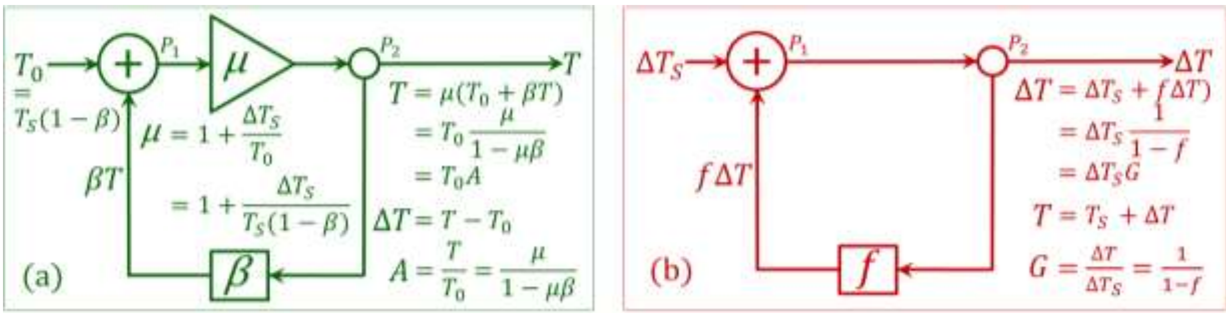

Fig. 1 A standard Bode feedback loop (a) compared with climatology's variant (b).

In Fig. 1(a), the full input signal T_0 passes via the summative input node P_1 and a gain block with simple-gain factor μ to the output node P_2, whence the feedback loop returns a fraction β

of the output signal T via P_1 and the μ gain block to node P_2, so that the system gain factor A ($:= T/T_0$), whose form is derived in (15), correctly combines the influences of the input signal T_0, the simple-gain factor μ, the feedback fraction β and the feedback factor $\mu\beta$.

$$\begin{aligned} T &= \mu(T_0 + \beta T) \\ &= \mu T_0 + \mu\beta T \\ \Rightarrow \quad T(1-\mu\beta) &= \mu T_0 \\ \Rightarrow \quad T &= T_0 \frac{\mu}{1-\mu\beta} := T_0 A, \end{aligned} \tag{15}$$

such that (16-17) compare the standard form A and the variant G of the system gain factor.

$$A := \frac{T}{T_0} = \frac{\mu}{1-\mu\beta}; \tag{16}$$

$$G := \frac{\Delta T}{\Delta T_S} = \frac{1}{1-f} \equiv \frac{\mu}{1-\mu\beta} \quad | \quad \mu := 1. \tag{17}$$

In Fig. 1(b), climatology's variant, the emission temperature T_0 is omitted in favor of ΔT_S (= 1.1 K) as the input signal. Thus, there is no μ gain block. The variant form f of the feedback factor (since $\mu := 1$, climatology's f is at once the feedback fraction and the feedback factor) replaces the Bode feedback factor $\mu\beta$, so that ΔT replaces T as the output signal, and the variant system gain factor G is as in (11). Authorities using the variant method include pedagogical works [e.g. 8-13, 15-16]; zero-dimensional models' diagnoses of equilibrium sensitivity for IPCC [e.g. 26, 36]; and IPCC itself [e.g. 4, p. 631 fn., quoted earlier].

Though the variant form of the feedback-loop diagram in Fig. 1(b) may be used to derive a somewhat overstated equilibrium sensitivity ΔT, its exclusive use in climatology has led to the erroneous assumption that the baseline emission temperature T_0 that is the input signal in the mainstream form of the feedback loop in Fig. 1(a) but is altogether omitted from Fig. 1(b) induces no feedbacks $\Delta T_{(0)}$ at all, so that the natural greenhouse effect ΔT_G, which is the sum of the contributions to T_N from the direct warming ΔT_B arising from the forcings caused by the non-condensing greenhouse gases and the feedbacks $\Delta T_{(b)}$ induced by ΔT_B, is mistakenly imagined to account for the entire difference between T_0 and the pre-industrial temperature T_N.

Owing to the above assumption it is incorrectly assumed that the feedback fraction f_{mid} is very large. In [28], for instance, f_{mid} is given as $\mathbf{0.75}$, six times the empirically-derived $\mathbf{0.12}$:

> "… water vapor accounts for ~50% of Earth's greenhouse effect, with clouds contributing 25%, CO_2 20%, and the minor GHGs and aerosols accounting for the remaining 5%. Because CO_2, O_2, N_2O, CH_4 and chlorofluorocarbons do not condense and precipitate, noncondensing GHGs constitute the key 25% of the radiative forcing that supports and sustains the entire terrestrial greenhouse effect, the remaining 75% coming as fast feedback contributions from water vapor and clouds …"

What is more, [28] concludes that f_{mid} today is also $\mathbf{0.75}$, a value at the upper end of the CMIP5 interval of f, implying that nonlinearities in feedbacks have had little effect over time:

> "The conclusion from this long-wave greenhouse flux attribution analysis is that (for current climate) approximately 75% of the greenhouse warming is the result of fast-feedback effects by water vapor and clouds."

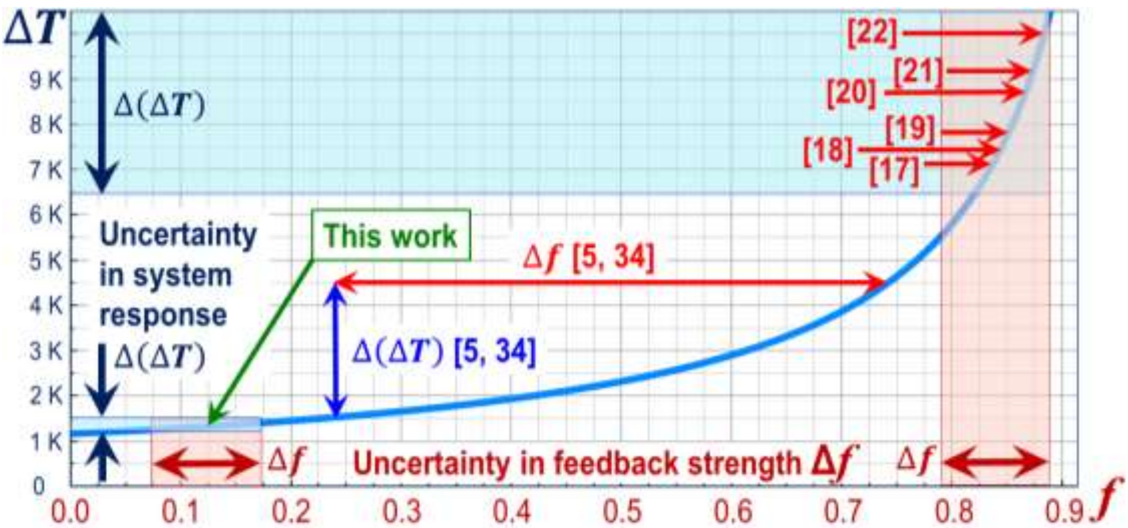

Fig. 2 The rectangular-hyperbolic response curve of Charney sensitivities ΔT against f. For $\Delta T_S = 1.1$ K, identical uncertainties Δf generate wider sensitivity intervals as $f \to 1$, since the uncertainty $\Delta(\Delta T)$ in the system response depends greatly on mean feedback strength. Based on [13, fig. 6]. High-end predicted sensitivities in [17-22] are contrasted with the interval in [5, 34] and with the empirically-derived value $\Delta T_{\text{mid}} = \mathbf{1.25\ K}$.

Since the error makes f_{mid} excessive, f_{max} is driven unduly close to unity, so that the non-linearity of the rectangular-hyperbolic response curve of Charney sensitivities ΔT against feedback factors f (Fig. 2) ensures that even very small increases in f_{max} as $f_{\text{max}} \longrightarrow 1$ drive disproportionately large apparent (but not real) "tipping-point" increases in ΔT_{max}.

Since $f \rightarrow 1 \Rightarrow \Delta T \rightarrow \infty$, implausibly elevated Charney sensitivities ΔT may be predicted, such as > 10 K in [22]. Yet in truth the interval of true feedback fractions β occupies a near-linear region of small slope near the origin of the response curve in Fig. 2, ruling out such extreme sensitivities. Where, as here, the interval of the output signal ΔT is determined from $f_{\text{mid}} \pm 2$ σ, not only the amplitude but also the interval breadth of the output signal will be well constrained where f_{mid} is $\sim\mathbf{0.12}$, but poorly constrained where f_{mid} is $\sim\mathbf{0.75}$.

6. Accounting for feedbacks $\Delta \boldsymbol{T}_{(\mathbf{0})}$ induced by emission temperature $\boldsymbol{T}_{\mathbf{0}}$

Three worked examples will be based on the set $\alpha \in \{0.293, \mathbf{0.418}, 0.6\}$, where albedos $\alpha \in \{0.6, 0.418, 0.293\}$ are as stated in [30, 28, 24] respectively, such that, from (2), net incoming radiative flux density $Q_0 \in \{136.5, \mathbf{198.6}, 241.2\}\ \text{W m}^{-2}$; from 5, the emission temperature $T_0 \in \{221.5, \mathbf{243.3}, 255.4\}$ K; and $(T_N - T_0) \in \{66.0, \mathbf{44.2}, 32.1\}$ K.

The feedback loops first in Fig. 1(a) and then in Fig. 1(b) will be deployed to account for feedbacks $\Delta T_{(0)}$ induced by the emission temperature T_0 that would obtain in the absence of the non-condensing greenhouse gases, whereupon the feedback fractions β or f (illustratively taken, as in [28], as invariant at all stages) will be derived.

6.1 First theoretical derivation of β, based on Fig. 1(a)

In these worked examples, the direct warming ΔT_B attributable to forcings from the presence of the naturally-occurring, non-condensing greenhouse gases will be taken, as in [28], as one-quarter of the difference between snowball-Earth emission temperature T_0 and the natural temperature T_N ($= 287.5$ K) that obtained in 1850.

Accordingly, (18) gives ΔT_B; (19, 20) give the natural direct-gain factor μ_N and the natural system-gain factor A_N; and rearranging (20) as (21) derives the feedback fraction β, whereupon the feedback factor $\mu_N\beta$ is given in (22).

$$\Delta T_B = (T_N - T_0)/4 \in \{16.5, \mathbf{11.1}, 8.0\} \text{ K}. \tag{18}$$

$$\mu_N = 1 + \frac{\Delta T_B}{T_0} \in \{1.0745, \mathbf{1.0455}, 1.0315\}. \tag{19}$$

$$A_N = \frac{\mu_N}{1 - \mu_N\beta} = \frac{T_N}{T_0} \in \{1.2980, \mathbf{1.1821}, 1.1260\}. \tag{20}$$

$$\beta = (A_N - \mu_N)/(A_N\mu_N) \in \{0.16, \mathbf{0.11}, 0.08\}. \tag{21}$$

$$f = \mu_N\beta \in \{0.17, \mathbf{0.12}, 0.09\}. \tag{22}$$

The first theoretical method thus coheres with the empirical method in that $f_{\text{mid}} = \mathbf{0.12}$.

6.2 Second theoretical derivation of β and ΔT, based on Fig. 1(b)

Using the variant feedback loop in Fig. 1(b) but substituting absolute for delta input and output signals in (23) gives pre-industrial f_{mid}, using which (24) gives ΔT.

$$f_{\text{mid}} = 1 - T_0/T_N \in \{0.23, \mathbf{0.15}, 0.11\}. \tag{23}$$

$$\Delta T = \Delta T_S/(1 - f_{\text{mid}}) \in \{1.4, \mathbf{1.3}, 1.2\} \text{ K}. \tag{24}$$

This second theoretical method somewhat overstates f, which is about 25% above its value $\mu_N\beta$ in the first theoretical method. Nevertheless, the coherence between the results from the two theoretical methods and the empirical method is as noteworthy as the absence of coherence between these results and the currently-imagined $f_{\text{mid}} = \mathbf{0.67}$ (CMIP5) or $\mathbf{0.75}$ [in 28].

The near-identity between pre-industrial and industrial-era feedback factors suggests that nonlinearities in individual feedbacks make little or no difference to equilibrium sensitivity ΔT. However, for completeness the method of treating nonlinearities in feedbacks is described in Appendix D, and explicit provision for nonlinearities will be made later herein.

The contributions to warming from feedbacks $\Delta T_{(0)}$ induced by emission temperature T_0 (stage 1 of 4 in deriving ΔT) and from subsequent natural and anthropogenic greenhouse forcings and their feedbacks (stages 2-4) will now be derived using the first theoretical method.

6.3 Stage 1: Derivation of the feedbacks $\Delta T_{(0)}$ induced by T_0.

If an input signal (such as emission temperature T_0) is present, feedbacks arise even in the absence of a forcing. Where $\beta \in \{0.16, \mathbf{0.11}, 0.08\}$, as derived in (21), and where $\mu_B \coloneqq 1$, a feedback-driven contribution $\Delta T_{(0)} \in \{43.2, \mathbf{30.2}, 22.6\}$ K induced by emission temperature $T_0 \in \{221.5, \mathbf{243.3}, 255.4\}$ K raises baseline temperature to $T_B \in \{263.8, \mathbf{273.5}, 278.0\}$ K as in (25-26). This contribution is induced not by the non-condensing greenhouse gases, as had been thought, but by T_0. Accordingly, the natural greenhouse effect ΔT_G is as shown in (27).

$$T_B = T_0 \frac{\mu_B}{1-\mu_B\beta} = T_0 \frac{1}{1-\beta} \in \{263.8, \mathbf{273.5}, 278.0\} \quad | \quad \mu_B \coloneqq 1. \tag{25}$$

$$\Delta T_{(0)} = T_B - T_0 \in \{42.3, \mathbf{30.2}, 22.6\}\text{ K}. \tag{26}$$

$$\Delta T_G = T_N - T_B \in \{23.8, \mathbf{14.1}, 9.6\}\text{ K}. \tag{27}$$

6.4 Stage 2. Apportionment of the corrected natural greenhouse effect ΔT_G.

The feedback-driven contribution $\Delta T_{(b)}$ to ΔT_G is given in (28).

$$\Delta T_{(b)} = \Delta T_G - \Delta T_B \in \{7.3, \mathbf{3.0}, 1.5\}\text{ K}. \tag{28}$$

6.5 Stage 3: Apportionment of the anthropogenic influence to date

For $T_S = 288.4$ K, rearranging (29) as (30) gives the direct-gain factor μ_S that accounts for anthropogenic as well as natural forcings to date.

$$T_S = T_0 \frac{\mu_S}{1-\mu_S\beta} = 288.4\text{ K}. \tag{29}$$

$$\mu_S = \frac{T_S}{T_0+\beta T_S} \in \{1.0772, \mathbf{1.0483}, 1.0343\}. \tag{30}$$

Since, by definition, (31) is also true, rearranging (31) as (32) derives the forcing-driven fraction ΔT_N of anthropogenic warming ΔT_A (= 0.9 K) from 1850 to date [31], whereupon (33) gives the feedback-driven contribution $\Delta T_{(n)}$ to ΔT_A.

$$\mu_S \coloneqq 1 + \frac{\Delta T_B + \Delta T_N}{T_0}. \tag{31}$$

$$\Delta T_N = T_0(\mu_S - 1) - \Delta T_B \in \{0.6, \mathbf{0.7}, 0.7\}\text{ K}. \tag{32}$$

$$\Delta T_{(n)} = \Delta T_A - \Delta T_N \in \{0.3, \mathbf{0.2}, 0.1\}\text{ K}. \tag{33}$$

6.6 Stage 4: Apportionment of the warming in response to doubled CO_2 concentration

Where $\Delta T_S = 1.1$ K at CO_2 doubling, the direct-gain factor μ is given in (34), whereupon Charney sensitivity ΔT to doubled CO_2 is derived in (35) and $\Delta T_{(s)}$ is given in (36).

$$\mu = 1 + \frac{\Delta T_B + \Delta T_N + \Delta T_S}{T_0} \in \{1.0821, \mathbf{1.0527}, 1.0386\}. \tag{34}$$

$$\Delta T = T - T_S = T_0 \frac{\mu}{1-\mu\beta} - T_S \in \{1.6, \mathbf{1.4}, 1.3\}\text{ K}. \tag{35}$$

$$\Delta T_{(s)} = \Delta T - \Delta T_S \in \{0.27, \mathbf{0.19}, 0.14\}\text{ K}. \tag{36}$$

Table 4. The worked examples, contrasting the theoretical, empirical and current methods

First theoretical method : $\alpha \in \{0.6, 0.418, 0.293\}$: $f = \mu_N\beta \in \{0.17, \mathbf{0.12}, 0.09\}$ (Fig. 1(a))			
Emission temperature T_0 $\{221.5, \mathbf{243.3}, 255.4\}$ K	Feedback contrib. $+\,\Delta T_{(0)}$ $\in \{42.3, \mathbf{30.2}, 22.6\}$ K	Post-feedback temp. $= T_B$ $\{263.8, \mathbf{273.5}, 278\}$ K	Feedback fraction β $\in \{0.16, \mathbf{0.11}, 0.08\}$
Natural ghg warming ΔT_B $\in \{16.5, \mathbf{11.1}, 8\}$ K	Feedback contrib. $+\,\Delta T_{(b)}$ $\in \{7.3, \mathbf{3}, 1.5\}$ K	Post-feedback warm. $= \Delta T_G$ $\in \{23.8, \mathbf{14.1}, 9.6\}$ K	Natural temp. $T_N = T_B + \Delta T_G$ 287.6 K
Anthropo. warming ΔT_N $\in \{0.6, \mathbf{0.7}, 0.7\}$ K	Feedback contrib. $+\,\Delta T_{(n)}$ $\in \{0.3, \mathbf{0.2}, 0.2\}$ K	Post-feedback warm. $= \Delta T_A$ $\mathbf{0.9}$ **K**	Today's surface temp. $T_S = T_N + \Delta T_A$ $\mathbf{288.4}$ **K**
2xCO_2: direct warm. ΔT_S $\mathbf{1.1}$ **K**	Feedback contrib. $+\,\Delta T_{(s)}$ $\in \{0.5, \mathbf{0.3}, 0.2\}$ K	Charney sensitivity $= \Delta T$ $\in \{1.6, \mathbf{1.4}, 1.3\}$ K	2xCO_2: surface temp. $T = T_S + \Delta T$ $\{290.0, \mathbf{289.8}, 289.7\}$ K

Second theoretical method: $f_{\text{mid}} = 1 - T_0/T_N \in \{0.23, \mathbf{0.15}, 0.11\}$ (Fig. 1(b))			
$\Delta T_S = \mathbf{1.1}$ **K**	$\Delta T_{(s)} \in \{0.3, \mathbf{0.2}, 0.1\}$ K	$\Delta T \in \{1.4, \mathbf{1.3}, 1.2\}$ K	$\{289.8, \mathbf{289.7}, 289.6\}$ K

Empirical method, based on Fig. 1(b) : $f_{\text{mid}} = \mathbf{0.12}$ (Table 3)			
$\Delta T_S = \mathbf{1.1}$ **K**	$\Delta T_{(s)} = \mathbf{0.15}$ **K**	$\Delta T = \mathbf{1.25}$ **K**	$T = \mathbf{289.65}$ **K**

Current method, misattributing $\Delta T_{(0)}$ to greenhouse gases: $f_{\text{mid}} = \mathbf{0.67}$ (CMIP5)			
$T_0 = 255.4$ K	$\Delta T_{(0)} = 0.0$ K	$T_B = 255.4$ K	$[\mu \coloneqq 1]$
$\Delta T_B = 10.7$ K	$\Delta T_{(b)} = 21.4$ K	$\Delta T_G = 32.1$ K	$T_N = 287.5$ K
$\Delta T_N = 0.3$ K	$\Delta T_{(n)} = 0.6$ K	$\Delta T_A = 0.9$ K	$T_S = 288.4$ K
$\Delta T_S = \mathbf{1.1}$ **K**	$\Delta T_{(s)} = \mathbf{2.2}$ **K**	$\Delta T = \mathbf{3.3}$ **K**	$T = \mathbf{291.7}$ **K**

Current method, misattributing $\Delta T_{(0)}$ to greenhouse gases: $f_{\text{mid}} = \mathbf{0.75}$ [28]			
$T_0 = 255.4$ K	$\Delta T_{(0)} = 0.0$ K	$T_B = 255.4$ K	$[\mu \coloneqq 1]$
$\Delta T_B = 8.0$ K	$\Delta T_{(b)} = 24.1$ K	$\Delta T_G = 32.1$ K	$T_N = 287.5$ K
$\Delta T_N = 0.2$ K	$\Delta T_{(n)} = 0.7$ K	$\Delta T_A = 0.9$ K	$T_S = 288.4$ K
$\Delta T_S = \mathbf{1.1}$ **K**	$\Delta T_{(s)} = \mathbf{3.3}$ **K**	$\Delta T = \mathbf{4.4}$ **K**	$T = \mathbf{292.8}$ **K**

Table 4 apportions forcings and feedbacks at all four stages. Some uncertainty arises because the fraction of the difference $T_N - T_0$ between emission and pre-industrial temperature attributable to the direct forcing from naturally-occurring greenhouse gases is unknown. Table 5 shows that, where ΔT_B is 50 to 100% of $T_N - T_0$, this uncertainty in ΔT is at most $\pm$0.2 K.

Table 5. Relevant parameters where greenhouse forcings represent 0-50% of $T_N - T_0$

α	Forcing %	ΔT_B	μ_N	β	μ_S	ΔT_N	μ	ΔT
0.6 [30]	50%	33.0 K	1.1491	0.10	1.1521	0.67 K	1.1571	1.4 K
	25%	**16.5 K**	**1.0746**	**0.16**	**1.0772**	**0.58 K**	**1.0821**	**1.6 K**
	0%	0.0 K	1.0000	0.23	1.0023	0.50 K	1.0072	1.8 K
0.418 [28]	50%	22.1 K	1.0910	0.07	1.0940	0.72 K	1.0985	1.3 K
	25%	**11.1 K**	**1.0455**	**0.11**	**1.0483**	**0.66 K**	**1.0527**	**1.4 K**
	0%	0.0 K	1.0000	0.15	1.0025	0.61 K	1.0070	1.5 K
0.293 [24]	50%	16.1 K	1.0630	0.05	1.0659	0.76 K	1.0702	1.2 K
	25%	**8.0 K**	**1.0315**	**0.08**	**1.0343**	**0.71 K**	**1.0386**	**1.3 K**
	0%	0.0 K	1.0000	0.11	1.0026	0.67 K	1.0069	1.4 K

Using the method of comparison in Table 3, assuming albedo $\alpha \in \{0.6, 0.418, 0.293\}$ and taking the corrected values of $\mu_S, \Delta T_A$ for each of ten periods as found in (38-39), the ratio of expected to observed warming is found to be $\{1.3, 1.1, 1.1\}$, against 2.6 in the current method.

$$\mu_S \coloneqq 1 + \frac{\Delta T_B + \Delta T_N}{T_0}. \tag{38}$$

$$\Delta T_A = T_0 \frac{\mu_S}{1 - \mu_S \beta} - T_N. \tag{39}$$

Uncertainties also arise from nonlinearities in individual feedbacks and hence in their sum. Since the interval of sensitivities in all of the corrected methods falls on the near-linear region of the hyperbolic response curve close to its origin, nonlinearities will be allowed for in (1) by assigning higher values to f_{mid}. Thus, if it were imagined, *per impossibile,* that after a CO_2 doubling compared with today f_{mid} would be as much as thrice the observationally-derived industrial-era $\mathbf{0.12}$ even though the theoretically-confirmed pre-industrial value is similar, Charney sensitivity ΔT would show very little change compared with the observationally-derived value $\mathbf{1.25\ K}$ (Table 6).

Table 6. Charney sensitivities ΔT at various values of f_{mid}

f_{mid}	ΔT	f_{mid}	ΔT
$0.00 - 0.04$	1.1 K	$0.25 - 0.29$	1.5 K
$\mathbf{0.05 - 0.12}$	**1.2 K**	$0.30 - 0.33$	1.6 K
$0.13 - 0.18$	1.3 K	$0.34 - 0.37$	1.7 K
$0.19 - 0.24$	1.4 K	$0.38 - 0.40$	1.8 K

7. The corrected interval of Charney sensitivities

In addition to the ± 0.2 K uncertainty in albedo and the ± 0.2 K uncertainty attributable to feedbacks (Table 5), in [5, p 676, §8.3.2.1, cf. 37] an uncertainty of $\pm 10\%$ (~ 1 K) in reference sensitivity ΔT_S is given. Taking the three uncertainties as simply additive, theoretical ΔT is $\mathbf{1.4} \pm 0.5$ K. By the observational method, where the bounds of f are $f_{\mathrm{mid}} \pm 40\%$ [35], ΔT is $\mathbf{1.25} \pm 0.05$ K, or $\mathbf{1.25} \pm 0.15$ K after allowing for the uncertainty in reference sensitivity.

These results from three distinct methods, which stand in contrast to the CMIP5 models' $\mathbf{3.3} \pm 1.2$ K, say nothing of the value of any individual feedback. Since the net influence of the sum λ of all feedbacks on final sensitivity has been demonstrated to be small, in practice either individual feedbacks must be small or negative feedbacks must substantially offset any strongly positive feedbacks, partly owing to the inherent stability of today's climate that arises from the near-invariance of the solar "constant" and the vast heat capacity of the global ocean and partly owing to the influence of such negative feedbacks as the lapse-rate feedback and the earlier onset of tropical afternoon convection and cloud formation in response to warming.

The feedbacks relevant to the determination of equilibrium sensitivity, listed in Table 1, are as applicable in the absence as in the presence of non-condensing greenhouse gases. There are some feedbacks relevant only in the presence of non-condensing gases: the CO_2-outgassing feedback is one instance. However, these feedbacks are by convention excluded from the calculation, perhaps because in the current understanding they would drive the feedback fraction f above unity, whereupon in (1) global cooling would be expected, indicating either that $f \ll 1$, as the observational and theoretical methods cohere in finding, or that feedback amplification is not an appropriate model for determining climate sensitivity. In either case, no justification for Charney sensitivities $\Delta T \gg 1.5$ K subsists.

On a snowball Earth, with no ice-free zone at the equator, the feedbacks in Table 1 would not operate. However, following a major volcanic or asteroidal event, conditions permitting these feedbacks to operate could arise, whereupon the feedbacks would be induced not only by the warming following the trigger event but by the entire emission temperature T_0, and surface temperature would rise to > 260 K even in the absence of feedbacks from the non-condensing greenhouse gases, whereupon the equatorial zone would be ice-free. Greenhouse-gas forcings and their consequential feedbacks would be sufficient to keep the tropical zone ice-free even during the era of the early, faint Sun. Owing to the operation of feedbacks even in the absence of the non-condensing greenhouse gases, threshold events triggered by control variables are hysteretic (i.e., persisting even after the control variable has ceased to act), accounting for the existence of the multiple stable climate states evident in the geological record.

Not only the feedback sum λ but also its constituent individual feedbacks have hitherto been exaggerated owing to the mistaken belief that the feedback-driven response $\Delta T_{(0)}$ to T_0 is part of the feedback-driven response $\Delta T_{(b)}$ to ΔT_B. Since the corrected natural greenhouse effect ΔT_G is a small fraction of absolute global temperature, the net feedback sum λ must be small, as (40) shows.

$$\lambda = \beta / \lambda_0 \in \{0.5, \mathbf{0.35}, 0.25\} \text{ W m}^{-2} \text{ K}^{-1}. \qquad (40)$$

Therefore, the CMIP5 models that have been shown to reflect closely the equilibrium sensitivities yielded by (1) are as likely as that equation to over-predict global warming to a much greater extent than suggested, for instance, in [51].

If due allowance were made for Hölder's inequality between integrals in determining T_0, its value would be somewhat less than in (5), whereupon β would be somewhat greater than in

(21) but λ_0 would be somewhat less than in (7, 40). On the other hand, if some fraction of the warming since 1950 were natural, β would be somewhat less than in (21). A multiplicity of such considerations renders the precise determination of equilibrium sensitivities an inexact science. Nevertheless, the present results show that, if climatology has made no error but that which has been identified here, it is unlikely that Charney sensitivity ΔT_{mid} will much exceed one-third of the $\mathbf{3.3}$ **K** current mid-range estimate.

8. Empirical verification in the laboratory

Feedback theory is in principle as applicable to climate as to electronic network analysis. Therefore, tests on an electronic circuit designed to represent features of the climate can indicate the extent to which climate feedback methodology conforms to theory.

In the climate, individual temperature feedbacks can neither be measured nor quantitatively distinguished by observation either from each other or from the forcings that induced them. However, since inputs and outputs in an electronic circuit can be directly measured, one of the authors (Whitfield) constructed a test rig to simulate the climate feedback loop electronically. The results were in all material respects consistent with the theory set out herein.

Based on the results from the test rig, a specification for a more sensitive circuit was drawn up and a government laboratory was commissioned to construct and test it. The input signal ΔT_S or T_0 shown as E_0 in [7], the simple-gain factor μ and the feedback fraction β could be varied, whereupon the resulting signal ΔT or T shown as E_R in [7], could be measured directly. T_S was usually used in place of T_0 to obtain the required precision.

The laboratory was given 23 sets of three numbers, in four test groups, and was asked to set the circuit using each triplet of values, and to measure the output in a temperature-controlled chamber. The results of all 23 tests conformed sufficiently closely to expectation to indicate that the understanding of feedback theory presented herein appears to be in substance correct.

The group results showed first that, for f on $f_{\text{mid}} \pm 40\%$ [35] and $\Delta T_S = 1.16$ K [25], and before correcting feedback-theory errors, ΔT falls on $\mathbf{2.3}$ [1.6, 3.6] K; secondly, that, where absolute signals T_S, T are used rather than ΔT_S, ΔT, the Charney sensitivity interval narrows to <1 K, and the upper bound again falls; thirdly, and crucially, that, even where $\mu \coloneqq 1$ (i.e., the input signal is unamplified), the output signal exceeds it by the expected margin in the presence of positive feedback, and, where $\mu > 1$, the output signal does not greatly exceed the value where $\mu \coloneqq 1$; and fourthly, that the magnitude and interval breadth of output responses to feedback fractions β are small.

The laboratory was sent an early draft of the present work and kindly confirmed that the text fairly represents its report (Appendix A). Appendix C provides more details of the tests.

9. A note on confidence intervals and precision

Though official estimates of climate sensitivity are often accompanied by confidence intervals (typically asserted to be 1 or 2 σ), the conditions precedent to the reliable estimation of such confidence intervals in climate-sensitivity studies are in truth absent. For instance, [1-5] assigned successively greater certainties to the proposition that recent global warming was chiefly anthropogenic: however, decisions on what confidence interval to assign to this proposition are made not by a recognized statistical technique but by a show of hands among government representatives. For this reason, no attempt has been made to assign confidence intervals to the climate sensitivity estimates reached herein. Likewise, some caution should be exercised in relying upon the precision with which various quantities are expressed.

10. Conclusion

It has been demonstrated that, insofar as the sensitivities predicted by the CMIP3/5 models reflect the current zero-dimensional model equation (1) as the calibration exercise indicates they do, they significantly exaggerate global warming, chiefly because emission temperature T_0

is not used as the input in (1) and, therefore, the substantial feedback $\Delta T_{(0)}$ induced by T_0 has been erroneously regarded as part of the feedback-driven contribution induced by the direct warming ΔT_B from the pre-industrial non-condensing greenhouse gases.

In the light of this result it is advisable greatly to reduce what [52] calls the "anticipated acceptable range" of equilibrium sensitivities that models have hitherto been tuned to deliver.

The present results, if found correct, make some contribution to the solution of four long-standing problems in climatology: constraint of the amplitudes of equilibrium sensitivities, including Charney sensitivity; constraint of the interval breadth of equilibrium sensitivities; the snowball-Earth deglaciation problem; and the faint-young-Sun paradox.

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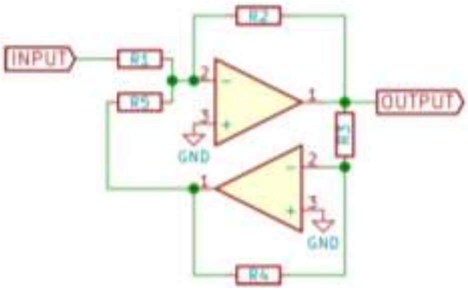
Appendix A

An investigation of feedback amplification

Report by a government laboratory

A.1 Circuit concept: To verify aspects of feedback amplifier theory as requested, a prototype circuit (Fig. A.1) was devised. It was expected to behave according to (A1.1).

$$E_R = E_0 \frac{\mu}{1-\mu\beta}. \tag{A.1}$$



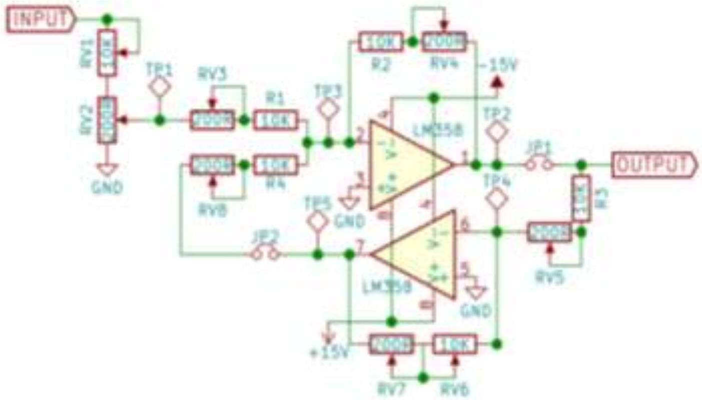


Fig. A.1 Above: circuit concept.
Right: Detailed circuit schematic.

In the circuit concept (Fig. A.1 left), the upper op-amp is configured as an inverting summing amplifier. Its output is then, in turn, fed back into the summing node of the first amplifier, establishing the feedback path. When this circuit is analyzed, it yields equations for μ and β:

$$\mu = \frac{R_2}{R_1};\ \beta = \frac{R_1}{R_5}\frac{R_4}{R_3} \tag{A.2}$$

From (A1.2), μ can be changed independently of β by adjusting R_2, and β can be changed by adjusting R_3, R_4 or R_5. In practice it is easiest to adjust R_2 and R_4 to vary μ and β respectively.

A.2 Assembled circuit: Fig. A.2 shows photographs of both sides of the assembled test circuit.

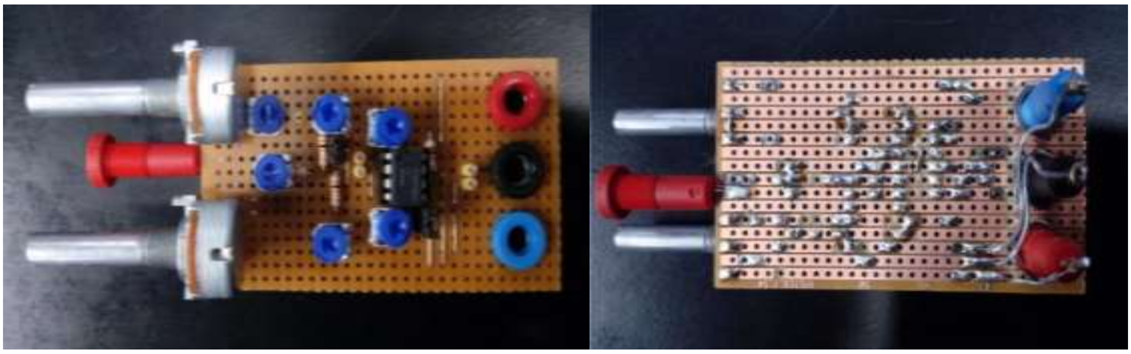


Fig. A.2 The test circuit

A.3 Measurement method: Before starting any measurements, the resistances of R_1, R_3 and R_5 were adjusted to be as close as possible to 10 KΩ. In the physical circuit, each of these resistances comprises a 10 KΩ resistor and a 200 Ω trimmer potentiometer in series. The multimeter was placed across each series pair and the trimmer turned to achieve as close as possible to 10 KΩ. All three resistances were measured as 10.00 KΩ.

In this circuit, the polarity of the output is always inverted: however, since this inversion has no effect on the absolute value of the output, for clarity it has been ignored in all tables.

A.4 Setting the direct gain factor μ and feedback fraction β: In all tests, μ was set by applying 1 V at TP1 and adjusting R_2 until the voltage at TP2 was as close as possible to either 1 or 1.004 V as required. The two jumpers on the circuit-board were removed to isolate the μ and β blocks, allowing the gain of each to be set and measured independently, and were replaced after μ was set.

To set β, R_4 was adjusted to the values in Table A.1. Gain on the return path was R_4/R_3, for $R_3 = 10$ K Ω. All settings were confirmed by measurement.

Table A.1

Test 1		Tests 2-3		Test 4	
Expected	**Measured**	Expected	**Measured**	Expected	**Measured**
0 Ω	**0 Ω**	0 Ω	**0 Ω**	0 Ω	**0 Ω**
2910 Ω	**2910 Ω**	20 Ω	**20 Ω**	1150 Ω	**1150 Ω**
4850 Ω	**4850 Ω**	30 Ω	**30 Ω**		
6790 Ω	**6790 Ω**	40 Ω	**40 Ω**		
		50 Ω	**50 Ω**		

Setting β to required values and confirming by measurement

Table A.2

Test No.		E_0		μ		β		E_R	
		Specified	**Measured**	Specified	**Measured**	Specified	**Measured**	Expected	**Measured**
1	1	1.159	**1.159**	1.000	**1.000**	0.000	**0.000**	1.159	**1.1590**
	2		**1.159**		**1.000**	0.291	**0.291**	1.63	**1.633**
	3		**1.159**		**1.000**	0.485	**0.485**	2.25	**2.249**
	4		**1.159**		**1.000**	0.679	**0.679**	3.61	**3.606**
2	1	288.4	**2.884**	1.004	**1.004**	0.000	**0.000**	289.6	**2.896 †**
	2		**2.884**		**1.004**	0.002	**0.002**	290.1	**2.901 †**
	3		**2.884**		**1.004**	0.003	**0.003**	290.4	**2.904 †**
	4		**2.884**		**1.004**	0.004	**0.004**	290.7	**2.907 †**
	5		**2.884**		**1.004**	0.005	**0.005**	291.0	**2.910 †**
3	1	288.4	**2.884**	1.000	**1.000**	0.000	**0.000**	288.4	**2.884 †**
	2	287.8	**2.878**		**1.000**	0.002	**0.002**	288.4	**2.884 †**
	3	287.5	**2.875**		**1.000**	0.003	**0.003**	288.4	**2.884 †**
	4	287.2	**2.872**		**1.000**	0.004	**0.004**	288.4	**2.884 †**
	5	287.0	**2.870**		**1.000**	0.005	**0.005**	288.4	**2.884 †**
	6	288.4	**2.884**	1.004	**1.004**	0.000	**0.000**	289.6	**2.896 †**
	7	287.8	**2.878**		**1.004**	0.002	**0.002**	289.5	**2.895 †**
	8	287.5	**2.875**		**1.004**	0.003	**0.003**	289.5	**2.895 †**
	9	287.2	**2.872**		**1.004**	0.004	**0.004**	289.5	**2.895 †**
	A	287.0	**2.870**		**1.004**	0.005	**0.005**	289.6	**2.896 †**
4	1	288.4	**2.884**	1.000	**1.000**	0.000	**0.000**	288.4	**2.884 †**
	2		**2.884**	1.004	**1.004**	0.000	**0.000**	289.6	**2.896 †**
	3	255.4	**2.554**	1.000	**1.000**	0.115	**0.115**	288.6	**2.886 †**
	4		**2.554**	1.004	**1.004**	0.115	**0.115**	289.9	**2.899 †**

Table of results. † For tests 2-4, voltages were divided by 100 to reach the specified precision.

A.5 Results and conclusion: Table A.2 gives the results of each test. In tests 2-4, voltages marked † were divided by 100 to approach the required precision. In the first section of test 2, the feedback fraction β was set as close as possible to zero by setting the resistance R_4 to the least possible value. In test 3, values $\mu > 1$ were obtained by setting R_2 to 10 times the required value and then dividing by R_1 ($= 10{,}000\ \Omega$). The last three runs of test 3 were repeated with μ set to the specified value.

Several amendments were made to the measurements in the original draft using a higher-precision multimeter capable of measuring up to 8 decimal places. As anticipated, the results obtained are now – to 2 or 3 decimal places – identical to the expected results based on amplifier feedback theory.

A.6 Bill of materials: Table A.3 lists the components used in assembling the experimental circuit.

Table A.3 Bill of materials and list of equipment used

Part	Qty.	RS Components stock no.	Equipment Used
LM 358 op amp	1	7615860	**Farnell TOPS2** ± 15 V stabilized power supply for op-amps
8-pin IC socket	1	6742435	**Iso-Tech IPS3303** DC power supply for test input voltages
10 KΩ resistor	4	7077745	**Datron Wavetek 1281** selfcal digital multimeter (calibrated)
10 KΩ potentiometer	2	2499238	To check the power supplies' output voltage
200 Ω potentiometer	6	4885287	To measure all voltages and resistances in the circuit
Header pins	2	2518086	To measure final output voltage
Test points	5	2622034	[Voltage measurements were to a resolution of 1 mV]
Red banana socket	3	4333326	To measure resistances: $< 600\ \Omega$: max. resolution 0.1 Ω
Blue banana socket	1	4333348	$< 6000\ \Omega$: max. resolution 1.0 Ω
Black banana socket	1	4333332	$< 60{,}000\ \Omega$: max. resolution 10 Ω
Veroboard	1	2065841	**Other equipment:** Test leads; screwdriver; jumpers

Appendix B

Response curve of the output signal in a feedback amplifier

Results from a test circuit constructed by a co-author

One of the authors (Whitfield) constructed a test circuit and obtained substantially the same results as those of the government laboratory given in Table A.2. The circuit was additionally used to confirm that the response curve of output in the presence of feedback is a hyperbola (Fig. B.1).

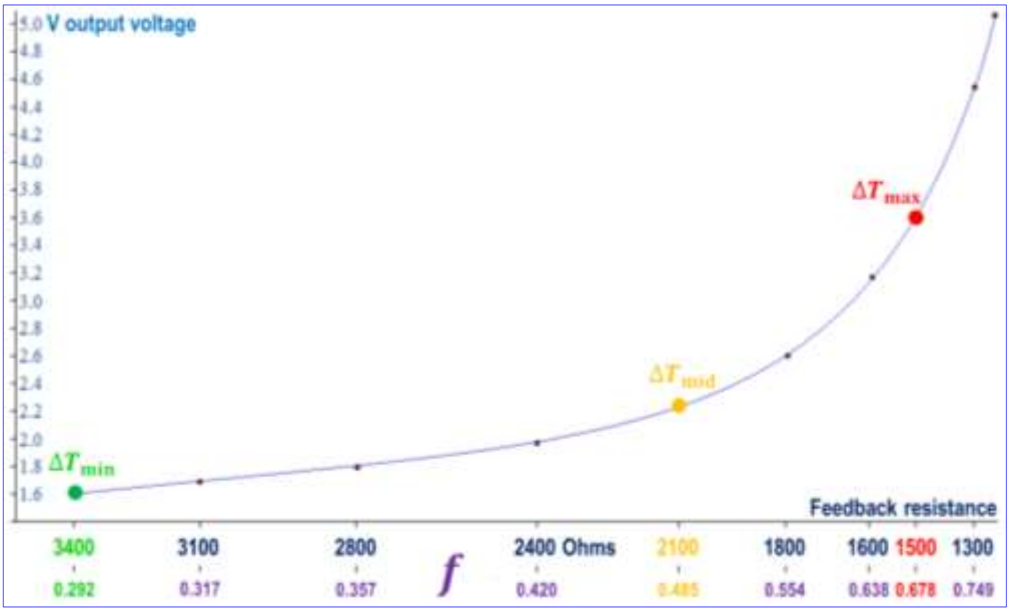


Fig. B.1 Response curve of the output signal in the presence of feedback factors f on $[0.28, 0.77]$

Appendix C

Empirical verification of the revised zero-dimensional model by testing at a government laboratory

Test group 1: Table C.1.

Result: For f on $f_{\text{mid}} \pm 40\%$ [35] and $\Delta T_S = 1.16$ K as in [25], and before correcting errors of feedback method, ΔT falls not on the $\mathbf{3.0} \pm 1.5$ K in [5, 33] but on $\mathbf{2.3}$ $[1.6, 3.6]$ K.

Here, $f_{\min}$ was taken as $1 - \Delta T_S/\Delta T_{\min}$ $(= 1 - 1.16/1.5 = 0.23)$; $f_{\max}$ was taken as $1 - 1.16/4.5$ $(= 0.74)$; f_{mid} $(= 0.49)$ as the mean of $f_{\min}$, $f_{\max}$; and, following [35], $\beta_{\min}$, $\beta_{\max}$ as $f_{\text{mid}} \pm 40\%$, i.e. 0.29 and 0.68 respectively. Then, reproducing the error of taking ΔT_S rather than T_S as equivalent to the input signal E_0 in [7, p. vii and ch. 3] and taking $\mu \coloneqq 1$, the interval of defective output signals ΔT (Bode's E_R) was measured as $\mathbf{2.3}$ $[1.6, 3.6]$ K.

Table C.1		E_0 Specified	E_0 **Measured**	μ Specified	μ **Measured**	β Specified	β **Measured**	E_R Expected	E_R **Measured**
1	1	1.159	**1.159**	1.000	**1.000**	0.000	**0.000**	1.16	**1.159**
	2		**1.159**		**1.000**	0.291	**0.291**	1.63	**1.633**
	3		**1.159**		**1.000**	0.485	**0.485**	2.25	**2.249**
	4		**1.159**		**1.000**	0.679	**0.679**	3.61	**3.606**

Test group 2: Table C.2.

Result: Where the input and output signals T_S, T are used rather than $\Delta T_S, \Delta T$, the Charney sensitivity interval narrows to < 1 K and the upper bound is further reduced.

In this test group, the full surface temperature T_S was taken as the input signal E_0, while μ in the gain block was set to $1 + \Delta T_S/T_S = 1 + 1.16/288.4$, or 1.004, T_S rather than T_E being taken as the input because μ could then be set as 1.004 exactly, since precisions beyond 3 d.p. were unattainable with the available equipment. To make ΔT_{mid} identical in both methods, β_{mid}, in (C1), was derived from f_{mid} (= 0.49 from test group 1) via (C2), in which $T_{\text{mid}} = T_S + \Delta T_{\text{mid}}$, i.e. 288.4 + 2.25 K (the 2.25 K comes from Table C.1, test 3).

$$\mu \beta_{\text{mid}} T_{\text{mid}} = f_{\text{mid}} \Delta T_{\text{mid}} \tag{C.1}$$

$$\Rightarrow \beta_{\text{mid}} = f_{\text{mid}} \frac{\Delta T_{\text{mid}}}{\mu T_{\text{mid}}} = 0.49 \left(\frac{2.25}{1.004 \times 290.7} \right) = 0.0037. \tag{C.2}$$

Since the 2 σ bounds of c diagnosed from the CMIP5 ensemble are $c_{\text{mid}} \pm 40\%$ [35, table 3], the 2 σ bounds of β are likewise $\beta_{\text{mid}} \pm 40\%$, for the feedback sum λ_{mid} is a factor common to

f_{mid} and β_{mid}. Thus, $\beta_{\min} = 0.002,\ \beta_{\max} = 0.005$, whereupon, as (C.3-C.4) show, ΔT falls on $\mathbf{2.3}$ [1.8, 2.7] K before correcting for the action of today's feedbacks on ΔT_S.

$$\Delta T_{\max} = T_S\left(\frac{\mu}{1-\mu\beta_{\max}} - 1\right) = 288.4\left(\frac{1.004}{1-1.004 \times 0.005} - 1\right) = 2.7 \text{ K; and} \tag{C.3}$$

$$\Delta T_{\min} = T_S\left(\frac{\mu}{1-\mu\beta_{\min}} - 1\right) = 288.4\left(\frac{1.004}{1-1.004 \times 0.002} - 1\right) = 1.8 \text{ K.} \tag{C.4}$$

Table C.2		E_0 Specified	E_0 **Measured**	μ Specified	μ **Measured**	β Specified	β **Measured**	E_R Expected	E_R **Measured**
2	1	288.4	**2.884**	1.004	**1.004**	0.000	**0.000**	289.6	**2.896** †
	2		**2.884**		**1.004**	0.002	**0.002**	290.1	**2.901** †
	3		**2.884**		**1.004**	0.003	**0.003**	290.4	**2.904** †
	4		**2.884**		**1.004**	0.004	**0.004**	290.7	**2.907** †
	5		**2.884**		**1.004**	0.005	**0.005**	291.0	**2.910** †

† For test groups 2-4, voltages were divided by 100 to reach the specified precision.

Values of β were specified to the nearest 0.001, since that was the maximum available precision of the laboratory equipment. The interval of Charney sensitivities, even before allowance is made for the action of today's feedbacks on T_S itself, is thus small.

Test group 3: Table C.3

Result: Even where $\mu \coloneqq 1$ (i.e., the input signal E_0 is unamplified), the output signal E_R exceeds E_0 by the expected margin after positive feedback; and, where $\mu > 1$, the output signal scarcely exceeds the value that obtains where $\mu \coloneqq 1$, confirming what climatology's current method inadvertently conceals: that the input signal induces feedback even in the absence of a μ amplification such as that from an anthropogenic forcing.

Sub-tests 1-5 of Test 3 progressively vary the non-feedback-driven fraction of T_S, whereupon, even in the absence of any direct-gain factor μ, the non-zero values of the feedback fraction β, after correctly accounting for the contribution of existing feedbacks to T_S, will restore the output signal E_R so that it is equal to T_S.

Sub-tests 6-10 show that, where $\mu > 1$ to account for a signal gain ΔT_S (in the climate, ΔT_S is the temperature response to a forcing such as that from doubled CO_2), the additional net contribution from temperature feedbacks to the output signal E_R (the label in [7] for the output signal T) in response to a small μ amplification of T_S is small, so that T barely exceeds T_S.

Table C.3		E_0		μ		β		E_R	
		Specified	**Measured**	Specified	**Measured**	Specified	**Measured**	Expected	**Measured**
3	1	288.4	**2.884**	1.000	**1.000**	0.000	**0.000**	288.4	**2.884 †**
	2	287.8	**2.878**		**1.000**	0.002	**0.002**	288.4	**2.884 †**
	3	287.5	**2.875**		**1.000**	0.003	**0.003**	288.4	**2.884 †**
	4	287.2	**2.872**		**1.000**	0.004	**0.004**	288.4	**2.884 †**
	5	287.0	**2.870**		**1.000**	0.005	**0.005**	288.4	**2.884 †**
	6	288.4	**2.884**	1.004	**1.004**	0.000	**0.000**	289.6	**2.896 †**
	7	287.8	**2.878**		**1.004**	0.002	**0.002**	289.5	**2.895 †**
	8	287.5	**2.875**		**1.004**	0.003	**0.003**	289.5	**2.895 †**
	9	287.2	**2.872**		**1.004**	0.004	**0.004**	289.5	**2.895 †**
	A	287.0	**2.870**		**1.004**	0.005	**0.005**	289.6	**2.896 †**

† For test groups 2-4, voltages were divided by 100 to reach the specified precision.

Test group 4: Table C.4.

Result: The magnitude and interval breadth of output responses to feedback fractions β are small when β is 0.12 (0.115 to facilitate precision).

Sub-tests 1-2 take $\beta = 0$, showing that, where $\mu > 0$ to account for ΔT_S, the output signal (Charney sensitivity in the climate) is equal to ΔT_S.

Sub-tests 3-4 show that, where $\beta_{\max} = 0.115$, the output signal is only 0.3 K greater than ΔT_S, demonstrating that, since $\beta_{\max}$ is only a small fraction, and since ΔT_S is a very small fraction of T_S, the enhancement $\beta \Delta T_S$ of the feedback contribution to the output is a very small fraction of a small fraction, so that feedbacks' contribution to final sensitivity must be small.

Table C.4		E_0		μ		β		E_R	
		Specified	**Measured**	Specified	**Measured**	Specified	**Measured**	Expected	**Measured**
4	1	288.4	**2.884**	1.000	**1.000**	0.000	**0.000**	288.4	**2.884 †**
	2		**2.884**	1.004	**1.004**	0.000	**0.000**	289.6	**2.896 †**
	3	255.4	**2.554**	1.000	**1.000**	0.115	**0.115**	288.6	**2.886 †**
	4		**2.554**	1.004	**1.004**	0.115	**0.115**	289.9	**2.899 †**

† For test groups 2-4, voltages were divided by 100 to reach the specified precision.

Even with the most sensitive equipment in a temperature-controlled laboratory, the mere presence of the operator was found to interfere with the very small signals that were obtained, inadvertently exemplifying the exiguity of feedbacks' contribution to ΔT.

Appendix D

Nonlinearities in individual feedbacks and in their sum

Where feedbacks are nonlinear, (1) is replaced by (D.1), derived in [14].

$$\Delta T = \lambda_0 \Delta Q_0 \left\{1 - f - \frac{\Delta T}{2}\left(\frac{df}{dT} + \frac{1-f}{\lambda_0}\frac{d\lambda_0}{dT}\right)\right\}^{-1}. \tag{D.1}$$

However, the fact that the linear zero-dimensional-model equation (1) faithfully reproduces the CMIP5 ensemble's interval of Charney sensitivities suggests at first blush that the nonlinearities in feedbacks that the models take into account have little bearing on equilibrium sensitivity. However, the amplitudes of the officially-estimated temperature feedbacks listed in Table 1 already reflect nonlinearities in the individual feedbacks.

For instance, owing to the Clausius-Clapeyron relation the atmospheric space can carry near-exponentially more water vapor as it warms, though this nonlinearity is to some extent offset both by a quasi-logarithmic temperature response to the feedback forcing and by the somewhat negative lapse-rate feedback, as well as by as-yet-unquantified negative feedbacks such as the earlier onset of tropical afternoon convection with warming.

Another countervailing influence is the Planck sensitivity parameter λ_0, on which equilibrium sensitivity ΔT has a squared dependency: λ_0 diminishes with albedo in response to warming. Also, the formidable heat capacity of the global ocean is a strongly thermostatic influence, as cryostratigraphy demonstrates [39]. Notwithstanding such thermostatic influences, the question arises whether the approach taken in [28] and similarly in the worked example, where the feedback fraction is taken as invariant at all stages, is appropriate.

The $\sim$34 K difference between the snowball-Earth emission temperature T_0 ($= 221.5$ K) and today's emission temperature T_E ($= 255.4$ K), an indication of the contribution of the albedo feedback to today's temperature T_S, represents more than two-thirds of the $\sim$50 K feedback-driven contribution $\Delta T_{(0)} + \Delta T_{(b)} + \Delta T_{(n)}$ to T_S. Today, however, as Table 1 shows, the albedo feedback represents little more than one-sixth of the feedback sum, since the vast land-based ice-sheets of the continental land masses of the Northern hemisphere have long since vanished, so that little extra-polar ice remains. Accordingly, the albedo feedback is today considerably below its value in the absence of the non-condensing greenhouse gases, suggesting the possibility that, notwithstanding the 7% K^{-1} Clausius-Clapeyron increase in column water vapor with temperature [50], the feedback fraction is, as [28] finds it to be, no greater today than formerly and may be less.

The Earth's top-of-atmosphere radiation budget R at equilibrium is given in (D.2).

$$R = Q_0 - L_0 = 0, \tag{D.2}$$

where L_0 is outgoing long-wave radiation. Where a forcing ΔQ_0 perturbs the equilibrium, the climate responds by changing surface temperature T_S. The equilibrium warming ΔT compared with the unperturbed value of today's surface temperature T_S is related to the forcing ΔQ_0 and to the radiative imbalance ΔR via the energy-balance equation (D.3),

$$(R - \Delta Q_0)[T_S + \Delta T] = (R - \Delta Q_0)[T_S] + \frac{\partial(R-\Delta Q_0)}{\partial T}\Delta T, \tag{D.3}$$

which is a Taylor-series expansion of $(R - \Delta Q_0)$ with higher-order terms omitted. Applying the substitutions in (D.4-D.5), (D>3) becomes (D.6), for which (D.7) defines the feedback parameter λ' [32, Appendix A].

$$(R - \Delta Q_0)(T_S + \Delta T) - (R - \Delta Q_0)T_S = \Delta R - \Delta Q_0. \tag{D.4}$$

$$\frac{\partial(R-\Delta Q_0)}{\partial T} = \lambda'. \tag{D.5}$$

$$\Delta R - \Delta Q_0 = \lambda' \Delta T. \tag{D.6}$$

$$\lambda' \coloneqq \lambda - \lambda_0^{-1}. \tag{D.7}$$

The feedback sum λ, currently a substantial contributor to the feedback parameter λ', is thus a key influence on the energy-balance equation (D.6) and on climate sensitivity.

The simple equation (1) for the zero-dimensional model assumes that only surface temperature T_S responded to radiative forcings, while water vapor, clouds and albedo are held fixed. However, as [13] explains, where α_i represents the i^{th} climate field, a Taylor-series expansion (D.8) describes the dependence of the climate system's radiative adjustment on the variances of these fields with changing climate:

$$\Delta R_{\alpha} = \frac{\Delta R_{\alpha}}{\Delta T} + \mathrm{O}(\Delta T^2) = \left\{ \sum_{i=1}^{n} \left[\left. \frac{\partial R}{\partial \alpha_i} \right)_{\alpha_{j,j\neq 1}} \frac{\mathrm{d}\alpha_i}{\mathrm{d}T} \right] \right\} \Delta T + \mathrm{O}(\Delta T^2) \qquad \text{(D.8)}$$

Accordingly, (D.9) is equivalent to (9).

$$\Delta T = \lambda_0 (\Delta Q_0 + \Delta R_{\alpha}) = \lambda_0 \left\{ \Delta Q_0 + \sum_{i=1}^{n} \left[\left. \frac{\partial R}{\partial \alpha_i} \right)_{\alpha_{j,j\neq 1}} \frac{\mathrm{d}\alpha_i}{\mathrm{d}T} \right] \right\} \Delta T + \mathrm{O}(\Delta T^2) \qquad \text{(D.9)}$$

Gathering the instances of ΔT, (D.9) may be recast as (D.10), where (D.11) gives f^i.

$$\Delta T = \Delta Q_0 \lambda_0 \frac{1}{1 - \sum_i f_i} \qquad \text{(D.10)}$$

$$f_i = \lambda_0 \left\{ \left. \frac{\partial R}{\partial \alpha_i} \right)_{\alpha_{j,j\neq 1}} \frac{\mathrm{d}\alpha_i}{\mathrm{d}T} \right\}. \qquad \text{(D.11)}$$

However, since the values of the feedback fractions obtained for the pre-industrial era by two theoretical methods and of the industrial-era feedback fraction obtained by an empirical method cohere, little nonlinearity is evident. In any event, nonlinearity in the feedback sum λ would make little difference, since λ is small.

References

[1] IPCC, Climate change – The IPCC Assessment (1990): Report prepared for the Intergovernmental Panel on Climate change by Working Group, in: J.T. Houghton, G.J. Jenkins, J.J. Ephraums (Eds.), First Assessment Report, Cambridge University Press, Cambridge, 1990.

[2] IPCC, Climate Change 1995 – The science of climate change: contribution of WG1 to the second assessment report, in: J.T. Houghton, L.G. Meira Filho, B.A. Callander et al. (Eds.), Second Assessment Report, Cambridge University Press, Cambridge, 1995.

[3] IPCC, Climate Change 2001: The scientific basis. Contribution of Working Group I to the Third Assessment Report of the Intergovernmental Panel on Climate Change, in: J.T. Houghton, Y. Ding, D.J. Griggs et al. (Eds.), Third Assessment Report, Cambridge University Press, Cambridge, 2001.

[4] IPCC, Climate Change 2007: The physical science basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change, in: S. Solomon, D. Qin, M. Manning et al. (Eds.), Cambridge University Press, Cambridge, 2007.

[5] IPCC, Climate change 2013: The physical science basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, in: T.F. Stocker, D. Qin, G.-K. Plattner et al. (Eds.), Cambridge University Press, Cambridge, 2013.

[6] J.R. Bates, Estimating climate sensitivity using two-zone energy-balance models, Earth & Space Sci. 3 (2016) 207–225, doi:10.1002/2015EA000154.

[7] H.W. Bode, Network Analysis and Feedback Amplifier Design, Van Nostrand Reinhold, New York, 1945.

[8] J. Hansen, A. Lacis, D. Rind et al., Climate sensitivity: Analysis of feedback mechanisms, in: J. Hansen, T. Takahashi (Eds.), Climate Processes and Climate Sensitivity, AGU Geophysical Monograph 29, Maurice Ewing Vol. 5, Amer. Geophys. Union, Washington DC (1984) 130–163.

[9] M.E. Schlesinger, Feedback analysis of results from energy balance and radiative-convective models, in: M.C. MacCracken, F. M. Luther (Eds.), The potential climatic effects of increasing carbon dioxide, U.S. Dept. of Energy, Washington DC (1985) 280–319.

[10] J.P. Peixoto, J.P. and A.H. Oort, Physics of climate, Amer. Inst. Phys., New York, 1992.

[11] J.A. Curry, P.J. Webster, Thermodynamics of atmospheres and oceans, Acad. Press, Amsterdam, 1999.

[12] J.M. Wallace, P.V. Hobbs, Atmospheric science: an introductory survey, 2nd ed., Elsevier/Acad. Press, Amsterdam, 2006.

[13] G. Roe, Feedbacks, timescales, and seeing red, Ann. Rev. Earth Planet. Sci. 37 (2009) 93–115

[14] R.S. Lindzen, Y.-S. Choi, On the observational determination of climate sensitivity and its implications, Asia-Pacific J. Atmos. Sci. 47 (2011) 377–390.

[15] C.W. Monckton of Brenchley, W. W.-H. Soon, D.N. Legates and W.M. Briggs, Why models run hot: results from an irreducibly simple climate model, Sci. Bull. 60:1 (2015) 122–135, doi:10.1007/s11434-014-0699-2.

[16] C.W. Monckton of Brenchley, The temperature feedback problem, Energy & Envir. 26:5 (2015) 829-840.

[17] K.C. Armour, Energy budget constraints on climate sensitivity in light of inconstant climate feedbacks. Nature Clim Ch [in press], doi:10.1038/NCLIMATE3278.

[18] T. Friedrich, A. Timmermann, M. Tigchelaar et al, Nonlinear climate sensitivity and its implications for future greenhouse warming. Sci Adv 2(11) (2016) e1501923, doi:10.1126/sciadv.1501923.

[19] D.J.A. Johansson, B.C. O'Neill, C. Tebaldi & O. Häggström, Equilibrium climate sensitivity in light of observations over the warming hiatus. Nat. Clim. Change 5 (2015) 449–453.

[20] C.E. Forest, P.H. Stone and A.P. Sokolov, Estimated PDFs of climate system properties including natural and anthropogenic forcings. Geophys. Res. Lett. 33 (2006) L01705.

[21] N.G. Andronova and M.E. Schlesinger, Objective estimation of the probability density function for climate sensitivity. J. Geophys. Res. Atmos. 106 (2001) 22605–22611.

[22] D.M. Murphy, S. Solomon, R.W. Portmann, et al., An observationally based energy balance for the Earth since 1950, J. Geophys. Res. 114 (2009) D17107, doi:10.1029/2009D012105.

[23] S. Mekaoui, S. Dewitte, C. Conscience et al., Total solar irradiance absolute level from DIRAD/SOVIM on the International Space Station, Adv Space Res 45 (2010) 1393–1406.

[24] G.N. Loeb, B.A. Wielicki, D.R. Dölling et al., Toward optimal closure of the Earth's top-of-atmosphere radiation budget, J Clim 22 (2009) 748–766, doi:10.1175/2008JCLI2637.1.

[25] G. Myhre, E.J. Highwood, K.P. Shine and F. Stordal, New estimates of radiative forcing due to well-mixed greenhouse gases, Geophys. Res. Lett. 25 (1998) 2715–2718.

[26] T. Andrews, J.M. Gregory, M.J. Webb et al., Forcing, feedbacks and climate sensitivity in CMIP5 coupled atmosphere-ocean climate models, Geophys. Res. Lett. 39 (2012) L09712, doi:10.1029/2012GL051607.

[27] G. Rybicki, A.P. Lightman, Radiative Processes in Astrophysics, Wiley-Interscience, New York, 1979.

[28] A.A. Lacis, G.A. Schmidt, D. Rind and R.A. Ruedy, Atmospheric CO_2: principal control knob governing Earth's temperature, Science 330 (2009), 356–359.

[29] A.A.Lacis, J.E. Hansen, G.L. Russell, V. Oinas, and J. Jonas, The role of long-lived greenhouse gases as principal LW control knob that governs the global surface temperature for past and future climate change. Tellus B 65 (2013), 19734, doi:10.3402/tellusb.v65i0.19734.

[30] R.T. Pierrehumbert, D.S. Abbot, A. Voigt, D.Koll, Climate of the Neoproterozoic. Ann. Rev. Earth Planet. Sci. 39 (2011), 417–460, doi:10.1146/annurev-ear-040809-152447.

[dataset] [31] C.P. Morice, J.J. Kennedy, N. Rayner, P.D. Jones, Quantifying uncertainties in global and regional temperature change using an ensemble of observational estimates: HadCRUT4 dataset, J. Geophys. Res. 117 (2012) D08101, http://www.metoffice.gov.uk/hadobs/hadcrut4/data/current/time_series/HadCRUT.4.5.0.0.monthly_ns_avg.txt.

[32] S. Bony, R. Colman, V. Kattsov et al., How well do we understand and evaluate climate change feedback processes? J. Clim. 19 (2006) 3445–3482, doi:10.1175/JCLI3819.1.

[dataset] [33] ISCCP (2016) Cloud analysis (part I): Climatology of global cloud and surface properties, http://isccp.giss.nasa.gov/climanal1.html, accessed 16 September 2016.

[34] J.G. Charney, A. Arakawa, D.J. Baker et al., Carbon dioxide and climate: a scientific Assessment. Report of an ad-hoc study group on carbon dioxide and climate, Climate Research Board, Assembly of Math. Phys. Sci., Nat. Research Coun., Woods Hole, 1979.

[35] J. Vial, J.-L. Dufresne, S. Bony, On the interpretation of inter-model spread in CMIP5 clmate sensitivity estimates, Clim. Dyn. 41 (2013) 3339–3362, doi:10.1007/s00382-013-1725-9.

[36] J.M. Gregory, W.J. Ingram, M.A. Palmer et al., A new method for diagnosing radiative forcing and climate sensitivity, Geophys. Res. Lett. 31 (2004) L03205, doi:10.1029/2003GL018747.

[37] R.D. Cess, M.-H. Zhang, G.L. Potter et al., Uncertainties in carbon dioxide radiative forcing in atmospheric general-circulation models. Science 262(5137) (1993) 1252–1255.

[38] H. Harde, Radiative transfer calculations and assessment of global warming by CO_2, Int. J. Atmos. Sci (2017), doi:10.1155/2017/9251034.

[39] J. Jouzel, V. Masson-Delmotte, O. Cattani et al., Orbital and millennial Antarctic climate variability over the past 800,000 years, Science 317 (2007) 793–796.

[40] R.L. Miller, G.A. Schmidt, L.S. Nazarenko et al., CMIP5 historical simulations (1850-2012) with Giss ModelE2. J. Adv. Model. Earth Syst. 6:2 (2014), 441–477, doi:10.1002/2013MS000265.

[41] G. Myhre, C.L. Myhre, P.M. Forster and K.P. Shine, Halfway to doubling of CO_2 radiative forcing. Nature Geoscience (2017) advance online publication, September 18.

[42] R. Knutti, T.F. Stocker, F. Joos et al., Constraints on radiative forcing and future climate change from observations and climate model ensembles. Nature 416 (2002), 719–723, doi:10.1038/416979a. [1750–2001: 1.9 1.9 W m^{-2}, 0.60 K obs.]

[43] J. Haywood, M. Schulz, Causes of the reduction in uncertainty in the anthropogenic radiative forcing of climate between IPCC (2001) and IPCC (2007). Geophys. Res. Lett. 34:10 (2007), L20701, 10.1029/2007GL030749. [1750–2006: 1.93 W m^{-2} (Fig. 1), 0.68 K obs.]

[44] R.B. Skeie, T.K. Berntsen, G. Myhre et al., Anthropogenic radiative forcing time series from pre-industrial times until 2010. Atmos. Chem. Phys. 11 (2011) 11827–11857, doi:10.5194/acp-11-11827-2011. [1750-2010: 1.4 W m^{-2}, 0.74 K obs.]

[45] O. Boucher, J. Haywood, On summing the components of radiative forcing of climate change. Clim. Dyn. 18 (2001), 293–302, doi:10.1007/s003820100185.

[46] D.J. Frame, D.A. Stone, Assessment of the first consensus prediction on climate change. Nature Clim. Change 3 (2013), 357–359, doi:10.1038/nclimate1763.

[47] M.R. Grose, J.S. Risbey, P.H. Whetton, Tracking regional temperature projections from the early 1990s in light of variations in regional warming, including 'warming holes', Climatic Change 140 (2017), 307–322, doi:10.1007/s10584-016-1840-9.

[48] S. Rahmstorf, G. Foster, A. Cazenave, Comparing climate projections to observations up to 2011, Environ. Res. Lett. 7 (2012), doi:10.1088/1748-9326/7/4/044035.

[49] S. Arrhenius, On the influence of carbonic acid in the air upon the temperature of the ground, Philos. Mag. & J. Sci. 5(41) (1896) 237–276

[50] F.J. Wentz, L. Ricciardulli, K. Hilburn, C. Mears, How much more rain will global warming bring? Science 317 (2007), 233–235, doi:10.1126/science.1140746.

[51] R.J. Millar, J.S. Fuglestvedt, P. Friedlingstein et al., Emission budgets and pathways consistent with limiting warming to 1.5 C°, Nature Geosci. 10 (2017), 741–747, doi:10.1038/ngeo3031.

[52] F. Hourdin, T. Mauritsen, A. Gettelman et al., The art and science of climate model tuning. Bull. Amer. Met Soc. 98:3 (2017) 589–603, doi:10.1175/BAMS-D-15-00135.1.